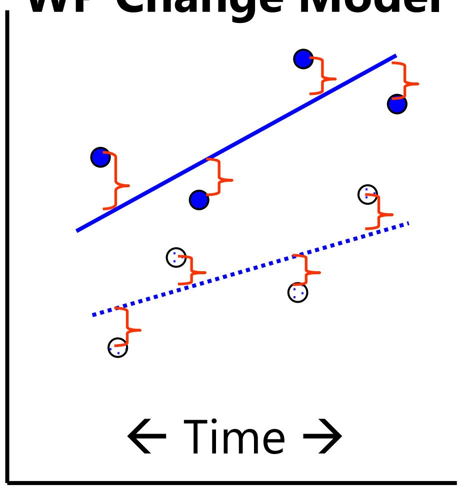
Time-Varying (TV) Predictors in Longitudinal Models of Within-Person Fluctuation

- Topics:
  - Concepts and what NOT to do with level-1 TV predictors
  - Univariate MLM strategies:
    - Person-(group/cluster)-mean-centering (*aka*, variable-centering)
    - Grand-mean-centering (*aka*, constant-centering)
  - Multivariate MLM strategies:
    - Latent centering (*aka*, turn the TV predictor into a TV outcome)
    - Implications for longitudinal (multilevel) mediation

The Joy of Time-Varying (TV) Predictors

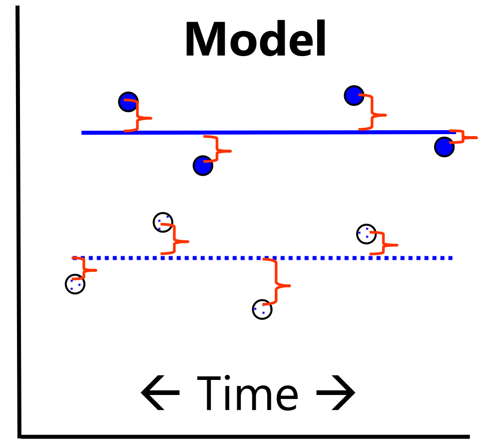
- TV predictors predict leftover **Level-1 WP (residual) variation**:

WP Change Model



If model for time works, then residuals should look like this →

WP Variation Model



- Modeling TV predictors (or any level-1 predictor) is complicated because they potentially contain **two different relations with y_{ti}**:
  - Relation of the *level-1 within-person* variation in the predictor x_{ti} with y_{ti}
  - Relation of the *level-2 between-person* variation in the predictor x_{ti} with y_{ti}
  - For now, we are assuming the predictor x_{ti} only **fluctuates** over time…
    - *We will need a **different model** when x_{ti} changes individually over time!*

The Joy of Time-Varying Predictors

- Time-varying (TV) predictors can usually have 2 levels of relations because **they are really 2 predictors in 1 variable**

- Example: Stress measured daily (to be used as predictor)
  - Some days are worse than others:
    - **Level-1 WP variation** *(can be captured using deviation from own mean)*
  - Some people just have more stress than others all the time:
    - **Level-2 BP variation** *(can be captured using person mean over time)*

- Can quantify relative sources of variation with an **ICC**
  - Intraclass Correlation ICC = (BP variance) / (BP variance + WP variance)
  - **ICC < 1?** TV predictor has **WP** variation (so it *could* have a **L1 WP** slope)
  - **ICC > 0?** TV predictor has **BP** variation (so it *could* have a **L2 BP** slope)
    - ICC specifically captures BP mean variation, but change variation is possible, too!

Between-Person vs. Within-Person Slopes

- Between- and within-person slopes could be in SAME direction
  - Time-Varying Stress → Time-Varying Health?
    - **Level-1 WP: People may feel worse than usual when they are currently under more stress than usual (regardless of what "usual" is)**
    - **Level-2 BP: People with more chronic stress than other people may have worse general health than people with less chronic stress**
- Between- and within-person slopes could be in OPPOSITE directions
  - Time-Varying Exercise → Time-Varying Blood pressure?
    - **Level-1 WP: During exercise, blood pressure is higher than during rest**
    - **Level-2 BP: People who exercise more often generally have lower blood pressure than people who are more sedentary**
- L1 within-person and L2 between-person slopes usually differ
  - Why? Because variables have different **meanings** at each level!
  - Why? Because variables have different **scales** at each level!

WAY WRONG: Within-Person Fluctuation Model with x_{ti} **represented at Level 1 Only:** → Its WP and BP Slopes are **Smushed Together**

x_{ti} is centered into TVx_{ti} WITHOUT representation at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{TVx_{ti}}) + \mathbf{e_{ti}}$

$TVx_{ti} = x_{ti} - C_1$ → it still has both Level-2 BP and Level-1 WP variation

Level 2: $\beta_{0i} = \boldsymbol{\gamma_{00}} + \mathbf{U_{0i}}$

$\beta_{1i} = \boldsymbol{\gamma_{10}}$

Because TVx_{ti} still contains its original 2 different kinds of variation (BP and WP), its 1 fixed slope has to do the work of 2 predictors!

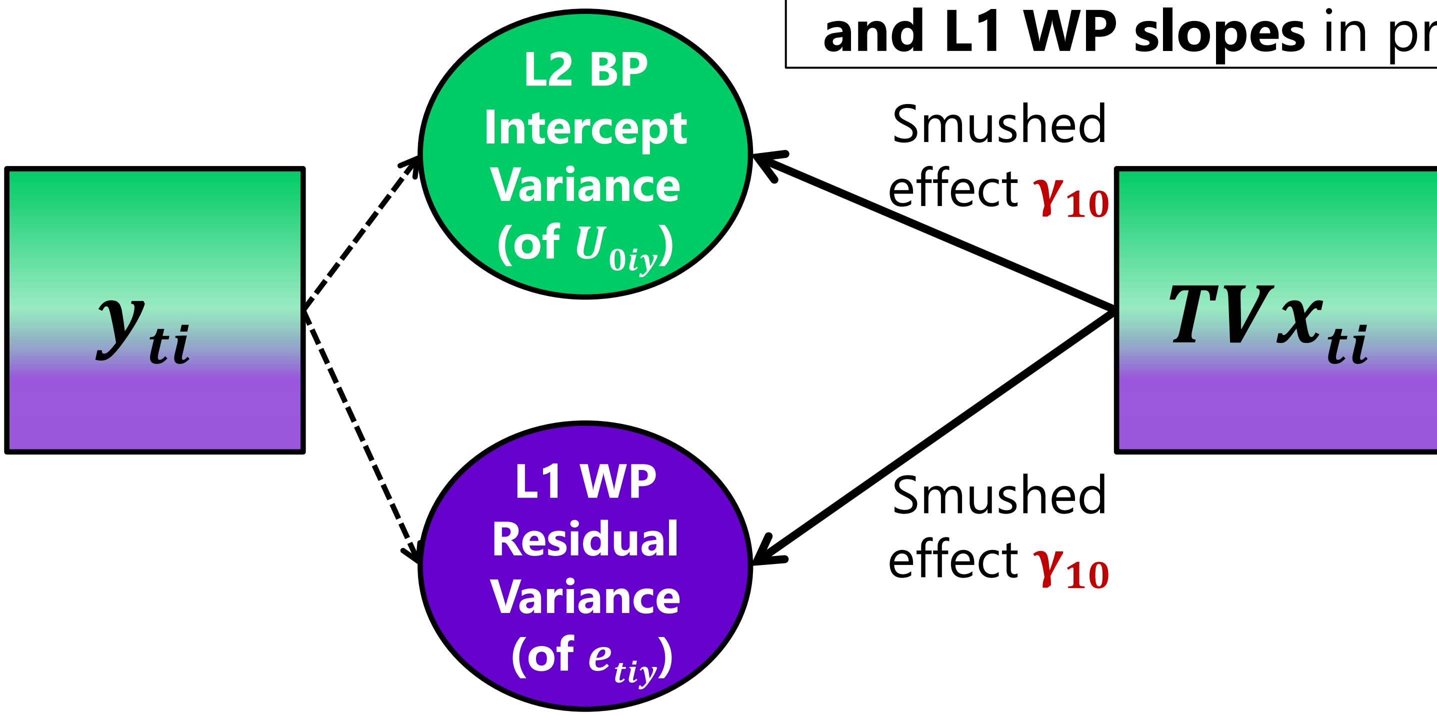
γ_{10} = *smushed* WP and BP effects

A *smushed* effect (to me) is also known as a *convergence*, *conflated*, or *composite* effect

Univariate MLM: Adding a Level-1 Predictor Without Level-2 Representation = Smushing

BP and WP variance in the **observed level-1 y_{ti} outcome** is partitioned by the **model** into estimated **variance components**

Observed level-1 TVx_{ti} predictor still has both BP and WP variance. AND given that TVx_{ti} has only **one fixed slope**, it captures a smushed effect that presumes **equal L2 BP and L1 WP slopes** in predicting y_{ti}!



3 Kinds of Fixed Slopes for TV Predictors

- **Is there a Level-1 Within-Person (WP) slope?**
  - When you have a higher x_{ti} predictor value than usual (*at this occasion*), do you also have a higher (or lower) y_{ti} outcome value than usual (*at same or later occasion*)?
  - If so, the **level-1 within-person *part* of the TV predictor** will reduce the level-1 residual variance (σ_e^2) of the TV outcome

- **Is there a Level-2 Between-Person (BP) slope?**
  - Do people with higher x_{ti} predictor values than other people (*on average over time*) also have higher (or lower) y_{ti} outcomes than other people (*on average over time*)?
  - If so, the **level-2 between-person *part* of the TV predictor** will reduce level-2 random intercept variance ($\tau_{U_0}^2$) of the TV outcome

- **Is there a Level-2 Contextual slope: Do the L2 BP and L1 WP slopes differ?**
  - After controlling for the actual value of TV predictor at that occasion, is there still **an incremental contribution** from the **level-2 between-person *part* of the TV predictor** (i.e., does one's general tendency matter beyond current TVP value)?
  - Equivalently, the **Level-2 Contextual slope = L2 BP slope − L1 WP slope**, so the Level-2 Contextual slope directly tests **if a smushed slope is ok (pry not!)**

3 Options to Prevent Smushed Slopes

- Within Univariate MLM framework (predict only one column):
  1. **Person-mean-centering**: manually carve up TV predictor into its level-specific parts using observed variables (1 predictor per level)
     - More generally, this is "**variable-centering**" because you are **subtracting a variable** (e.g., the cluster/group/person mean or person baseline value)
     - Will always yield **level-1 within slopes** and **level-2 between slopes**!

  2. **Grand-mean-centering**: do NOT carve up TV predictor into its level-specific parts, but add level-2 mean to distinguish level-specific slopes
     - More generally, this is "**constant-centering**" because you are **subtracting a constant** but still keeping all levels of variance in level-1 TV predictor
     - Choice of constant is irrelevant (changes where 0 is, not what variance it has)
     - Will always yield **level-1 within slopes** and **level-2 contextual slopes**!

- Within Multivariate MLM framework (via M-SEM or SEM):
  3. **Latent-centering**: Treat the TV predictor as another outcome → let the model carve it up into **level-specific latent variables**
     - Best in theory, but the type of level-2 slope provided (between or contextual) depends on type of model syntax (and the estimator in Mplus)! (Hoffman, 2019)

Option 1. Person-Mean-Centering (P-MC)

- In **P-MC**, we turn the TV predictor x_{ti} into **2 observed variables** that directly represent its BP (level-2) and WP (level-1) sources of variation and **include these 2 predictors instead of original** x_{ti}:

- **Level-2, BP predictor = person mean of** x_{ti}
  - $\mathbf{PMx_i} = \bar{x}_i - C_2$
  - $\mathrm{PMx_i}$ is centered at constant C_2, chosen for meaningful 0 (e.g., sample mean)
  - $\mathrm{PMx_i}$ is positive? Above sample mean → "more than other people"
  - $\mathrm{PMx_i}$ is negative? Below sample mean → "less than other people"

- **Level-1, WP predictor = deviation from person mean of** x_{ti}
  - $\mathbf{WPx_{ti}} = x_{ti} - \bar{x}_i$ *(note: uncentered person mean $\bar{x}_i$ is used to center x_{ti})*
  - $\mathrm{WPx_{ti}}$ is NOT centered at a constant – **we subtract a VARIABLE**
  - $\mathrm{WPx_{ti}}$ is positive? Above your own mean → "more than usual"
  - $\mathrm{WPx_{ti}}$ is negative? Below your own mean → "less than usual"

Within-Person Fluctuation Model with **Person-Mean-Centered Level-1 x_{ti}**

→ WP and BP slopes directly as **separate** parameters

x_{ti} is person-mean-centered into WPx_{ti}, with PMx_i at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(WPx_{ti}) + e_{ti}$

$WPx_{ti} = x_{ti} - \overline{x}_i$ → it has only Level-1 WP variation

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$PMx_i = \overline{x}_i - C_2$ → it has only Level-2 BP variation

γ_{10} = L1 WP main effect of having *more x_{ti} than usual*

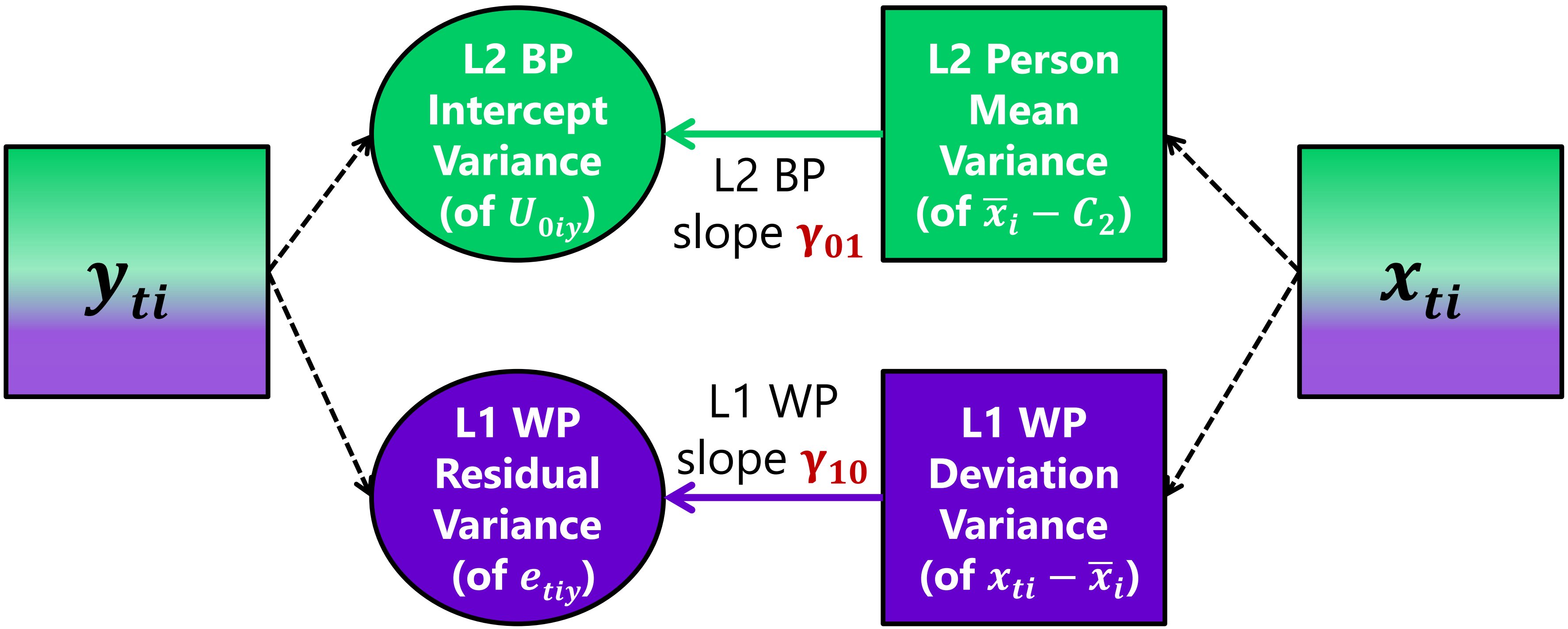
γ_{01} = L2 BP main effect of having *more $\overline{x}_i$ than other people*

Because WPx_{ti} and PMx_i are uncorrelated, each gets the total effect for its level (WP=L1, BP=L2)

Univariate MLM: Variable-Centering*

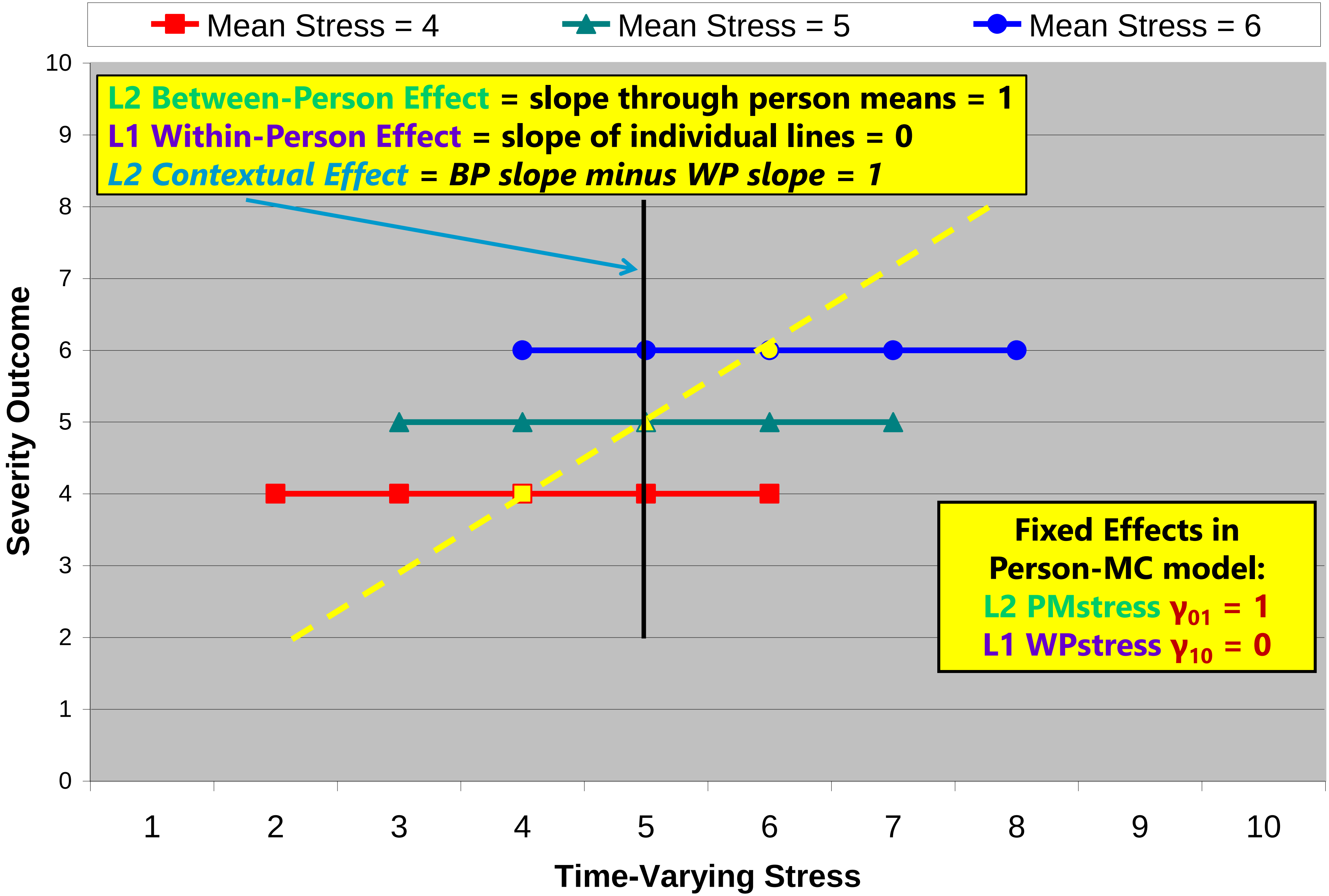
Model-based partitioning of level-1 y_{ti} outcome into level-specific **latent variables**

Manual partitioning of level-1 x_{ti} predictor into level-specific **observed variables**

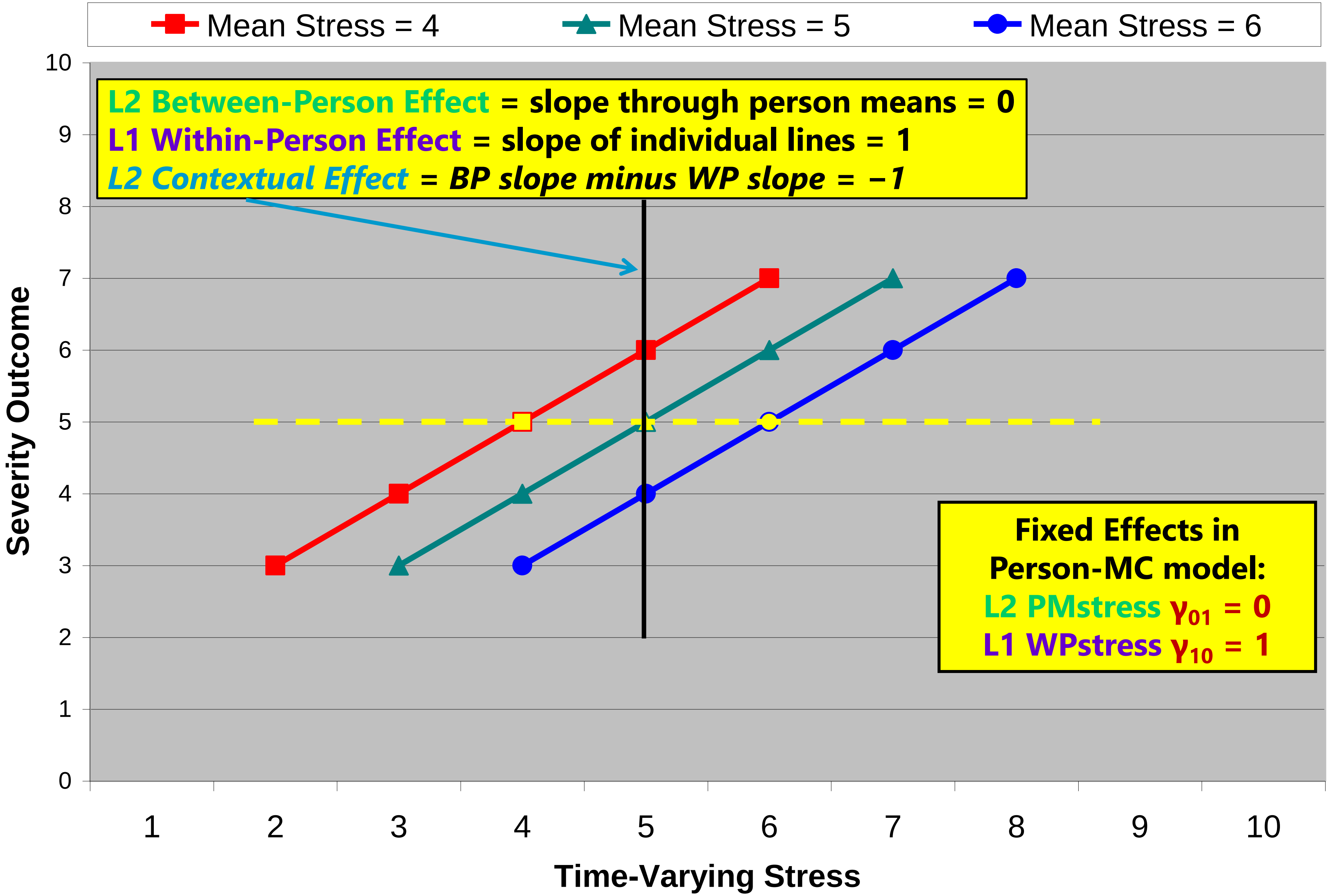


* Known as "person-mean-centering" more generally directly analogous to cluster/group-mean-centering in multilevel models for clustered data)

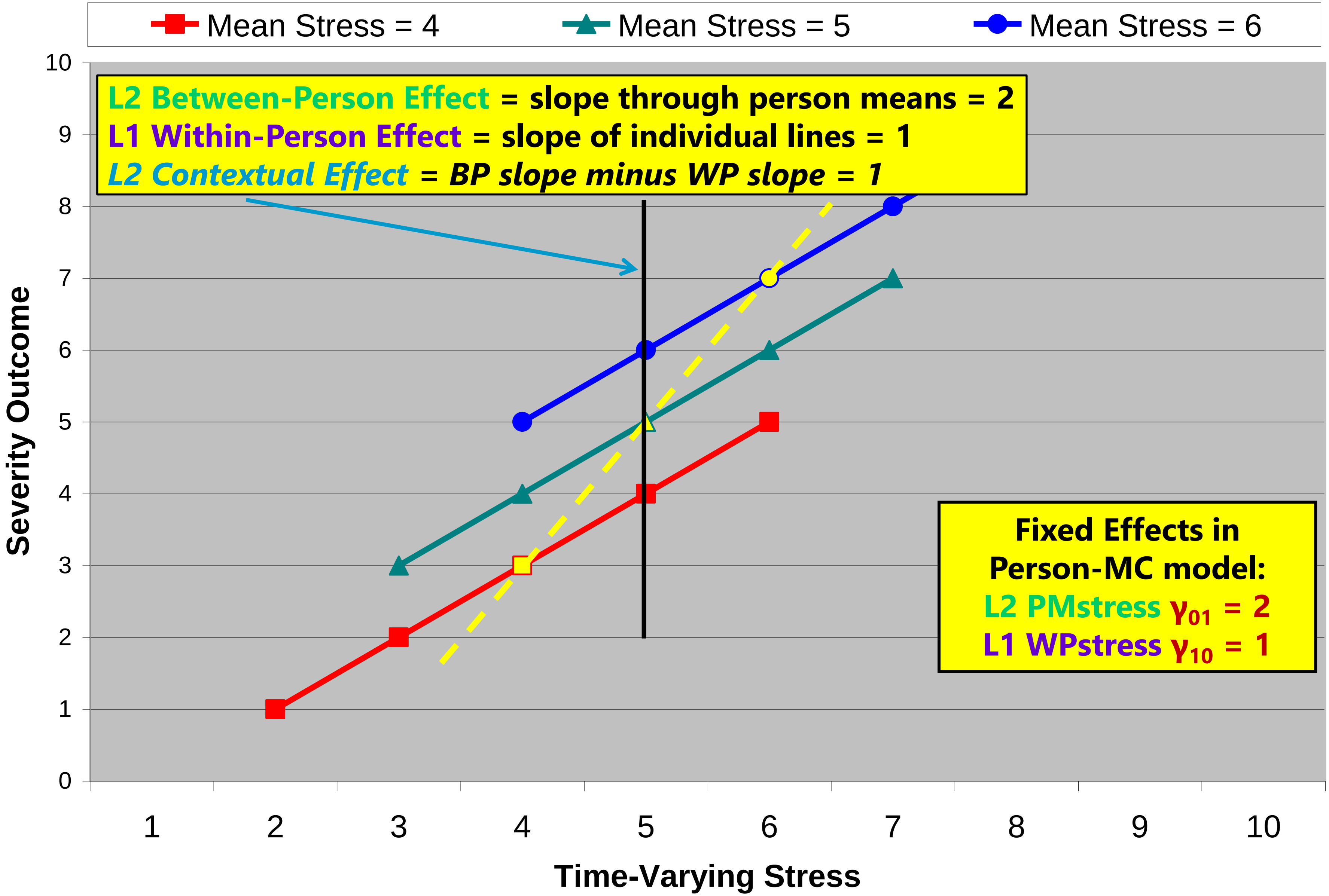
ALL Between-Person Effect, NO Within-Person Effect



<u>NO</u> Between-Person Effect, <u>ALL</u> Within-Person Effect

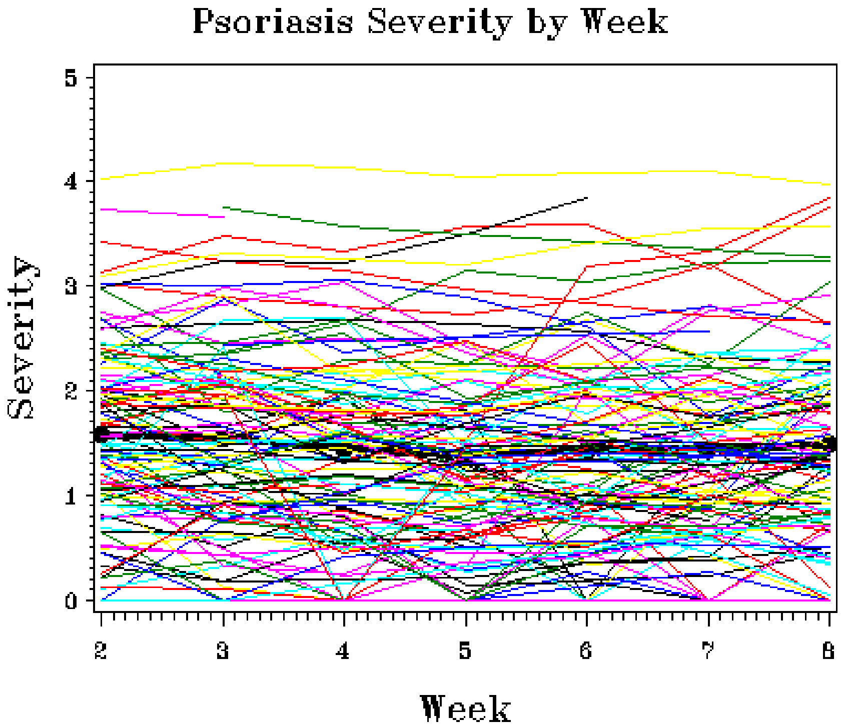


Between-Person Effect > Within-Person Effect



Example: Weekly Stress and Psoriasis

- 127 psoriasis patients, 8 weekly assessments (only last 7 used)
- How does perceived stress predict psoriasis severity? And is there a time lag for these processes to occur?
- No change in treatment → only WP fluctuation over time
- Analysis plan:
  - ICCs for stress and severity—how much variance is at each level?
  - Assess pattern of variance and covariance in severity over time
    - This was [PSQF 6271 Example 4]
  - Evaluate prediction of severity by stress at lag 0 and lag 1 weeks… without smushing!



Example: Weekly Stress and Psoriasis

- Empty means, random intercept model to get ICCs → proportion of total variance due to BP mean differences
  - For each variable: $y_{ti} = \gamma_{00} + U_{0i} + e_{ti}$, $\text{ICC} = \frac{\tau^2_{U_0}}{\tau^2_{U_0} + \sigma^2_e} = \frac{\text{BP}}{\text{BP+WP}}$
  - Severity outcome: ICC = .83; stress predictor: ICC = .56

- For the severity outcome, the best-fitting unconditional time model for the variance had a level-2 random intercept (in G), along with heterogeneous level-1 residual variances and a Toeplitz (banded) correlation structure up to lag 3 (in R, below)

Estimated R Correlation Matrix for ID 1 → WP residual correlation

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.5115	0.3566	0.1112			
2	0.5115	1.0000	0.5115	0.3566	0.1112		
3	0.3566	0.5115	1.0000	0.5115	0.3566	0.1112	
4	0.1112	0.3566	0.5115	1.0000	0.5115	0.3566	0.1112
5		0.1112	0.3566	0.5115	1.0000	0.5115	0.3566
6			0.1112	0.3566	0.5115	1.0000	0.5115
7				0.1112	0.3566	0.5115	1.0000

Example: Weekly Stress and Psoriasis

Level 1: $\text{severity}_{ti} = \beta_{0i} + \beta_{1i}(\text{WPstressLag0}_{ti}) + \beta_{2i}(\text{WPstressLag1}_{ti}) + e_{ti}$

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{PMstress}_{i}) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\beta_{2i} = \gamma_{20}$

WP effects are <u>fixed</u> (no random slopes) → same for everyone

$\text{WPx}_{ti} = x_{ti} - \bar{x}_i$ → **it has only Level-1 WP variation**

$\text{PMx}_{i} = \bar{x}_i - 2$ → **it has only Level-2 BP variation**

Model for the Means:

- γ_{00} → expected severity for someone with person mean stress = 2, and who had severity = 2 last week and currently
- γ_{01} → BP difference in *average* severity per unit person mean stress
- γ_{10} and γ_{20} → WP change in *current* severity per unit more stress than usual this week (lag 0) and last week (lag 1)

Example: Weekly Stress and Psoriasis

Level 1: $\text{severity}_{ti} = \beta_{0i} + \beta_{1i}(\text{WPstressLag0}_{ti}) + \beta_{2i}(\text{WPstressLag1}_{ti}) + e_{ti}$

Level 2: $\beta_{0i} = 1.96 + 0.48^{*}(\text{PMstress}_{i}) + U_{0i}$

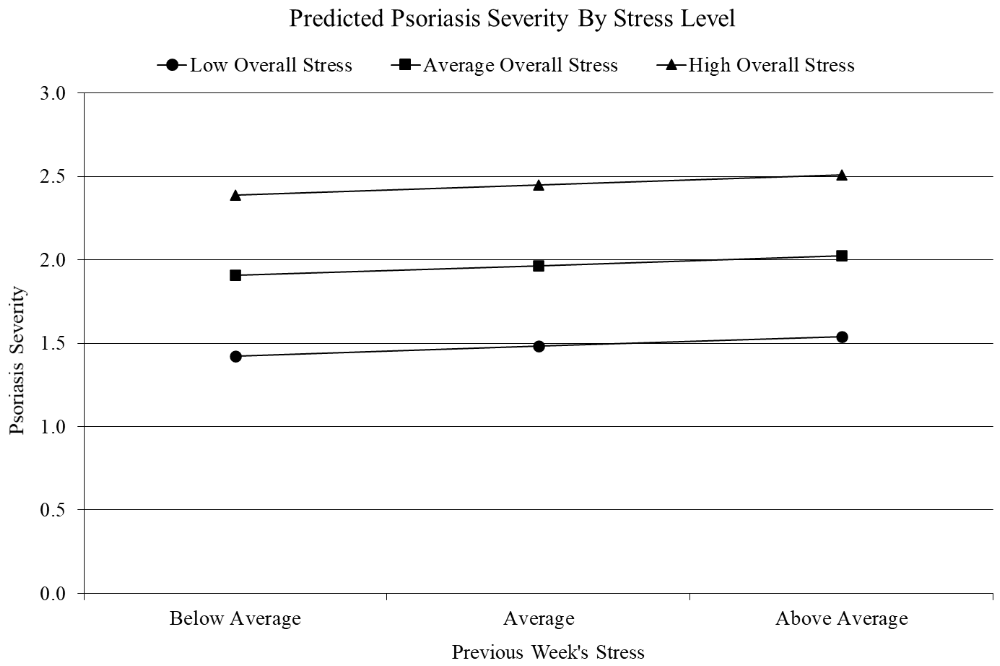
$\beta_{1i} = 0.02$

$\beta_{2i} = 0.06^{*}$

WP effects are fixed (no random slopes) → same for everyone

$\text{WPx}_{ti} = x_{ti} - \bar{x}_i$ → **it has only Level-1 WP variation**

$\text{PMx}_{i} = \bar{x}_i - 2$ → **it has only Level-2 BP variation**



Example: Syntax by Univariate MLM Program (Using Long Data)

SAS:

```
PROC MIXED DATA=work.Example COVTEST METHOD=REML;
     CLASS ID;
     MODEL severity = PMstress WPstressLag0 WPstressLag1 / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT / TYPE=UN SUBJECT=ID;
     REPEATED week / RCORR TYPE=TOEPH(4) SUBJECT=ID;
RUN;
```

R (lmer from lme4 package)—using lmertest package, which does provide correct denominator DF, but custom R matrix structures are not available (might be possible using gls from nlme instead), so RI only here:

```
modelname = lmer(data=Example, REML=TRUE,
                formula=severity~1+PMstress+WPstressLag0+WPstressLag1+(1+|ID))
summary(modelname, ddf="Satterthwaite")
```

STATA—I don't think custom Toeplitz structure with heterogeneous residual variances is possible, so I used RI + a homogeneous residual variance version here:

```
mixed severity c.PMstress c.WPstressLag0 c.WPstressLag1, || ID: , ///
      variance reml covariance(un) residuals(toeplitz3,t(week)) ///
      dfmethod(satterthwaite) dftable(pvalue)
```

SPSS—I don't think custom Toeplitz structure with heterogeneous variances is possible, so RI only here :

```
MIXED severity BY ID WITH PMstress WPstressLag0 WPstressLag1
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV
      /FIXED  = PMstress WPstressLag0 WPstressLag1
      /RANDOM = INTERCEPT | COVTYPE(UN) SUBJECT(ID).
```

3 Kinds of Fixed Slopes for TV Predictors

- **2 kinds of slopes Person-Mean-Centering tells us <u>directly</u>:**

- **Is there a Level-1 Within-Person (WP) slope?**
  - When you have higher predictor values <u>than usual</u> (*at this occasion*), do you also have higher outcomes values <u>than usual</u> (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance (σ_e^2)?
  - **Given directly by fixed slope of $\mathbf{WPx_{ti}}$ regardless of whether $\mathbf{PMx_i}$ is there**
  - Note: L1 slope multiplies the **relative** value of $\boldsymbol{x_{ti}}$, NOT the **original** $\boldsymbol{x_{ti}}$

- **Is there a Level-2 Between-Person (BP) slope?**
  - Do people with higher predictor values <u>than other people</u> (*on average over time*) also have higher outcomes <u>than other people</u> (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random int var ($\tau_{U_0}^2$)?
  - **Given directly by fixed slope of $\mathbf{PMx_i}$ regardless of whether $\mathbf{WPx_{ti}}$ is there**
  - Note: BP slope is NOT controlling for the original value of x_{ti} at each occasion

3rd Kind of Slope for TV Predictors

- **What Person-Mean-Centering DOES NOT tell us directly:**

- **Is there a Level-2 Contextual effect: Do the BP and WP slopes differ?**
  - After controlling for the original value of the TV predictor at that occasion, is there still **an incremental contribution from having a higher person mean** of the TV predictor (i.e., does one's general tendency for the predictor explain more $\tau^2_{U_0}$ above and beyond just the time-specific value of the predictor)?
  - If there is no contextual effect, then the TV predictor's **L2 BP** and **L1 WP** slopes show **convergence**, which means their effects are of equivalent magnitude

- **To answer this question about the Level-2 Contextual effect for the incremental contribution of the person mean, we have two options:**
  - Use Person-MC, and ask for the **contextual slope = between – within** (via SAS ESTIMATE, R contest1D, SPSS TEST, STATA LINCOM, Mplus NEW)
  - Use **"constant-centering"** for time-varying x_{ti} instead: $\mathbf{TVx_{ti}} = \boldsymbol{x_{ti}} - \boldsymbol{C_1}$
    → **centered at CONSTANT C_1, NOT A LEVEL-2 VARIABLE**
    - Which constant only matters for the reference point; it could be the grand mean or any (even 0)

Why the Difference in the Level-2 Slope? Remember Regular Old Regression…

- In this model: $y_i = \beta_0 + \beta_1(x1_i) + \beta_2(x2_i) + e_i$
- If $x1_i$ and $x2_i$ **ARE NOT** correlated:
  - β_1 carries **ALL the relationship** between $x1_i$ and y_i
  - β_2 carries **ALL the relationship** between $x2_i$ and y_i

- If $x1_i$ and $x2_i$ **ARE** correlated:
  - β_1 is **different than** the bivariate relationship between $x1_i$ and y_i
    - "Unique" effect of $x1_i$ *controlling for* $x2_i$ (or *holding* $x2_i$ *constant)*
  - β_2 is **different than** the bivariate relationship between $x2_i$ and y_i
    - "Unique" effect of $x2_i$ *controlling for* $x1_i$ (or *holding* $x1_i$ *constant)*

- Hang onto that idea…

Person-MC vs. Grand-MC: Variable- vs. Constant-Centering for TV Predictors

Level 2		Original	Person-MC Level 1	Grand-MC Level 1
$\bar{x}_i$	$\mathrm{PMx_i} = \bar{x}_i - 5$	x_{ti}	$\mathrm{WPx_{ti}} = x_{ti} - \bar{x}_i$	$\mathrm{TVx_{ti}} = x_{ti} - 5$
3	−2	2	−1	−3
3	−2	4	1	−1
7	2	6	−1	1
7	2	8	1	3

Same L2 $\mathrm{PMx_i}$ goes into the model given either way of centering the level-1 variable x_{ti}

In **variable-centering** (P-MC), the level-2 BP mean variation is gone from $\mathrm{WPx_{ti}}$, so it is NOT correlated with $\mathrm{PMx_i}$

In **constant-centering** (G-MC), the level-2 BP mean variation is still inside $\mathrm{TVx_{ti}}$, so it IS STILL CORRELATED with $\mathrm{PMx_i}$

So the effects of $\mathrm{PMx_i}$ and $\mathrm{TVx_{ti}}$ when included together under constant-centering will be different than if either predictor were included by itself…

Within-Person Fluctuation Model with **Constant-Centered Level-1 x_{ti}**

→ Model tests difference of WP vs. BP slopes (it's been fixed!)

x_{ti} is constant-centered into TVx_{ti}, <u>WITH</u> PMx_i at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{TVx}_{ti}) + e_{ti}$

$\text{TVx}_{ti} = x_{ti} - C_1$ → it still has both Level-2 BP and Level-1 WP variation

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{PMx}_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\text{PMx}_i = \bar{x}_i - C_2$ → it has only Level-2 BP variation

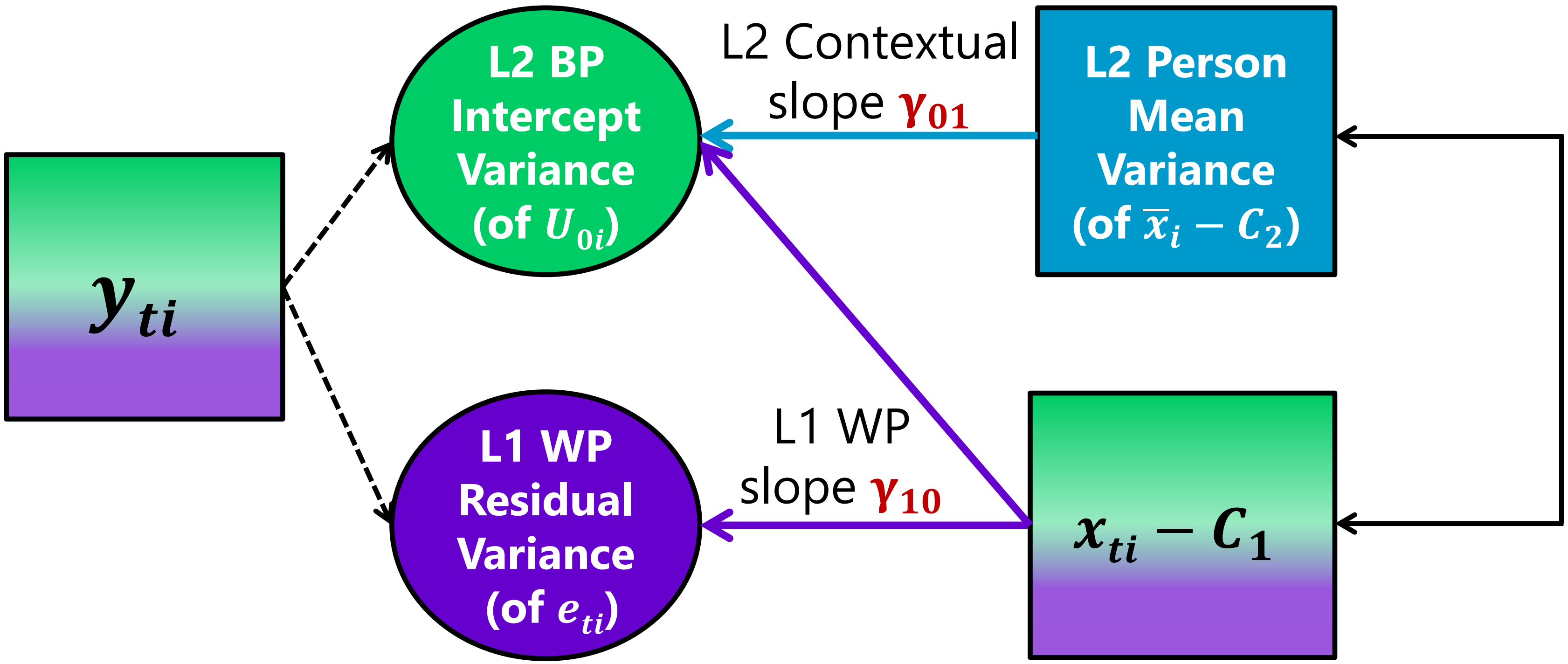
γ_{10} becomes the L1 WP slope → *unique* level-1 effect after controlling for PMx_i

γ_{01} *becomes* the L2 Contextual slope that indicates <u>how the L2 BP effect differs from the L1 WP effect</u>
→ *unique* level-2 slope after controlling for TVx_{ti}
→ does usual level matter beyond current level?

Univariate: Constant-Centering WITH Level-2 Predictor → OK now!

Model-based partitioning of y_{ti} outcome into level-specific **latent variables**

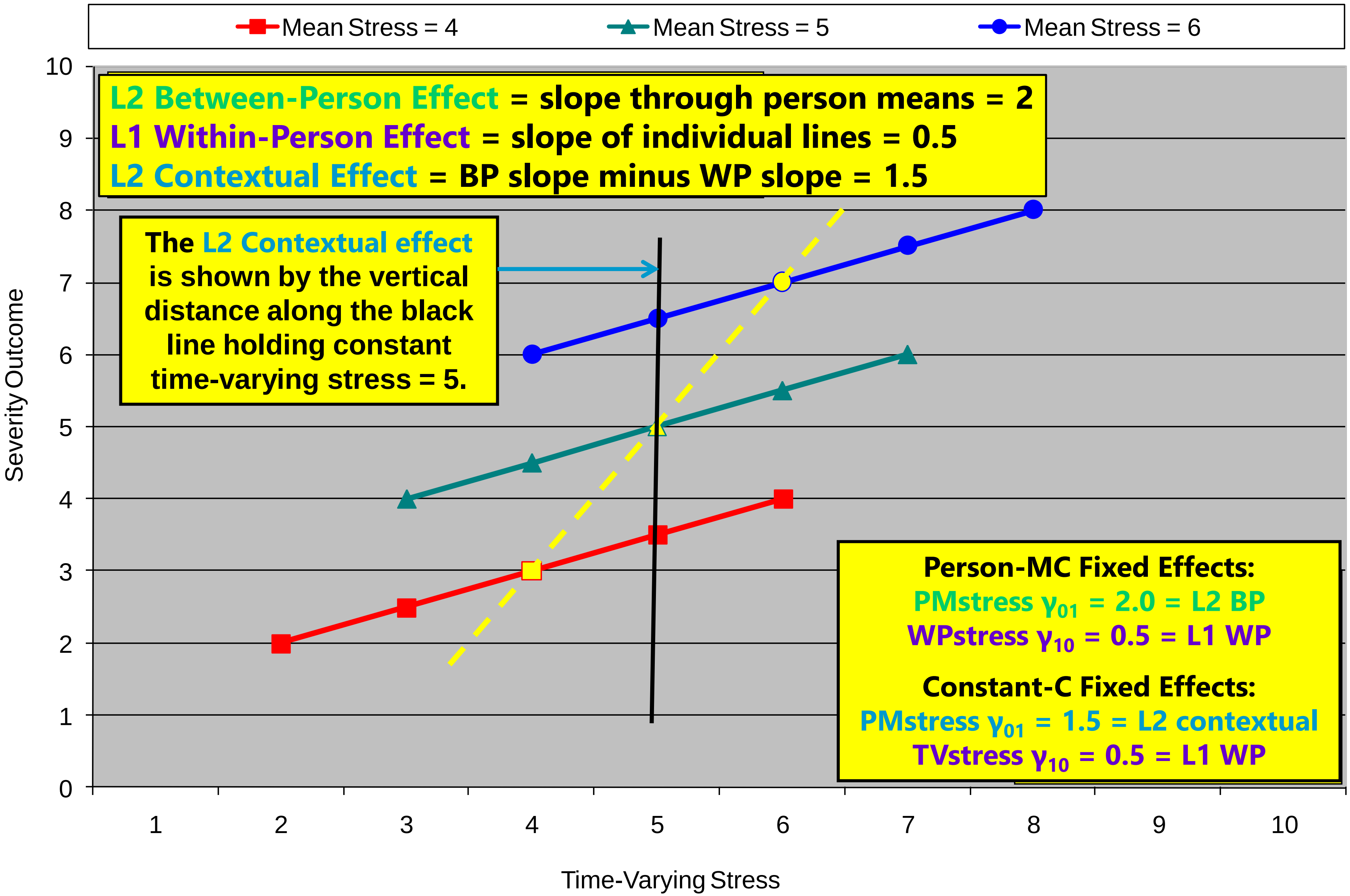
Level-1 x_{ti} is still NOT partitioned, but person mean $\bar{x}_i - C_2$ is added to allow an **incremental L2 effect**



L2 BP slope = L1 WP slope + Level-2 Contextual slope

Because original x_{ti} still has L2 BP variance, it still carries *some* of the L2 BP effect…

Person-MC vs. Constant-C: Example



Person-MC and Constant-C Models are Equivalent Given Only a **Fixed** Level-1 Main Effect Slope

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Btw, I am using a centering constant = 0 at both levels to simplify the notation

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

→ $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$

→ $y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

Composite Model:
← In terms of P-MC
← In terms of Const-C

Constant-C: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

→ $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

Effect	P-MC	Const-C
Intercept	γ_{00}	γ_{00}
L1 WP	γ_{10}	γ_{10}
L2 Context	$\gamma_{01} - \gamma_{10}$	γ_{01}
L2 BP	γ_{01}	$\gamma_{01} + \gamma_{10}$

The Joy of Interactions Involving Time-Varying Predictors

- **<u>Must consider interactions with both its L2 BP and L1 WP parts:</u>**
- Example: Does time-varying stress (x_{ti}) interact with group ($\mathbf{Grp_i}$)?

- <u>Person-Mean-Centering (Variable-Centering):</u>
  - $\mathbf{WPx_{ti}} * \mathbf{Grp_i}$ → Does the **L1 WP** stress slope differ between groups?
  - $\mathbf{PMx_i} * \mathbf{Grp_i}$ → Does the **L2 BP** stress slope differ between groups?
    - Level-2 interaction is not controlling for current levels of stress
    - If forgotten, then $\mathbf{Grp_i}$ moderates the stress effect only at level 1 WP (not L2 BP)

- <u>Constant-Centering:</u>
  - $\mathbf{TVx_{ti}} * \mathbf{Grp_i}$ → Does the **L1 WP** slope effect differ between groups?
  - $\mathbf{PMx_i} * \mathbf{Grp_i}$ → Does the **L2 Contextual** slope effect differ between groups?
    - Incremental L2 stress effects *after controlling for current levels of stress*
    - **If forgotten**, then although the L1 main effect of stress has been unsmushed via the main effect of $\mathbf{PMx_i}$, **the interaction of $\mathbf{TVx_{ti}} * \mathbf{Grp_i}$ is still smushed**

Interactions with Time-Varying Predictors: Example: TV Stress (x_{ti}) by Group (Grp_i)

Btw, I am using a centering constant = 0 at both levels

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(Grp_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
$+ \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + \gamma_{11}(Grp_i)(x_{ti} - PMx_i)$

Constant-C: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(Grp_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + \gamma_{11}(Grp_i)(x_{ti})$

Interactions Involving Time-Varying Predictors Belong at Both Levels of the Model

On the left below → Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti} + \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + \gamma_{11}(Grp_i)(x_{ti} - PMx_i)$$

← Composite model as Person-MC

$$y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti} + \gamma_{02}(Grp_i) + (\gamma_{03} - \gamma_{11})(Grp_i)(PMx_i) + \gamma_{11}(Grp_i)(x_{ti})$$

← Composite model as Constant-C

On the right below → Constant-C: $TVx_{ti} = x_{ti}$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti} + \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + \gamma_{11}(Grp_i)(x_{ti})$$

After adding an interaction for Grp_i with stress at both levels, the Person-MC and Constant-MC models are equivalent

---

Intercept: $\gamma_{00} = \gamma_{00}$ **BP Slope:** $\gamma_{01} = \gamma_{01} + \gamma_{10}$ **Context:** $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WP Slope: $\gamma_{10} = \gamma_{10}$ **BP*Grp Slope:** $\gamma_{03} = \gamma_{03} + \gamma_{11}$ **Context*Grp:** $\gamma_{03} = \gamma_{03} - \gamma_{11}$

Grp Slope: $\gamma_{20} = \gamma_{20}$ **BP*WP or Context*WP is the same:** $\gamma_{11} = \gamma_{11}$

Intra-Variable* Interactions

- **Still must consider interactions with both its BP and WP parts!**
- Example: Interaction of TV stress (x_{ti}) with person mean stress (PMx_i), such that person mean stress is also a moderator (like $\mathbf{Grp_i}$ before)

- Person-Mean-Centering (Variable-Centering):
  - $\mathbf{WPx_{ti}} * \mathbf{PMx_i}$ → Does the **L1 WP** stress slope differ by overall stress?
  - $\mathbf{PMx_i} * \mathbf{PMx_i}$ → Does the **L2 BP** stress slope differ by overall stress?
    - Level-2 interaction is not controlling for current levels of stress
    - If forgotten, then $\mathbf{PMx_i}$ moderates the stress effect only at level 1 WP (not L2 BP)

- Constant-Centering:
  - $\mathbf{TVx_{ti}} * \mathbf{PMx_i}$ → Does the **L1 WP** stress slope differ by overall stress?
  - $\mathbf{PMx_i} * \mathbf{PMx_i}$ → Does the **L2 Contextual** stress slope differ by overall stress?
    - Incremental BP stress effect *after controlling for current levels of stress*
    - **If forgotten**, then although the L1 main effect of stress has been unsmushed via the main effect of $\mathbf{PMx_i}$, **the interaction of** $\mathbf{TVx_{ti}} * \mathbf{PMx_i}$ **is still smushed**

** Btw, this idea was also seen in controlling age slopes for age cohort…*

Intra-Variable Interactions:
Example: TV Stress (x_{ti}) by Person Mean Stress (PMx_i)

Btw, I am using a centering constant = 0 at both levels

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(PMx_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti} - PMx_i)$

Constant-C: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(PMx_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$

Intra-Variable Interactions:
Example: TV Stress (x_{ti}) by Person Mean Stress (PMx_i)

On the left below → Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$$
$$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti} - PMx_i)$$

← **Composite model as Person-MC**

$$y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$$
$$+ (\gamma_{02} - \gamma_{11})(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$$

← **Composite model as Constant-C**

On the right below → Constant-C: $TVx_{ti} = x_{ti}$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$$
$$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$$

After adding an interaction for PMx_i with stress at both levels, the Person-MC and Constant-C models are equivalent

Intercept: $\gamma_{00} = \gamma_{00}$ **BP Slope:** $\gamma_{01} = \gamma_{01} + \gamma_{10}$ **Contextual:** $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WP Slope: $\gamma_{10} = \gamma_{10}$ **BP² Slope:** $\gamma_{02} = \gamma_{02} + \gamma_{11}$ **Contextual²:** $\gamma_{02} = \gamma_{02} - \gamma_{11}$

BP*WP or Contextual*WP is the same: $\gamma_{11} = \gamma_{11}$

When Person-MC ≠ Constant-Centering: Random Slopes of TV Predictors

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

Variance due to PMx_i is removed from the random slope in Person-MC.

→ $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + U_{1i}(x_{ti} - PMx_i) + e_{ti}$

Constant-C: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

PMx_i variance is still part of the Constant-C random slope → smushed random effect! Thus, the level-1 predictor to be given a random slope should be P-MC to prevent this problem.

→ $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + U_{1i}(x_{ti}) + e_{ti}$

Preventing Smushed (BP=WP) Slopes

- **Fixed side: 2 strategies to prevent smushed slopes**
  - If using variable-centered (P-MC) L1 TVP ($\mathbf{WPx_{ti}}$), it can only have a **L1 WP slope**, and its L2 $\mathbf{PMx_i}$ can only have a **L2 BP slope** (so no problem)
  - If using constant-C L1 TVP ($\mathbf{TVx_{ti}}$), its L1 slope will be smushed (BP=WP) if you don't add its L2 $\mathbf{PMx_i}$ to allow a **L2 contextual slope = BP − WP**

- **Random side: Only 1 strategy is likely possible!** *(see Rights & Sterba, MBR in press, for details)*
  - If using variable-centered (P-MC) L1 TVP ($\mathbf{WPx_{ti}}$), its L2 random slope variance **only** captures L2 BP differences in its L1 WP slope (so no problem)
    - Creates a pattern of quadratic heterogeneity of variance **across** $\mathbf{WPx_{ti}}$ **ONLY**
  - If using constant-C L1 TVP ($\mathbf{TVx_{ti}}$), its L2 random slope variance **also** creates **intercept heterogeneity of variance** (beyond BP diffs in L1 WP slope)
    - Enforces **SAME** pattern of quadratic heterogeneity of variance across **L1** $\mathbf{WPx_{ti}}$ and **L2** $\mathbf{PMx_i}$
  - If using $\mathbf{TVx_{ti}}$, you need a "contextual" random slope to allow a different pattern of variance heterogeneity across $\mathbf{PMx_i}$ than $\mathbf{WPx_{ti}}$ (for BP − WP)
    - Requires a L2 BP random "slope ?" variance for **L2** $\mathbf{PMx_i}$ – good luck estimating it!

Modeling Time-Varying Categorical Predictors

- Person-MC and Constant-C usually refer to *quantitative* TV predictors, but the need to separate BP and WP effects applies to *categorical* TV predictors too
- Binary level-1 predictors do not lend themselves intuitively to Person-MC
  - e.g., x_{ti} = 0 or 1 per occasion, person mean = .40 across occasions → impossible values (if x_{ti} = 0, then WPx_{ti} = 0 − 0.40 = −0.40; if x_{ti} = 1, then WPx_{ti} = 1 − 0.40 = +0.60)
  - Easier: Leave x_{ti} uncentered in estimating its fixed slope and include person mean as level-2 predictor so that results = Const-C (but still use Person-MC in estimating its random slope)
- For >2 categories, person means of multiple dummy codes may start to break down, but we can think about types of people, and code BP effects accordingly
- Example: Dementia present/not at each time point?
  - **BP effects →** Ever diagnosed with dementia (no, yes) rather than person mean
    - People who will eventually be diagnosed may differ prior to diagnosis (a BP stable effect)
  - **TV effect →** Diagnosed with dementia at each time point (no, yes)?
    - Acute differences of before/after diagnosis logically can only exist in the "ever" people
- Other examples: Mentor status, father absence, type of shift work (AM/PM)

Summary: Univariate MLM for Specifying Effects of Time-Varying Predictors

- "Univariate" approach to MLM is possible for time-varying predictors that *fluctuate* over time (and lower-level predictors with only mean differences across higher levels in general)

- Level-1 predictor can be created two different ways:
  - Easier to understand is variable-centering: $\mathbf{WPx_{ti}} = x_{ti} - \bar{x}_i$
    - Directly isolates level-1 within variance, so $\mathbf{WPx_{ti}}$ → L1 within effects
  - More common is constant-centering: $\mathbf{TVx_{ti}} = x_{ti} - C_1$
    - Does NOT remove level-2 BP variance, so $\mathbf{TVx_{ti}}$ will have smushed (BP=WP) effects **unless** you add the necessary slopes for its level-2 predictor analog

- Level-2 predictor is always constant-centered: $\mathbf{PMx_i} = \bar{x}_i - C_2$
  - $\mathbf{PMx_i}$ slope is **L2 Between** effect when paired with **L1** $\mathbf{WPx_{ti}}$
  - $\mathbf{PMx_i}$ slope is **L2 Contextual** effect when paired with **L1** $\mathbf{TVx_{ti}}$
    - Within + Contextual = Between; Between − Within = Contextual

I Prefer Variable-Centering…

- …because constant-centering is much easier to screw up! ☺
- See Table 1 from: Hoffman, L., & Walters, R. W. (2022). Catching up on multilevel modeling. *Annual Review of Psychology, 73*, 629-658.

Table 1 Predictor effect type by model specification

Centering strategy for level-1 predictor (constant-centered level-2 predictor)	Fixed effect type by predictors included		
	Level-1 only	Level-2 only	Both levels
Variable-centered level-1			
Level-1 predictor: $L1x_{wb} = x_{wb} - \bar{x}_b$	Within	(= 0)	Within
Level-2 predictor: $L2x_b = \bar{x}_b - C_2$	(= 0)	Between	Between
Constant-centered level-1			
Level-1 predictor: $L1x_{wb} = x_{wb} - C_1$	Smushed	(= 0)	Within
Level-2 predictor: $L2x_{wb} = \bar{x}_b - C_2$	(= Within)	Between	Contextual

Abbreviations: w, within; b, between; C_1, level-1 centering constant; C_2, level-2 centering constant.
Parentheses indicate assumptions about the fixed slopes of omitted predictors.

Variance Accounted For By Level-1 Predictors

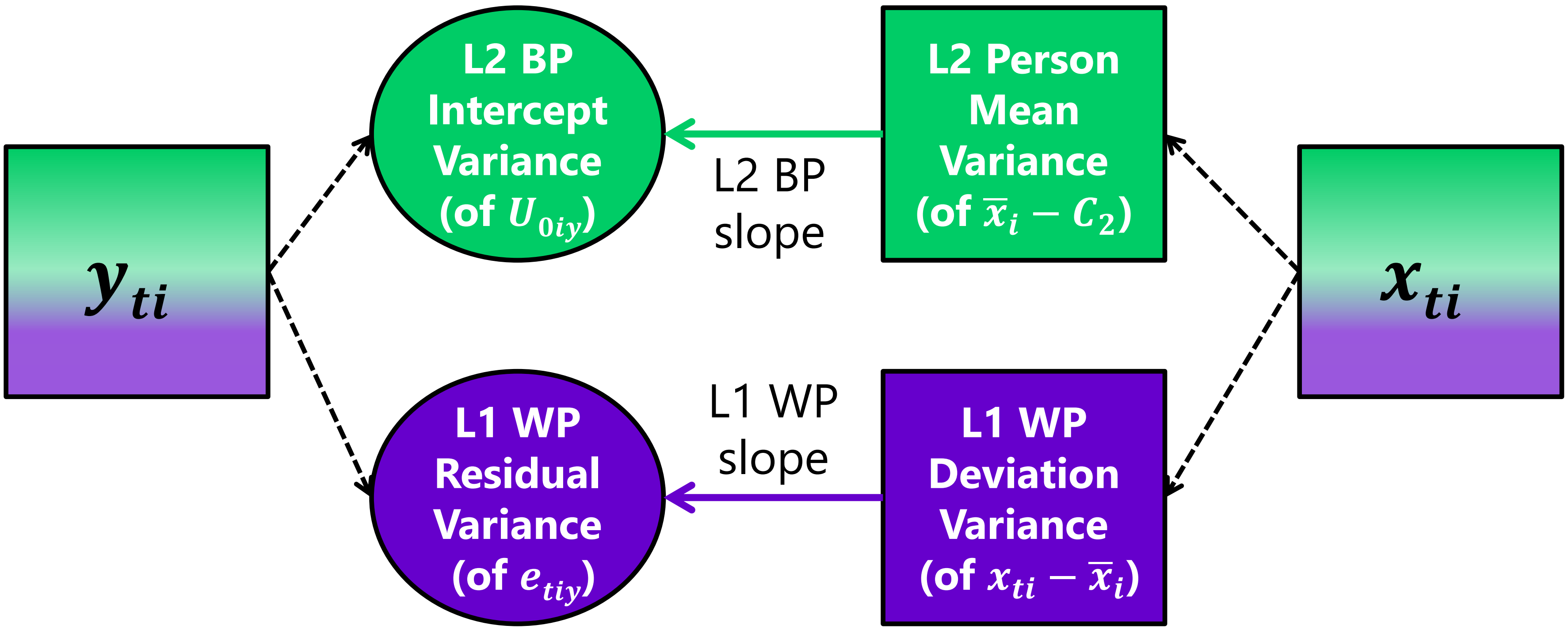
- **Fixed effects of level-1 TV predictors:**
  - Level-1 WP part of TV predictors (as main effects by themselves or as part of interactions with other TV predictors) reduce Level-1 (WP) residual variance σ_e^2
- **What happens to the level-2 random intercept variance depends on what levels of variance the level-1 TV predictor still has:**
  - If the level-1 TV predictor STILL has level-2 variance (e.g., Grand-MC predictors), then its level-2 part can reduce level-2 random intercept variance $\tau_{U_0}^2$
    - But badly smushed effects could increase level-2 random intercept variance instead!
  - If the level-1 TV predictor DOES NOT have level-2 variance (e.g., Person-MC or latent-centered predictors), then any reduction in the level-1 residual variance σ_e^2 will cause an INCREASE in level-2 random intercept variance $\tau_{U_0}^2$
    - Same thing happens with Grand-MC level-1 predictors, but you don't generally see it
  - It's just an artifact that the estimate of true random intercept variance is:

    True $\tau_{U_0}^2$ = observed $\tau_{U_0}^2 - \frac{\sigma_e^2}{L1n}$ → so if only σ_e^2 decreases, then $\tau_{U_0}^2$ increases

Univariate MLM: Variable-Centering

Model-based partitioning of level-1 y_{ti} outcome into level-specific **latent variables**

Manual partitioning of level-1 x_{ti} predictor into level-specific **observed variables**

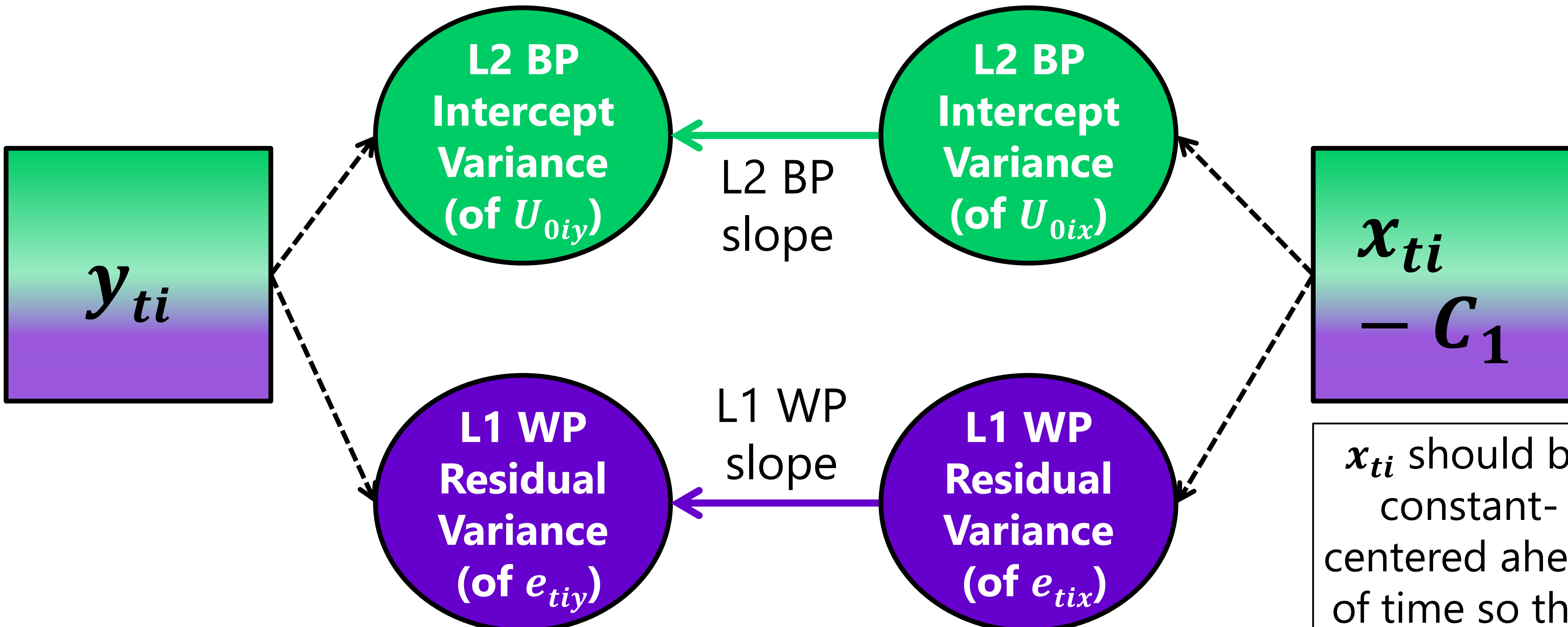


Why not let the model estimate variance components for x_{ti}, too? We can do so using multivariate MLM (via SEM or M-SEM).

Multivariate MLM: Latent-Centering

Model-based partitioning of level-1 y_{ti} outcome into level-specific **latent variables**

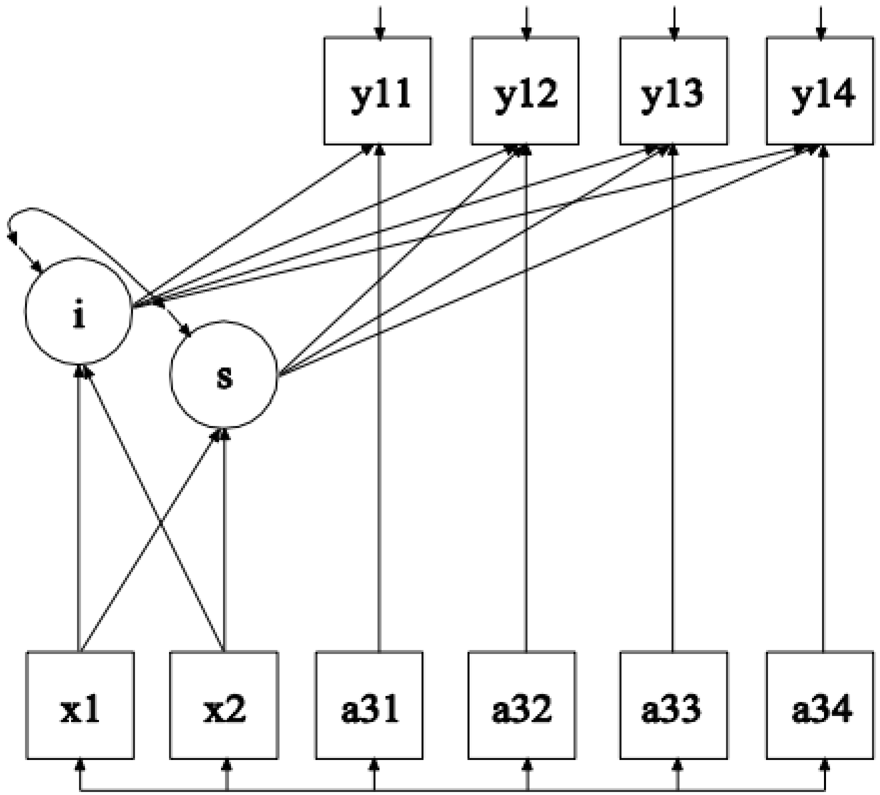
Model-based partitioning of level-1 x_{ti} predictor (= outcome now) into level-specific **latent variables**



x_{ti} should be constant-centered ahead of time so that 0 is meaningful

Univariate MLM software can be tricked into multivariate MLM if the relationships of X and Y at each level are phrased as covariances, but not if you want directed regressions (or moderators thereof)

Time-Varying Predictors in Single-Level SEM: What *Not* to Do… in Mplus



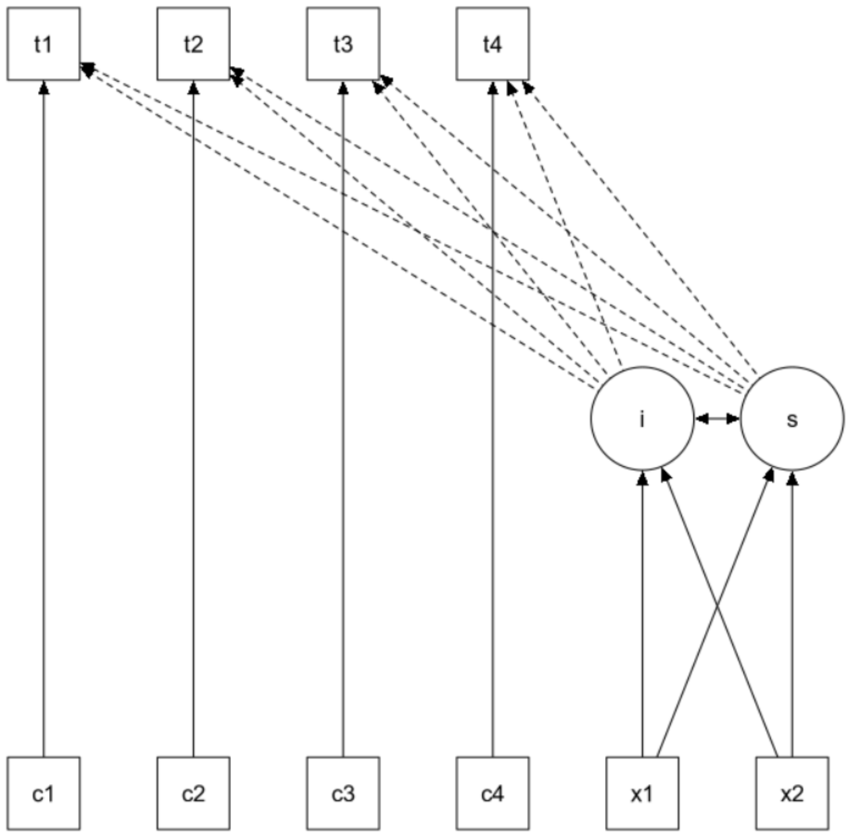

```
TITLE:     this is an example of a linear growth
           model for a continuous outcome with time-
           invariant and time-varying covariates
DATA:      FILE IS ex6.10.dat;
VARIABLE:  NAMES ARE y11-y14 x1 x2 a31-a34;
MODEL:     i s | y11@0 y12@1 y13@2 y14@3;
           i s ON x1 x2;
           y11 ON a31;
           y12 ON a32;
           y13 ON a33;
           y14 ON a34;
```

This diagram is from the (current) Mplus v. 8 Users Guide example 6.10.

Although the y11–y14 outcomes are predicted by latent intercept and time slope factors (separating two kinds of BP variance from WP variance), this is not the case for the a31–a34 TVPs.

Consequently, in the model shown here, the a→y paths will be smushed.

Time-Varying Predictors in Single-Level SEM: What *Not* to Do… in R lavaan



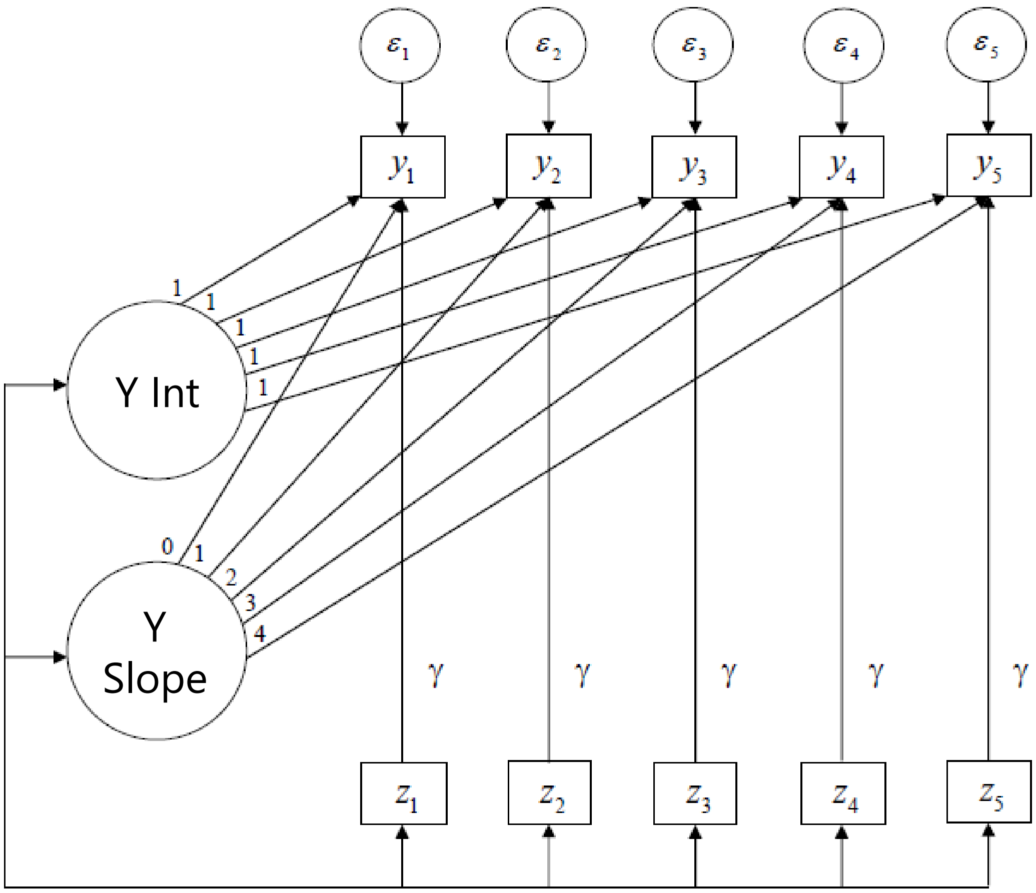

```
# a linear growth model with a time-varying covariate
model <- '
  # intercept and slope with fixed coefficients
    i =~ 1*t1 + 1*t2 + 1*t3 + 1*t4
    s =~ 0*t1 + 1*t2 + 2*t3 + 3*t4
  # regressions
    i ~ x1 + x2
    s ~ x1 + x2
  # time-varying covariates
    t1 ~ c1
    t2 ~ c2
    t3 ~ c3
    t4 ~ c4
'
fit <- growth(model, data = Demo.growth)
summary(fit)
```

This diagram is from the (current) [lavaan tutorial on growth curves]

Although the t1–t4 outcomes are predicted by latent intercept and time slope factors (separating two kinds of BP variance from WP variance), this is not the case for the c1–c4 TVPs.

Consequently, in the model shown here, the c→y paths will be smushed.

Time-Varying Predictors in Single-Level SEM: What *Should* We Do?

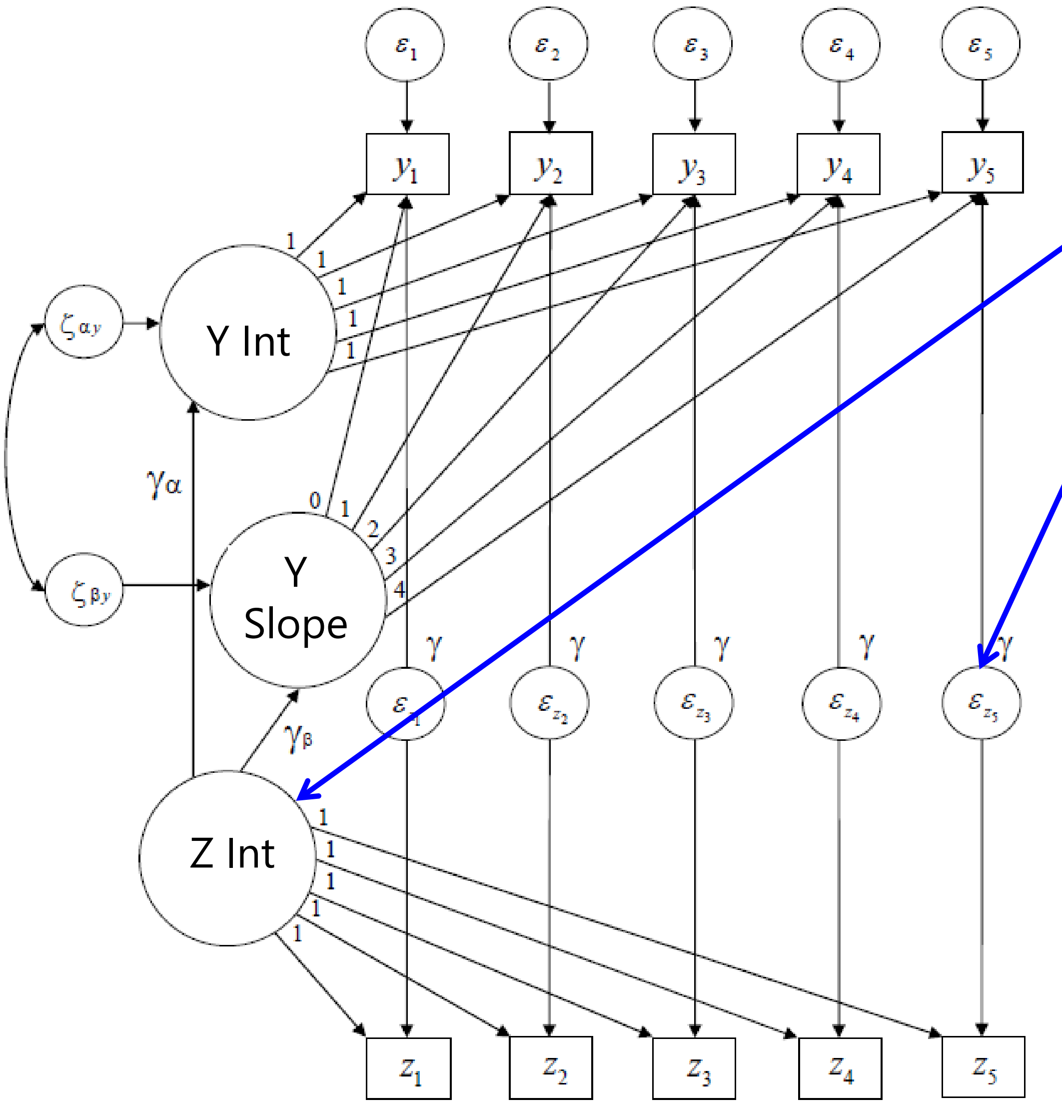


This diagram is from Curran et al. (2012). The time-varying predictors z1–z5 boxes have directed effects onto the y1–y5 outcomes at the same time.

If you constrain these paths to be equal (as γ), you get **a smushed effect** (they call it an "aggregate" effect).

IF you add covariances of the z's with the intercept, then γ becomes **the WP effect**. But the BP effect is not in here! And you cannot add PMz to get it like in MLM because it will be redundant (→ ipsative).

How to Fix Your SEM: for TV Predictors with WP Fluctuation Only (from Curran et al., 2012)



The z1–z5 time-varying predictors now have their own random intercept factor, which directly represents their level-2 BP intercept variance.

The **BP intercept effect of z→y** is given by $\boldsymbol{\gamma_\alpha}$ because of the **structured residuals**: the new ε_z latent variables to which the level-1 residual variances of z1–z5 have been moved. The **WP effect** is now given by **γ** from $\varepsilon_{z1\text{-}z5}$ → y1–y5.

If z1–z5 had predicted y1–y5 directly, the **z→y** intercept path would have held a contextual effect instead of a BP effect.

Univariate vs. Multivariate MLM (SEM or M-SEM)

- If your time-varying predictors have only BP intercept variance, their piles of variance can be reasonably approximated in univariate MLM OR by truly multivariate MLMs (via SEM or Multilevel SEM)
  - It's called "SEM" because random effects = latent variables, but there is no latent variable measurement model as in traditional SEM, which is why I don't like the term M-SEM, and prefer "(Truly) Multivariate MLM" (where "truly" to me distinguishes which software is used)

- <u>Pros of Truly Multivariate MLMs (SEM or M-SEM):</u>
  - Univariate MLM uses observed variables for variance in X, but fits a model for the variance in Y; truly multivariate MLMs fit a model for both X and Y, which makes more sense
  - Simulations suggest that the L2 fixed slopes in M-SEM are less biased (because person means are not perfectly reliable as assumed), but the L2 fixed slopes also less precise, particularly for variables with lower ICCs (little intercept info) and small level-1 *n*

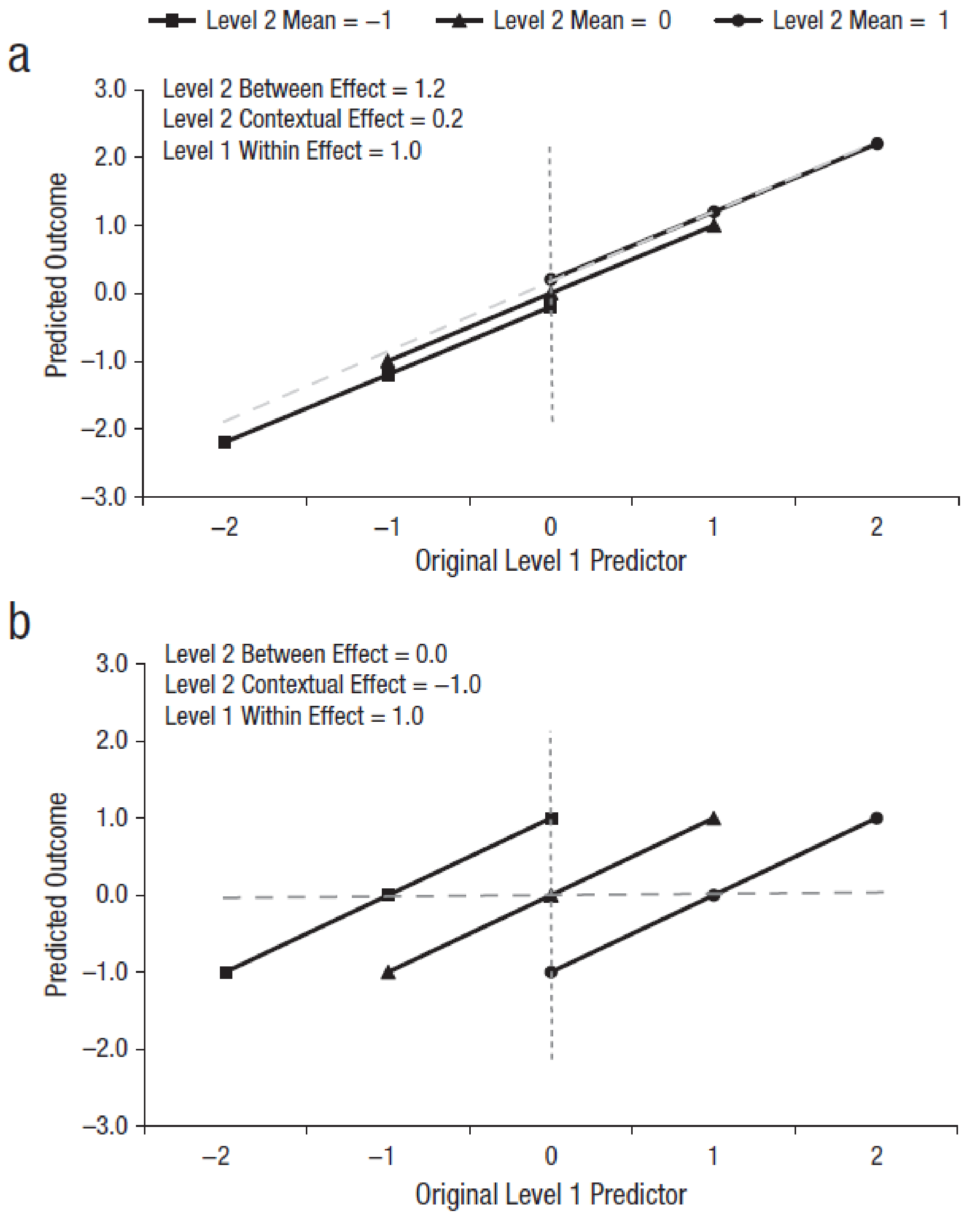
- <u>Cons of Truly Multivariate MLMs (SEM or M-SEM):</u>
  - Current software does not have REML or denominator DF → not good for small samples
  - Interactions among what used to be person means in univariate MLM instead become interactions among latent variables (random effects) in multivariate MLM (hard to estimate)
  - Whether your level-2 slopes are between or contextual varies by software used, syntax specification, and method of estimation! (see details in [Hoffman 2019, AMPPS])

Implications for Longitudinal Mediation

- Mediation is more complex in multilevel samples and only logically possible at both levels for **one combination**, as shown below
  - By mediation, I mean "M is part of the reason why X → Y" theoretically
  - Although indirect effects can always be computed, they may not make sense
  - Notation: each variable measured at Level 2 or Level 1 (= both L1+L2)

X predictor	M mediator	Y outcome	L1 mediation?	L2 mediation?
2	2	2	no	yes
2	2	1	no	yes
2	1	2	no	yes
2	1	1	no	yes
1	2	2	no	yes
1	2	1	no	yes
1	1	2	no	yes
1	**1**	**1**	**yes**	**yes**

Bonus: Between vs. Contextual Effects



- Image from Hoffman (2019), example using clustered data
- *Top:* Contextual effect is minimal—there is no added benefit to going to a high-SES school when comparing across schools *at same level of student SES*
- *Bottom:* Contextual effect is negative—at the same student SES level, relatively high students from low-SES schools do better than relatively low students from high-SES schools