Example 3: Models for Change over Time in Latent Factors using Single-Level Structural Equation Modeling (SEM)
(complete syntax and output available for Mplus electronically)

These real data (N = 653) come from the Octogenarian Twin Study of Aging in Sweden. I am analyzing three measures of cognition—block design, digit–symbol substitution, and prose recall—whose pattern of correlation is consistent with a single latent factor at each occasion. For the sake of this example, I am only using four occasions (collected at two-year intervals) and pretending these occasions are completely balanced (given that these models are more difficult to estimate for unbalanced occasions). Likewise, I am ignoring the nesting of individuals in twin pairs to use as many observations as possible. This analysis will involve three main steps: (1) verifying the factor structure across occasions as a *configural invariance* model (model 1), (2) testing *longitudinal measurement invariance* to ensure comparable meaning of the latent factor over time (models 2a–4b), and (3) examining whether higher-order factors for an intercept and latent basis change can adequately describe the pattern of means, variances, and covariances over time in the latent factor (models 5a–5b).

Model 1. Mplus Syntax for Configural Invariance—all measurement model parameters estimated separately over time, with all factor means=0 and factor variances=1 fixed for identification:

```
DATA:    FILE = OCTO.csv;  ! Data in same folder as input
         FORMAT = free; TYPE = INDIVIDUAL;  ! Defaults
VARIABLE:
! Unique ID, baseline age, block design, digit symbol, prose recall
  NAMES = case ageT0 block1-block5 digit1-digit5 prose1-prose5;
! Variables to be used in the model (first four occasions only)
  USEVARIABLES =  block1-block4 digit1-digit4 prose1-prose4;
! Missing data indicator
  MISSING ARE ALL (-999);

ANALYSIS:    TYPE = GENERAL; ESTIMATOR = MLR;  ! Robust FIML estimation
OUTPUT:      RESIDUAL MODINDICES(6.635);       ! Help troubleshoot misfit
             STDYX TECH4; ! Standardized solution and latent variable corrs
MODEL:
```

From Grimm et al. (2016), adapted for three instead of four outcomes:

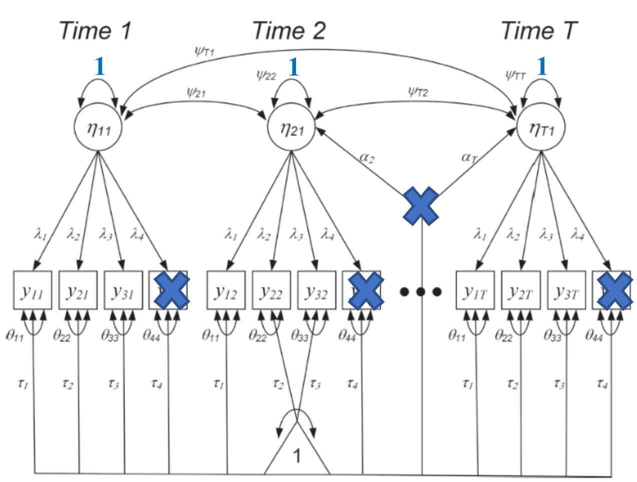


FIGURE 14.2. Path diagram of a longitudinal factor model with strict invariance.

```
!!!!!! 1. Configural Invariance Model !!!!!!
! Define latent factors (Factor = indicator loadings)
  T1 BY block1* digit1* prose1*;
  T2 BY block2* digit2* prose2*;
  T3 BY block3* digit3* prose3*;
  T4 BY block4* digit4* prose4*;

! Indicator intercepts
  [block1-block4*];
  [digit1-digit4*];
  [prose1-prose4*];

! Indicator residual variances
  block1-block4*;
  digit1-digit4*;
  prose1-prose4*;

! Same-outcome residual covariances over time
  block1-block4 WITH block1-block4*;
  digit1-digit4 WITH digit1-digit4*;
  prose1-prose4 WITH prose1-prose4*;

! Latent factor means fixed to 0 for
! identification
  [T1@0 T2@0 T3@0 T4@0];
! Latent factor variances fixed to 1 for identification
   T1@1 T2@1 T3@1 T4@1;
! Latent factor covariances (all possible pairs)
   T1 T2 T3 T4 WITH T1* T2* T3* T4*;
```

Model 1. Mplus Output for Configural Invariance:

```
Number of Free Parameters                       60 → 12 load, 12 int, 12 resvar, 18 res cov,
Loglikelihood                                         and 6 factor cov
          H0 Value                       -13135.677 → Our configural invariance model LL
          H0 Scaling Correction Factor      1.0873 → Deviation from multiv normality=1
            for MLR
          H1 Value                       -13121.771 → Saturated=best model LL
          H1 Scaling Correction Factor      1.0595 → Deviation from multiv normality=1
            for MLR

Information Criteria → Smaller is better (because they start with −2LL)
          Akaike (AIC)                   26391.355
          Bayesian (BIC)                 26660.250
          Sample-Size Adjusted BIC       26469.750
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                             27.704* → LRT for configural against saturated=best
          Degrees of Freedom                    30
          P-Value                           0.5861
          Scaling Correction Factor         1.0039
            for MLR
```

MLR estimation requires a modified LRT formula using the scaling correlation factors given above

```
*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
    for chi-square difference testing in the regular way.  MLM, MLR and WLSM
    chi-square difference testing is described on the Mplus website.  MLMV, WLSMV,
    and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation) → How much worse than saturated model=0
          Estimate                           0.000
          90 Percent C.I.             0.000  0.027
          Probability RMSEA <= .05           1.000

CFI/TLI
          CFI                                1.000 → How much better than null model=0
          TLI                                1.000

Chi-Square Test of Model Fit for the Baseline Model
          Value                           3516.779 → LRT for null vs saturated (don't need)
          Degrees of Freedom                    66
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual) → How much worse than saturated model=0
          Value                              0.010


                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
FACTOR LOADINGS → SLOPE OF FACTOR PREDICTING EACH OUTCOME
T1        BY
    BLOCK1             6.046      0.239     25.275      0.000
    DIGIT1            10.648      0.434     24.522      0.000
    PROSE1             3.272      0.147     22.209      0.000
 T2        BY
    BLOCK2             6.449      0.220     29.371      0.000
    DIGIT2            10.975      0.416     26.400      0.000
    PROSE2             3.558      0.152     23.400      0.000
 T3        BY
    BLOCK3             6.610      0.253     26.118      0.000
    DIGIT3            11.624      0.453     25.672      0.000
    PROSE3             3.866      0.177     21.809      0.000
 T4        BY
    BLOCK4             6.976      0.286     24.373      0.000
    DIGIT4            12.787      0.596     21.464      0.000
    PROSE4             4.690      0.194     24.172      0.000
```

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR COVARIANCES (= CORRELATIONS BECAUSE FACTOR VARIANCES=1)				
T1 WITH				
T2	0.952	0.014	66.221	0.000
T3	0.871	0.030	28.985	0.000
T4	0.825	0.032	25.386	0.000
T2 WITH				
T3	0.942	0.022	41.877	0.000
T4	0.911	0.022	40.934	0.000
T3 WITH				
T4	0.954	0.014	69.532	0.000
RESIDUAL COVARIANCES FOR SAME OUTCOME OVER TIME				
BLOCK1 WITH				
BLOCK2	7.565	1.274	5.940	0.000
BLOCK3	7.778	1.261	6.169	0.000
BLOCK4	5.987	1.441	4.155	0.000
BLOCK2 WITH				
BLOCK3	6.900	1.256	5.492	0.000
BLOCK4	4.118	1.287	3.200	0.001
BLOCK3 WITH				
BLOCK4	5.432	1.473	3.687	0.000
DIGIT1 WITH				
DIGIT2	9.279	3.496	2.654	0.008
DIGIT3	7.746	3.521	2.200	0.028
DIGIT4	8.503	3.979	2.137	0.033
DIGIT2 WITH				
DIGIT3	8.249	3.404	2.423	0.015
DIGIT4	8.766	3.571	2.455	0.014
DIGIT3 WITH				
DIGIT4	4.525	3.863	1.171	0.241
PROSE1 WITH				
PROSE2	5.181	0.647	8.011	0.000
PROSE3	4.403	0.708	6.218	0.000
PROSE4	3.932	0.767	5.127	0.000
PROSE2 WITH				
PROSE3	5.568	0.736	7.566	0.000
PROSE4	4.697	0.857	5.480	0.000
PROSE3 WITH				
PROSE4	5.233	0.779	6.720	0.000
FACTOR MEANS (IS "MEAN" FOR ANY VARIABLE IN THE LIKELIHOOD NOT PREDICTED)				
Means				
T1	0.000	0.000	999.000	999.000
T2	0.000	0.000	999.000	999.000
T3	0.000	0.000	999.000	999.000
T4	0.000	0.000	999.000	999.000
OUTCOME INTERCEPTS (EXPECTED OUTCOME WHEN FACTOR PREDICTOR=0)				
Intercepts				
BLOCK1	10.173	0.302	33.647	0.000
BLOCK2	9.564	0.311	30.723	0.000
BLOCK3	8.752	0.321	27.305	0.000
BLOCK4	7.519	0.364	20.653	0.000
DIGIT1	21.039	0.511	41.135	0.000
DIGIT2	19.923	0.526	37.908	0.000
DIGIT3	18.714	0.573	32.682	0.000
DIGIT4	15.602	0.710	21.974	0.000
PROSE1	8.503	0.187	45.513	0.000
PROSE2	8.097	0.211	38.412	0.000
PROSE3	7.274	0.239	30.412	0.000
PROSE4	6.521	0.289	22.582	0.000
FACTOR VARIANCES (IS "VARIANCE" FOR ANY VARIABLE IN THE LIKELIHOOD NOT PREDICTED)				
Variances				
T1	1.000	0.000	999.000	999.000
T2	1.000	0.000	999.000	999.000
T3	1.000	0.000	999.000	999.000
T4	1.000	0.000	999.000	999.000

```
OUTCOME LEFTOVER VARIANCES (IS "RESIDUAL VARIANCE" FOR ANY PREDICTED OUTCOME)
Residual Variances
    BLOCK1            19.334      1.707     11.329      0.000
    BLOCK2            14.178      1.456      9.736      0.000
    BLOCK3            12.465      1.739      7.168      0.000
    BLOCK4            12.533      1.807      6.935      0.000
    DIGIT1            32.716      4.583      7.138      0.000
    DIGIT2            24.595      3.834      6.414      0.000
    DIGIT3            24.554      4.088      6.006      0.000
    DIGIT4            24.878      4.918      5.058      0.000
    PROSE1             9.981      0.680     14.686      0.000
    PROSE2            10.664      0.774     13.778      0.000
    PROSE3             9.803      1.017      9.643      0.000
    PROSE4             7.431      0.960      7.739      0.000
```

Given the excellent fit of this model, it appears that the outcome means, variances, and covariances are well recreated by the four correlated factors (one for each occasion), along with residual covariances for the same outcome over time. Next, we examine **longitudinal measurement invariance** for each parameter separately: **loadings** (called **metric** or weak), **intercepts** (called **scalar** or strong), and **residual variances** (called **residual** or strict). To compare each layer of constraints as nested models, we will use **rescaled likelihood ratio tests**, which is the −2ΔLL accounting for the scaling correction factors. At each layer, we will hope that global model fit is **not significantly worse** from enforcing the invariance constraints, and we will also examine modification indices to see if any specific parameters want to be noninvariant (different) over time (as local fit). For more explanation and examples of testing invariance, please see Lecture 7 and Examples 7a–7d from my SEM class.

Model 2a. Mplus Syntax for Full Metric Invariance—Model 1 except the factor loadings for the same outcome are now constrained equal over time, and the factor variance =1 at T1 for identification but is free at T2–T4:

```
MODEL:  ! DATA, VARIABLE, ANALYSIS, OUTPUT are same

!!!!!! 2a. Full Metric Invariance Model !!!!!!
! Define latent factors (Factor = indicator loadings)
  T1 BY block1* digit1* prose1* (BL DL PL);
  T2 BY block2* digit2* prose2* (BL DL PL);
  T3 BY block3* digit3* prose3* (BL DL PL);
  T4 BY block4* digit4* prose4* (BL DL PL);

! Indicator intercepts
  [block1-block4*];
  [digit1-digit4*];
  [prose1-prose4*];

! Indicator residual variances
  block1-block4*;
  digit1-digit4*;
  prose1-prose4*;

! Same-outcome residual covariances over time
  block1-block4 WITH block1-block4*;
  digit1-digit4 WITH digit1-digit4*;
  prose1-prose4 WITH prose1-prose4*;

! Latent factor means fixed to 0 for
! identification
  [T1@0 T2@0 T3@0 T4@0];
! Latent factor variance=1 at T1 for identification, free otherwise
   T1@1 T2* T3* T4*;
! Latent factor covariances (all possible pairs)
   T1 T2 T3 T4 WITH T1* T2* T3* T4*;
```

From Grimm et al. (2016), adapted for three instead of four outcomes:

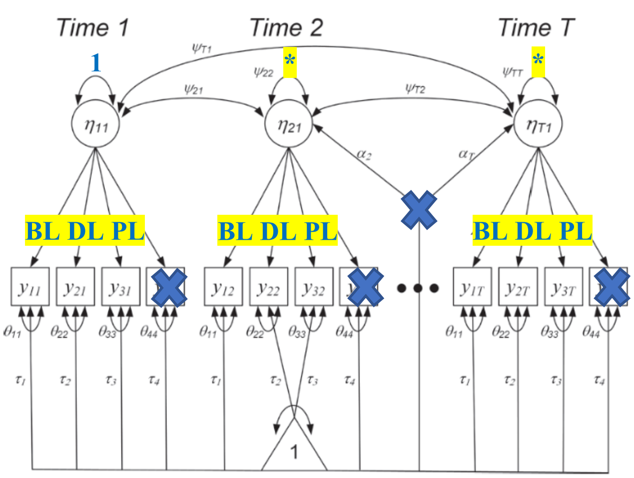


FIGURE 14.2. Path diagram of a longitudinal factor model with strict invariance.

Model 2a. Mplus Output for Full Metric Invariance:

```
MODEL FIT INFORMATION
Number of Free Parameters                        54 → Saved DF=6 (12load vs. 3load + 3FactVar)
Loglikelihood
          H0 Value                        -13141.701 → Our metric invariance model LL
          H0 Scaling Correction Factor        1.1194
            for MLR
          H1 Value                        -13121.771 → Saturated=best model LL
          H1 Scaling Correction Factor        1.0595
            for MLR

Information Criteria
          Akaike (AIC)                     26391.403
          Bayesian (BIC)                   26633.408
          Sample-Size Adjusted BIC         26461.958
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                               41.112*
          Degrees of Freedom                      36
          P-Value                             0.2566
          Scaling Correction Factor           0.9696
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                             0.015
          90 Percent C.I.                0.000  0.033
          Probability RMSEA <= .05             1.000

CFI/TLI
          CFI                                  0.999
          TLI                                  0.997

SRMR (Standardized Root Mean Square Residual)
          Value                                0.028
```

Does the full metric invariance model (2a) fit *worse* than the configural model (1)?
Yes, $-2\Delta LL(df=6) = 15.09$, $p = .0196$

In examining why the constrained model fits worse, modification indices (below) suggest the loading of prose wants to be greater at T4, so we can free that loading to create a **partial metric** invariance model to move forward.*

```
MODEL MODIFICATION INDICES (truncated)
                     M.I.     E.P.C.
BY Statements
T2   BY PROSE4       7.372     0.510
T3   BY PROSE4       7.879     0.506
T4   BY PROSE4       7.285     0.348
```

If we freed the factor loading at T4, the rescaled $-2\Delta LL$ should improve by 7.285, and the T4 loading should be greater by 0.348.

```
MODEL RESULTS (RELEVANT PARAMETERS ONLY)
                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME
 T1       BY
    BLOCK1             5.917      0.215     27.569      0.000 = BL
    DIGIT1            10.484      0.388     27.047      0.000 = DL
    PROSE1             3.455      0.121     28.641      0.000 = PL
 T2       BY
    BLOCK2             5.917      0.215     27.569      0.000 = BL
    DIGIT2            10.484      0.388     27.047      0.000 = DL
    PROSE2             3.455      0.121     28.641      0.000 = PL
 T3       BY
    BLOCK3             5.917      0.215     27.569      0.000 = BL
    DIGIT3            10.484      0.388     27.047      0.000 = DL
    PROSE3             3.455      0.121     28.641      0.000 = PL
 T4       BY
    BLOCK4             5.917      0.215     27.569      0.000 = BL
    DIGIT4            10.484      0.388     27.047      0.000 = DL
    PROSE4             3.455      0.121     28.641      0.000 = PL

FACTOR VARIANCES FREE AFTER T1 → INCREASING VARIABILITY OVER TIME
 Variances
    T1                 1.000      0.000    999.000    999.000
    T2                 1.124      0.055     20.307      0.000
    T3                 1.233      0.072     17.149      0.000
    T4                 1.522      0.108     14.053      0.000
```

* Note: Although one could argue that the metric model is "good enough" based on its absolute fit, I wanted to show an example of how to trouble-shoot sources of noninvariance and create partial invariance models.

Model 2b. Mplus Syntax for Partial Metric Invariance—Model 2a except the factor loading for prose at T4 is now allowed to differ from its factor loadings at T1–T3:

```
MODEL:  ! DATA, VARIABLE, ANALYSIS, OUTPUT are same

!!!!!! 2b. Patrial Metric Invariance Model !!!!!!
! Define latent factors (Factor = indicator loadings)
  T1 BY block1* digit1* prose1* (BL DL PL);
  T2 BY block2* digit2* prose2* (BL DL PL);
  T3 BY block3* digit3* prose3* (BL DL PL);
  T4 BY block4* digit4* prose4* (BL DL PL4);

! Indicator intercepts
  [block1-block4*];
  [digit1-digit4*];
  [prose1-prose4*];

! Indicator residual variances
  block1-block4*;
  digit1-digit4*;
  prose1-prose4*;

! Same-outcome residual covariances over time
  block1-block4 WITH block1-block4*;
  digit1-digit4 WITH digit1-digit4*;
  prose1-prose4 WITH prose1-prose4*;

! Latent factor means fixed to 0 for
! identification
  [T1@0 T2@0 T3@0 T4@0];
! Latent factor variance=1 at T1 for identification, free otherwise
   T1@1 T2* T3* T4*;
! Latent factor covariances (all possible pairs)
   T1 T2 T3 T4 WITH T1* T2* T3* T4*;
```

From Grimm et al. (2016), adapted for three instead of four outcomes:

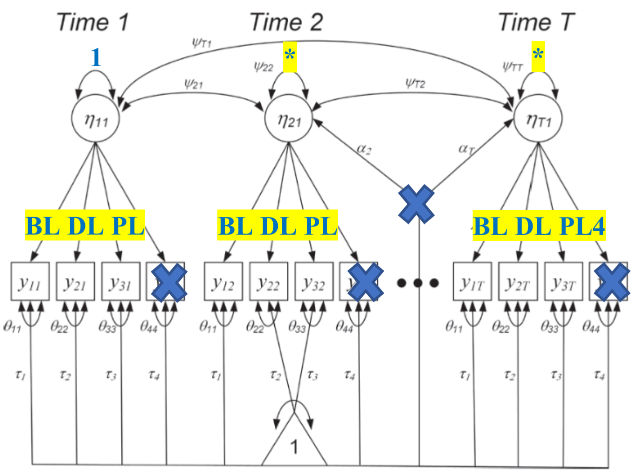


FIGURE 14.2. Path diagram of a longitudinal factor model with strict invariance.

Model 2b. Mplus Output for Partial Metric Invariance:

```
Number of Free Parameters                         55
Loglikelihood
          H0 Value                        -13137.301
          H0 Scaling Correction Factor        1.1146
            for MLR
          H1 Value                        -13121.771
          H1 Scaling Correction Factor        1.0595
            for MLR
Information Criteria
          Akaike (AIC)                     26384.603
          Bayesian (BIC)                   26631.089
          Sample-Size Adjusted BIC         26456.465
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                               31.925*
          Degrees of Freedom                      35
          P-Value                             0.6173
          Scaling Correction Factor           0.9729
            for MLR
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                             0.000
          90 Percent C.I.                0.000  0.025
          Probability RMSEA <= .05             1.000
CFI/TLI
          CFI                                  1.000
          TLI                                  1.000
SRMR (Standardized Root Mean Square Residual)
          Value                                0.017
```

Does the partial metric invariance model (2b) still fit *worse* than the configural model (1)?
No, $-2\Delta LL(df=\mathbf{5}) = 4.127$, $p = .5313$

This means that differences in the factor variances over time were sufficiently responsible for the prior differences in the factor loadings over time. In other words, outcomes are related to the latent factor equivalently across time.

Now we can move forward to test equality of the outcome intercepts (scalar).

```
MODEL RESULTS (RELEVANT PARAMETERS ONLY)
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME EXCEPT PROSE4
 T1        BY
    BLOCK1             5.987      0.214     28.027      0.000
    DIGIT1            10.553      0.387     27.288      0.000
    PROSE1             3.361      0.126     26.618      0.000
 T2        BY
    BLOCK2             5.987      0.214     28.027      0.000
    DIGIT2            10.553      0.387     27.288      0.000
    PROSE2             3.361      0.126     26.618      0.000
 T3        BY
    BLOCK3             5.987      0.214     28.027      0.000
    DIGIT3            10.553      0.387     27.288      0.000
    PROSE3             3.361      0.126     26.618      0.000
 T4        BY
    BLOCK4             5.987      0.214     28.027      0.000
    DIGIT4            10.553      0.387     27.288      0.000
    PROSE4             3.915      0.194     20.158      0.000 = PL4 at T4 is > T1,T2,T3
 Variances
    T1                 1.000      0.000    999.000    999.000
    T2                 1.119      0.055     20.486      0.000
    T3                 1.231      0.071     17.345      0.000
    T4                 1.410      0.107     13.228      0.000
```

Model 3a. Mplus Syntax for Full Scalar Invariance—Model 2b except the intercepts for the same outcome are constrained equal (including prose4, given how few outcomes there are per factor), and the factor mean = 0 at T1 for identification but is free at T2–T4:

```
MODEL:  ! DATA, VARIABLE, ANALYSIS, OUTPUT are same

!!!!!! 3a. Full Scalar Invariance Model !!!!!!
! Define latent factors (Factor = indicator loadings)
  T1 BY block1* digit1* prose1* (BL DL PL);
  T2 BY block2* digit2* prose2* (BL DL PL);
  T3 BY block3* digit3* prose3* (BL DL PL);
  T4 BY block4* digit4* prose4* (BL DL PL4);

! Indicator intercepts
  [block1-block4*]  (BI);
  [digit1-digit4*]  (DI);
  [prose1-prose4*]  (PI);

! Indicator residual variances
  block1-block4*;
  digit1-digit4*;
  prose1-prose4*;

! Same-outcome residual covariances over time
  block1-block4 WITH block1-block4*;
  digit1-digit4 WITH digit1-digit4*;
  prose1-prose4 WITH prose1-prose4*;

! Latent factor mean=0 at T1 for
! identification, free otherwise
  [T1@0 T2* T3* T4*];
! Latent factor variance=1 at T1 for identification, free otherwise
   T1@1 T2* T3* T4*;
! Latent factor covariances (all possible pairs)
   T1 T2 T3 T4 WITH T1* T2* T3* T4*;
```

From Grimm et al. (2016), adapted for three instead of four outcomes:

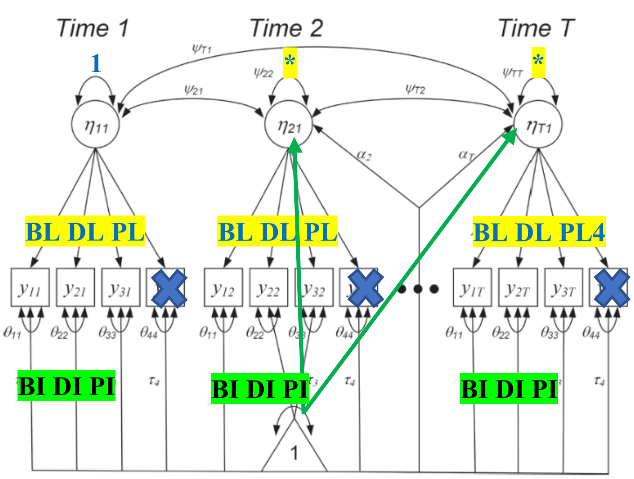


FIGURE 14.2. Path diagram of a longitudinal factor model with strict invariance.

Model 3a. Mplus Output for Full Scalar Invariance:

```
Number of Free Parameters                        49 → Saved DF=6 (12int vs. 3int + 3FactMean)
Loglikelihood
          H0 Value                        -13140.311
          H0 Scaling Correction Factor        1.1311
            for MLR
          H1 Value                        -13121.771
          H1 Scaling Correction Factor        1.0595
            for MLR
Information Criteria
          Akaike (AIC)                     26378.621
          Bayesian (BIC)                   26598.219
          Sample-Size Adjusted BIC         26442.644
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                               38.075*
          Degrees of Freedom                      41
          P-Value                             0.6014
          Scaling Correction Factor           0.9739
            for MLR
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                             0.000
          90 Percent C.I.              0.000   0.024
          Probability RMSEA <= .05             1.000
CFI/TLI
          CFI                                  1.000
          TLI                                  1.000
SRMR (Standardized Root Mean Square Residual)
          Value                                0.020
```

Does the full scalar model (3a) fit *worse* than the partial metric model (2a)?
No, −2ΔLL(df=**6**) = 6.144, p = .4073

This means that differences in the factor means over time were sufficiently responsible for the differences in the outcome means (now intercepts) over time.

Now we can move forward to test equality of the outcome residual variances.

```
MODEL RESULTS (RELEVANT PARAMETERS ONLY)
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
FACTOR MEANS SHOW DECLINE OVER TIME
  Means
    T1                 0.000      0.000    999.000    999.000
    T2                -0.110      0.027     -4.030      0.000
    T3                -0.255      0.037     -6.936      0.000
    T4                -0.479      0.049     -9.741      0.000

OUTCOME INTERCEPTS NOW EQUAL FOR SAME OUTCOME OVER TIME
 Intercepts
    BLOCK1            10.232      0.285     35.949      0.000 = BI
    BLOCK2            10.232      0.285     35.949      0.000
    BLOCK3            10.232      0.285     35.949      0.000
    BLOCK4            10.232      0.285     35.949      0.000
    DIGIT1            21.067      0.480     43.919      0.000 = DI
    DIGIT2            21.067      0.480     43.919      0.000
    DIGIT3            21.067      0.480     43.919      0.000
    DIGIT4            21.067      0.480     43.919      0.000
    PROSE1             8.422      0.176     47.835      0.000 = PI
    PROSE2             8.422      0.176     47.835      0.000
    PROSE3             8.422      0.176     47.835      0.000
    PROSE4             8.422      0.176     47.835      0.000
```

Model 4a. Mplus Syntax for Full Residual Variance Invariance—Model 3a except the residual variances for the same outcome are constrained equal over time (including prose4 to start with):

```
MODEL:  ! DATA, VARIABLE, ANALYSIS, OUTPUT are same

!!!!!! 4a. Full Residual Variance Invariance Model !!!!!!
! Define latent factors (Factor = indicator loadings)
  T1 BY block1* digit1* prose1* (BL DL PL);
  T2 BY block2* digit2* prose2* (BL DL PL);
  T3 BY block3* digit3* prose3* (BL DL PL);
  T4 BY block4* digit4* prose4* (BL DL PL4);

! Indicator intercepts
  [block1-block4*] (BI);
  [digit1-digit4*] (DI);
  [prose1-prose4*] (PI);

! Indicator residual variances
  block1-block4* (BR);
  digit1-digit4* (DR);
  prose1-prose4* (PR);

! Same-outcome residual covariances over time
  block1-block4 WITH block1-block4*;
  digit1-digit4 WITH digit1-digit4*;
  prose1-prose4 WITH prose1-prose4*;

! Latent factor mean=0 at T1 for
! identification, free otherwise
  [T1@0 T2* T3* T4*];
! Latent factor variance=1 at T1 for identification, free otherwise
   T1@1 T2* T3* T4*;
! Latent factor covariances (all possible pairs)
   T1 T2 T3 T4 WITH T1* T2* T3* T4*;
```

From Grimm et al. (2016), adapted for three instead of four outcomes:

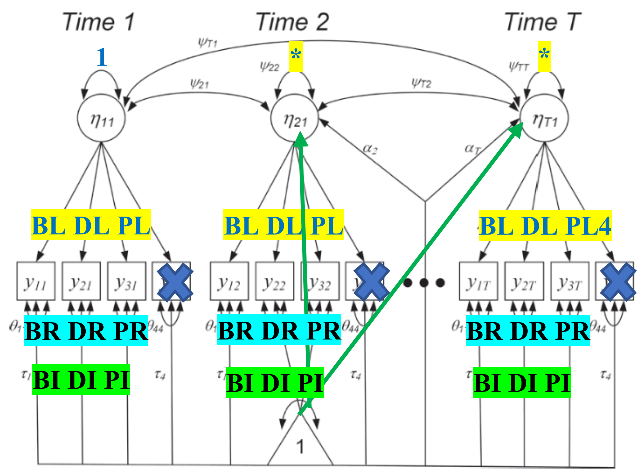


FIGURE 14.2. Path diagram of a longitudinal factor model with strict invariance.

Model 4a. Mplus Output for Full Residual Variance Invariance:

```
Number of Free Parameters                       40 → Saved DF=9 (12resvar vs. 3resvar)
Loglikelihood
          H0 Value                      -13157.694
          H0 Scaling Correction Factor      1.1780
            for MLR
          H1 Value                      -13121.771
          H1 Scaling Correction Factor      1.0595
            for MLR
Information Criteria
          Akaike (AIC)                   26395.388
          Bayesian (BIC)                 26574.651
          Sample-Size Adjusted BIC       26447.651
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                            74.477*
          Degrees of Freedom                    50
          P-Value                           0.0140
          Scaling Correction Factor         0.9647
            for MLR
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.027
          90 Percent C.I.             0.013  0.040
          Probability RMSEA <= .05           0.999
CFI/TLI
          CFI                                0.993
          TLI                                0.991
SRMR (Standardized Root Mean Square Residual)
          Value                              0.032
```

Does the full residual variance model (4a) fit *worse* than the full scalar model (3a)?
Yes, $-2\Delta LL(df=\mathbf{9}) = 37.680$, $p < .0001$

```
MODEL MODIFICATION INDICES (truncated)
                M.I.     E.P.C.
Variances/Residual Variances
BLOCK1        21.897      4.079
DIGIT1        10.763      7.267
```

If we freed the block residual variance at T1, the rescaled $-2\Delta LL$ would improve by 21.897, and the residual variance should be greater by 4.079. To save a step, I will free both of these residual variances at once.

```
MODEL RESULTS (RELEVANT PARAMETERS ONLY)
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
RESIDUAL VARIANCES = AMOUNT OF "NOT THE FACTOR" VARIANCE EQUAL OVER TIME
    BLOCK1            15.848      1.193     13.282      0.000 = BR
    BLOCK2            15.848      1.193     13.282      0.000
    BLOCK3            15.848      1.193     13.282      0.000
    BLOCK4            15.848      1.193     13.282      0.000
    DIGIT1            26.480      3.211      8.246      0.000 = DR
    DIGIT2            26.480      3.211      8.246      0.000
    DIGIT3            26.480      3.211      8.246      0.000
    DIGIT4            26.480      3.211      8.246      0.000
    PROSE1            10.032      0.538     18.661      0.000 = PR
    PROSE2            10.032      0.538     18.661      0.000
    PROSE3            10.032      0.538     18.661      0.000
    PROSE4            10.032      0.538     18.661      0.000
```

Model 4b. Mplus Syntax for Partial Residual Variance Invariance—Model 4a except the residual variances for block and digit at T1 can differ from those at T2–T4:

```
MODEL:  ! DATA, VARIABLE, ANALYSIS, OUTPUT are same

!!!!!! 4b. Partial Residual Variance Invariance Model !!!!!!
! Define latent factors (Factor = indicator loadings)
  T1 BY block1* digit1* prose1* (BL DL PL);
  T2 BY block2* digit2* prose2* (BL DL PL);
  T3 BY block3* digit3* prose3* (BL DL PL);
  T4 BY block4* digit4* prose4* (BL DL PL4);

! Indicator intercepts
  [block1-block4*] (BI);
  [digit1-digit4*] (DI);
  [prose1-prose4*] (PI);

! Indicator residual variances
  block1* (BR1); block2-block4* (BR);
  digit1* (DR1); digit2-digit4* (DR);
  prose1-prose4* (PR);

! Same-outcome residual covariances over
time
  block1-block4 WITH block1-block4*;
  digit1-digit4 WITH digit1-digit4*;
  prose1-prose4 WITH prose1-prose4*;

! Latent factor mean=0 at T1 for
! identification, free otherwise
  [T1@0 T2* T3* T4*];
! Latent factor variance=1 at T1 for identification, free otherwise
   T1@1 T2* T3* T4*;
! Latent factor covariances (all possible pairs)
   T1 T2 T3 T4 WITH T1* T2* T3* T4*;
```

From Grimm et al. (2016), adapted for three instead of four outcomes:

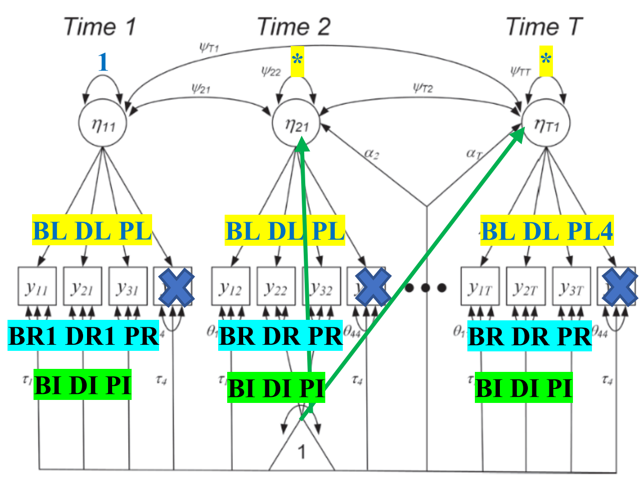


FIGURE 14.2. Path diagram of a longitudinal factor model with strict invariance.

Model 4b. Mplus Output for Partial Residual Variance Invariance:

```
Number of Free Parameters                      42 → Saved DF=7 (12resvar vs. 3+2resvar)
Loglikelihood
          H0 Value                      -13144.753
          H0 Scaling Correction Factor      1.1650
            for MLR
          H1 Value                      -13121.771
          H1 Scaling Correction Factor      1.0595
            for MLR
Information Criteria
          Akaike (AIC)                   26373.506
          Bayesian (BIC)                 26561.732
          Sample-Size Adjusted BIC       26428.382
```

```
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                              47.525*
          Degrees of Freedom                      48
          P-Value                             0.4922
          Scaling Correction Factor           0.9672
            for MLR
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                             0.000
          90 Percent C.I.               0.000  0.025
          Probability RMSEA <= .05             1.000
CFI/TLI
          CFI                                  1.000
          TLI                                  1.000
Chi-Square Test of Model Fit for the Baseline Model
          Value                             3516.779
          Degrees of Freedom                      66
          P-Value                             0.0000
SRMR (Standardized Root Mean Square Residual)
          Value                                0.025
```

Does the partial residual variance model (4b) still fit *worse* than the full scalar model (3a)?
No, −2ΔLL(df=7) = 9.576, *p* = .2139

This will be our new baseline moving forward with respect to the structural model, which is saturated here (all possible means, variances, and covariances are estimated except where constrained for identification).

But we will need to change the method of identification for our change model so that all the lower-order factor variances can be estimated instead…

```
MODEL RESULTS
                                                        Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
FACTOR LOADINGS EQUAL FOR SAME OUTCOME OVER TIME EXCEPT PROSE4
T1       BY
    BLOCK1             5.972      0.215     27.823      0.000 = BL → TO BE USED NEXT
    DIGIT1            10.579      0.385     27.475      0.000 = DL
    PROSE1             3.371      0.125     26.973      0.000 = PL
 T2       BY
    BLOCK2             5.972      0.215     27.823      0.000 = BL
    DIGIT2            10.579      0.385     27.475      0.000 = DL
    PROSE2             3.371      0.125     26.973      0.000 = PL
 T3       BY
    BLOCK3             5.972      0.215     27.823      0.000 = BL
    DIGIT3            10.579      0.385     27.475      0.000 = DL
    PROSE3             3.371      0.125     26.973      0.000 = PL
 T4       BY
    BLOCK4             5.972      0.215     27.823      0.000 = BL
    DIGIT4            10.579      0.385     27.475      0.000 = DL
    PROSE4             3.911      0.195     20.103      0.000 = PL4
FACTOR COVARIANCES ALLOWED TO DIFFER OVER TIME (NOT CORRELATIONS HERE)
T1       WITH
    T2                 1.009      0.028     36.545      0.000
    T3                 0.966      0.042     23.020      0.000
    T4                 0.983      0.052     19.024      0.000
 T2       WITH
    T3                 1.109      0.059     18.727      0.000
    T4                 1.150      0.067     17.099      0.000
 T3       WITH
    T4                 1.263      0.077     16.482      0.000
RESIDUAL COVARIANCES FOR SAME OUTCOME OVER TIME (FREELY ESTIMATED)
 BLOCK1   WITH
    BLOCK2             7.453      1.193      6.247      0.000
    BLOCK3             8.263      1.248      6.620      0.000
    BLOCK4             6.584      1.448      4.548      0.000
 BLOCK2   WITH
    BLOCK3             7.159      1.198      5.978      0.000
    BLOCK4             4.482      1.319      3.398      0.001
 BLOCK3   WITH
    BLOCK4             6.331      1.359      4.658      0.000
 DIGIT1   WITH
    DIGIT2             8.909      3.339      2.668      0.008
    DIGIT3             7.459      3.531      2.113      0.035
    DIGIT4             7.823      3.728      2.099      0.036
 DIGIT2   WITH
    DIGIT3             7.398      3.483      2.124      0.034
    DIGIT4             7.779      3.446      2.257      0.024
 DIGIT3   WITH
```

```
    DIGIT4              2.729       3.671       0.743       0.457
 PROSE1   WITH
    PROSE2              4.916       0.619       7.944       0.000
    PROSE3              4.368       0.681       6.418       0.000
    PROSE4              4.717       0.848       5.560       0.000
 PROSE2   WITH
    PROSE3              5.261       0.622       8.461       0.000
    PROSE4              5.325       0.853       6.240       0.000
 PROSE3   WITH
    PROSE4              6.301       0.680       9.261       0.000
FACTOR MEANS SHOW INCREASING DECLINE OVER TIME
 Means
    T1                  0.000       0.000     999.000     999.000
    T2                 -0.110       0.027      -4.032       0.000
    T3                 -0.256       0.037      -6.944       0.000 → Δ T2 = -.146
    T4                 -0.484       0.049      -9.791       0.000 → Δ T3 = -.228
INTERCEPTS FOR SAME OUTCOME HELD EQUAL OVER TIME (SO CHANGE IS DUE TO FACTORS ONLY!)
 Intercepts
    BLOCK1             10.238       0.284      35.996       0.000 = BI
    BLOCK2             10.238       0.284      35.996       0.000
    BLOCK3             10.238       0.284      35.996       0.000
    BLOCK4             10.238       0.284      35.996       0.000
    DIGIT1             21.086       0.481      43.876       0.000 = DI
    DIGIT2             21.086       0.481      43.876       0.000
    DIGIT3             21.086       0.481      43.876       0.000
    DIGIT4             21.086       0.481      43.876       0.000
    PROSE1              8.423       0.176      47.934       0.000 = PI
    PROSE2              8.423       0.176      47.934       0.000
    PROSE3              8.423       0.176      47.934       0.000
    PROSE4              8.423       0.176      47.934       0.000
FACTOR VARIANCES SHOW INCREASING VARIABILITY OVER TIME
 Variances
    T1                  1.000       0.000     999.000     999.000
    T2                  1.126       0.054      20.887       0.000
    T3                  1.231       0.070      17.630       0.000
    T4                  1.415       0.105      13.534       0.000
RESIDUAL VARIANCES = AMOUNT OF "NOT THE FACTOR" VARIANCE EQUAL EXCEPT BLOCK1 AND DIGIT1
    BLOCK1             19.552       1.624      12.041       0.000 = BR1
    BLOCK2             13.573       1.220      11.127       0.000 = BR
    BLOCK3             13.573       1.220      11.127       0.000 = BR
    BLOCK4             13.573       1.220      11.127       0.000 = BR
    DIGIT1             32.968       4.390       7.510       0.000 = DR1
    DIGIT2             23.577       3.147       7.492       0.000 = DR
    DIGIT3             23.577       3.147       7.492       0.000 = DR
    DIGIT4             23.577       3.147       7.492       0.000 = DR
    PROSE1              9.918       0.542      18.283       0.000 = PR
    PROSE2              9.918       0.542      18.283       0.000 = PR
    PROSE3              9.918       0.542      18.283       0.000 = PR
    PROSE4              9.918       0.542      18.283       0.000 = PR
```

Model 5a. Mplus Syntax for Latent Basis Change Model—Keeping non-invariant parameters from prior measurement models, but using a "marker item" identification method for the factor variances:

```
MODEL:  ! DATA, VARIABLE, ANALYSIS, OUTPUT are same

!!!!!! 5a. Latent Basis Change Model !!!!!!
! Define latent factors (Factor = indicator loadings)
! Factor loadings held equal over time except prose4
  T1 BY block1@5.972; T1 BY digit1* prose1* (DL PL);
  T2 BY block2@5.972; T2 BY digit2* prose2* (DL PL);
  T3 BY block3@5.972; T3 BY digit3* prose3* (DL PL);
  T4 BY block4@5.972; T4 BY digit4* prose4* (DL PL4);

! Indicator intercepts all held equal over time
  [block1-block4*] (BI);
  [digit1-digit4*] (DI);
  [prose1-prose4*] (PI);
```

Because our time-specific factor variances need to be free to become leftover (= "disturbances"), we need to change our model identification to use a "marker item" whose factor loading is fixed (and still equal over time). Rather than fixing that loading to 1, we are fixing it to the value corresponding to the previous T1 factor (with mean=0 and variance=1), that way the total SD ~= 1 for the T1 factor.

```
! Indicator residual variances held equal over time
! except block1 and digit1
  block1* (BR1); block2-block4* (BR);
  digit1* (DR1); digit2-digit4* (DR);
  prose1-prose4* (PR);

! Same-outcome residual covariances over time
  block1-block4 WITH block1-block4*;
  digit1-digit4 WITH digit1-digit4*;
  prose1-prose4 WITH prose1-prose4*;

! Latent factor mean=0 at all occasions so that all mean change
! is captured by the intercept and slope factors' fixed effects
  [T1@0 T2@0 T3@0 T4@0];
! Latent factor variance held equal over time (like diagonal R matrix)
! so all heterogeneity of variance is captured by slope factor variance
   T1* T2* T3* T4* (ResVar);
! Latent factor covariances (all possible pairs) SHUT OFF @0 so that
! all covariance over time is captured by intercept and slope factor variances
   T1 T2 T3 T4 WITH T1@0 T2@0 T3@0 T4@0;
```

```
! Define new higher-order intercept and latent basis change factors
  Int BY T1@1 T2@1 T3@1 T4@1;
  Slp BY T1@0 T2*  T3*  T4@1;
! Higher-order factor means = fixed effects
  [Int@0 Slp*]; ! Fixed int = 0 for identification
! Higher-order factor variances = random effect variances
  Int* Slp*;
! Higher-order factor covariance = random effects covariance
  Int WITH Slp*;
```

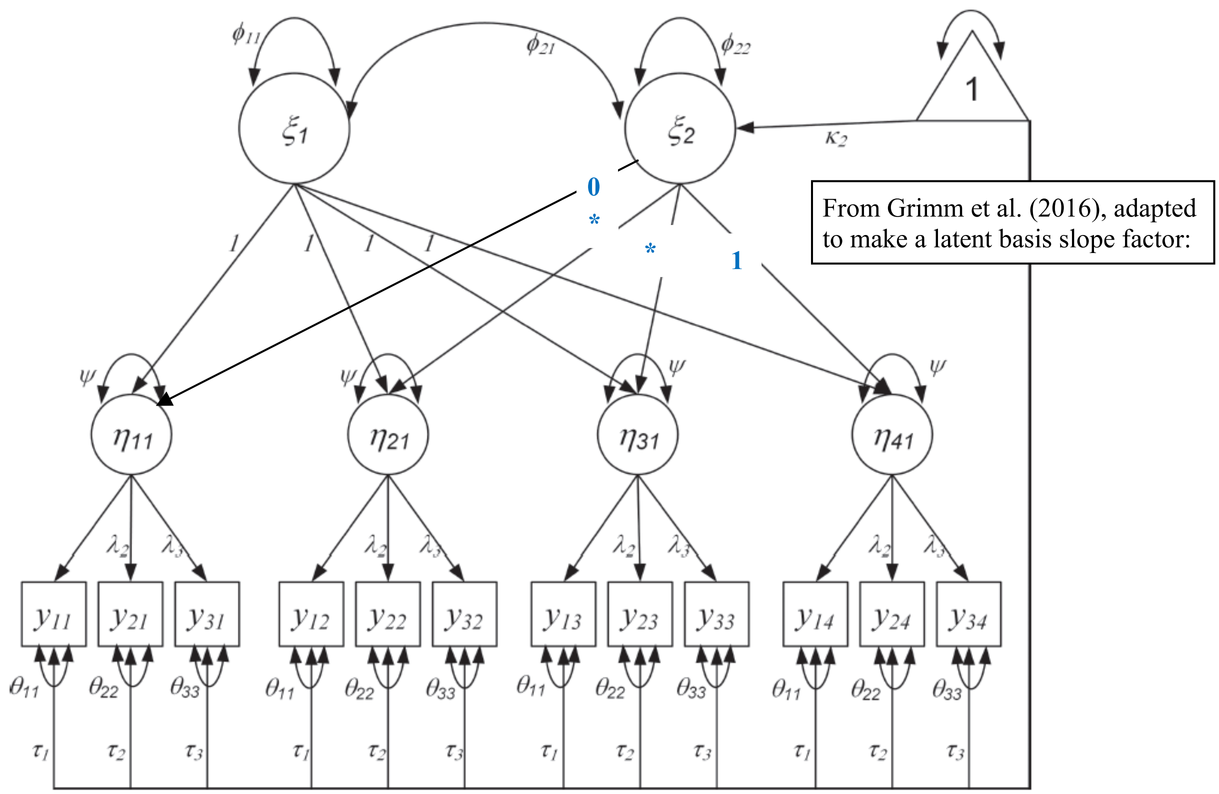


FIGURE 14.3. Path diagram of a second-order growth model.

Model 5a. Mplus Output for Latent Basis Change Model:

```
Number of Free Parameters                        36 → Saved DF=6
Loglikelihood
          H0 Value                        -13151.623
          H0 Scaling Correction Factor        1.1915
            for MLR
          H1 Value                        -13121.771
          H1 Scaling Correction Factor        1.0595
            for MLR
Information Criteria
          Akaike (AIC)                     26375.247
          Bayesian (BIC)                   26536.583
          Sample-Size Adjusted BIC         26422.284
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                               61.458*
          Degrees of Freedom                      54
          P-Value                             0.2265
          Scaling Correction Factor           0.9715
            for MLR
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                             0.015
          90 Percent C.I.               0.000  0.030
          Probability RMSEA <= .05             1.000
CFI/TLI
          CFI                                  0.998
          TLI                                  0.997
SRMR (Standardized Root Mean Square Residual)
          Value                                0.028
```

Saved DF=6… how?
3 factor means → 1 fixed change slope
3 factor variances and 6 covariances → 2 loadings, 1 intercept factor variance, 1 slope factor variance, and 1 covariance

Does the latent basis change model (5a) fit *worse* than the partial residual variance model (4b)?
Yes, $-2\Delta LL(df=\mathbf{6}) = 13.658$, $p = .0337$

```
MODEL MODIFICATION INDICES (truncated)
                 M.I.     E.P.C.
Means/Intercepts/Thresholds
[T4]          10.295     -0.194
```

If we freed the factor intercept at T4, the rescaled −2ΔLL would improve by 10.295, and the factor intercept should be lower by 0.194. (And no, moving the fixed loading of 1 for the change factor to T2 instead of T4 doesn't solve the problem…)

```
MODEL RESULTS – NEW PARAMETERS ONLY:
                                                 Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
NEW HIGHER-ORDER FACTOR LOADINGS
 INT      BY
    T1                 1.000      0.000    999.000    999.000
    T2                 1.000      0.000    999.000    999.000
    T3                 1.000      0.000    999.000    999.000
    T4                 1.000      0.000    999.000    999.000
 SLP      BY
    T1                 0.000      0.000    999.000    999.000
    T2                 0.270      0.045      6.057      0.000 → 27.0% of change by T2
    T3                 0.629      0.074      8.439      0.000 → 62.6% of change by T3
    T4                 1.000      0.000    999.000    999.000
HIGHER-ORDER FACTOR COVARIANCE = RANDOM EFECT COVARIANCE (IN G MATRIX)
 INT      WITH
    SLP                0.025      0.056      0.441      0.659
HIGHER-ORDER FACTOR MEANS = FIXED INTERCEPT=0 FOR IDENTIFICATION, FIXED SLOPE
 Means
    INT                0.000      0.000    999.000    999.000
    SLP               -0.466      0.047     -9.890      0.000 → Total mean decline over time
FACTOR VARIANCES = RANDOM EFFECTS VARIANCES (IN G MATRIX)
 Variances
    INT                0.993      0.069     14.304      0.000
    SLP                0.372      0.083      4.494      0.000
Residual Variances = RESIDUAL VARIANCE OF LOWER-ORDER FACTORS (IN R MATRIX DIAGONAL)
    T1                 0.044      0.011      4.110      0.000
    T2                 0.044      0.011      4.110      0.000
    T3                 0.044      0.011      4.110      0.000
    T4                 0.044      0.011      4.110      0.000
```

Model 5b. Mplus Syntax for Revised Latent Basis Change Model—Model 5a, except freeing the factor intercept at T4:

```
MODEL:  ! DATA, VARIABLE, ANALYSIS, OUTPUT are same

!!!!!! 5b. Revised Latent Basis Change Model !!!!!!
! Define latent factors (Factor = indicator loadings)
! Factor loadings held equal over time except prose4
  T1 BY block1@5.972; T1 BY digit1* prose1* (DL PL);
  T2 BY block2@5.972; T2 BY digit2* prose2* (DL PL);
  T3 BY block3@5.972; T3 BY digit3* prose3* (DL PL);
  T4 BY block4@5.972; T4 BY digit4* prose4* (DL PL4);

! Indicator intercepts all held equal over time
  [block1-block4*] (BI);
  [digit1-digit4*] (DI);
  [prose1-prose4*] (PI);

! Indicator residual variances held equal over time
! except block1 and digit1
  block1* (BR1); block2-block4* (BR);
  digit1* (DR1); digit2-digit4* (DR);
  prose1-prose4* (PR);

! Same-outcome residual covariances over time
  block1-block4 WITH block1-block4*;
  digit1-digit4 WITH digit1-digit4*;
  prose1-prose4 WITH prose1-prose4*;

! Latent factor mean=0 at all occasions so that all mean change
! is captured by the intercept and slope factors' fixed effects
  [T1@0 T2@0 T3@0 T4*]; ! T4 int now free
! Latent factor variance held equal over time (like diagonal R matrix)
! so all heterogeneity of variance is captured by slope factor variance
   T1* T2* T3* T4* (ResVar);
! Latent factor covariances (all possible pairs) SHUT OFF @0 so that
! all covariance over time is captured by intercept and slope factor variances
   T1 T2 T3 T4 WITH T1@0 T2@0 T3@0 T4@0;

! Define new higher-order intercept and latent basis change factors
  Int BY T1@1 T2@1 T3@1 T4@1;
  Slp BY T1@0 T2*  T3*  T4@1;
! Higher-order factor means = fixed effects
  [Int@0 Slp*]; ! Fixed int = 0 for identification
! Higher-order factor variances = random effect variances
  Int* Slp*;
! Higher-order factor covariance = random effects covariance
  Int WITH Slp*;
```

Model 5b. Mplus Output for Revised Latent Basis Change Model:

```
Number of Free Parameters                          37 → Saved DF=5 now
Loglikelihood
          H0 Value                          -13146.993
          H0 Scaling Correction Factor          1.1808
            for MLR
          H1 Value                          -13121.771
          H1 Scaling Correction Factor          1.0595
            for MLR
Information Criteria
          Akaike (AIC)                       26367.987
          Bayesian (BIC)                     26533.805
          Sample-Size Adjusted BIC           26416.330
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                                51.749*
          Degrees of Freedom                       53
          P-Value                              0.5230
```

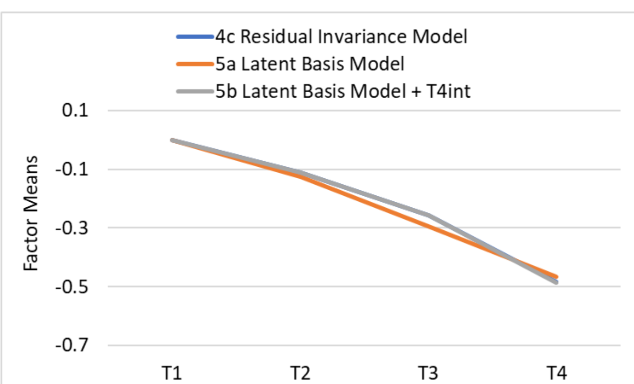
Saved DF=5… how?
3 factor means → 1 fixed change slope **+1 int**
3 factor variances and 6 covariances →
2 loadings, 1 intercept factor variance, 1 slope factor variance, and 1 covariance

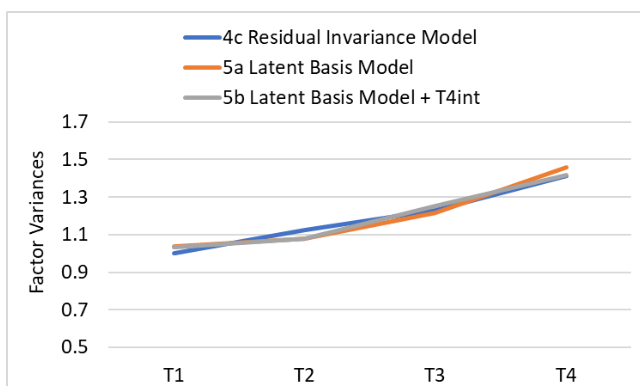
Does the revised latent basis change model (5b) fit *worse* than the partial residual variance model (4b)?
No, −2ΔLL(df=**5**) = 4.274, p = .5106

```
          Scaling Correction Factor         0.9748
            for MLR
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                          0.000
          90 Percent C.I.                   0.000  0.024
          Probability RMSEA <= .05          1.000
CFI/TLI
          CFI                               1.000
          TLI                               1.000
SRMR (Standardized Root Mean Square Residual)
          Value                             0.027

FULL MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

FACTOR LOADINGS EQUAL FOR SAME OUTCOME OVER TIME EXCEPT PROSE4
T1       BY
    BLOCK1             5.972      0.000    999.000    999.000
    DIGIT1            10.574      0.347     30.459      0.000
    PROSE1             3.362      0.128     26.327      0.000
 T2       BY
    BLOCK2             5.972      0.000    999.000    999.000
    DIGIT2            10.574      0.347     30.459      0.000
    PROSE2             3.362      0.128     26.327      0.000
 T3       BY
    BLOCK3             5.972      0.000    999.000    999.000
    DIGIT3            10.574      0.347     30.459      0.000
    PROSE3             3.362      0.128     26.327      0.000
 T4       BY
    BLOCK4             5.972      0.000    999.000    999.000
    DIGIT4            10.574      0.347     30.459      0.000
    PROSE4             3.921      0.177     22.130      0.000 = PL4
NEW HIGHER-ORDER FACTOR LOADINGS
 INT      BY
    T1                 1.000      0.000    999.000    999.000
    T2                 1.000      0.000    999.000    999.000
    T3                 1.000      0.000    999.000    999.000
    T4                 1.000      0.000    999.000    999.000
 SLP      BY
    T1                 0.000      0.000    999.000    999.000
    T2                 0.329      0.057      5.792      0.000 → 32.9% of change by T2
    T3                 0.752      0.084      8.977      0.000 → 75.2% of change by T3
    T4                 1.000      0.000    999.000    999.000
DISTURBANCES COVARIANCES FOR FACTORS SHUT OFF (LIKE NO RESIDUAL COVARIANCE IN R)
T1       WITH
    T2                 0.000      0.000    999.000    999.000
    T3                 0.000      0.000    999.000    999.000
    T4                 0.000      0.000    999.000    999.000
 T2       WITH
    T3                 0.000      0.000    999.000    999.000
    T4                 0.000      0.000    999.000    999.000
 T3       WITH
    T4                 0.000      0.000    999.000    999.000
HIGHER-ORDER FACTOR COVARIANCE = RANDOM EFFECTS COVARIANCE (IN G MATRIX)
 INT      WITH
    SLP                0.009      0.052      0.182      0.856
RESIDUAL COVARIANCES FOR SAME OUTCOME OVER TIME (FREELY ESTIMATED)
 BLOCK1   WITH
    BLOCK2             7.535      1.199      6.282      0.000
    BLOCK3             8.122      1.251      6.490      0.000
    BLOCK4             6.569      1.455      4.516      0.000
 BLOCK2   WITH
    BLOCK3             7.210      1.176      6.130      0.000
    BLOCK4             4.510      1.304      3.458      0.001
 BLOCK3   WITH
    BLOCK4             6.207      1.367      4.539      0.000
 DIGIT1   WITH
    DIGIT2             9.229      3.285      2.809      0.005
    DIGIT3             6.952      3.520      1.975      0.048
    DIGIT4             7.552      3.622      2.085      0.037
 DIGIT2   WITH
    DIGIT3             7.658      3.452      2.218      0.027
    DIGIT4             7.915      3.410      2.321      0.020
```

```
 DIGIT3   WITH
    DIGIT4             2.184      3.642      0.600      0.549
 PROSE1   WITH
    PROSE2             4.942      0.618      8.001      0.000
    PROSE3             4.335      0.677      6.403      0.000
    PROSE4             4.732      0.846      5.596      0.000
 PROSE2   WITH
    PROSE3             5.273      0.618      8.537      0.000
    PROSE4             5.327      0.849      6.275      0.000
 PROSE3   WITH
    PROSE4             6.274      0.673      9.317      0.000
HIGHER-ORDER FACTOR MEANS = FIXED INTERCEPT=0 FOR IDENTIFICATION, FIXED SLOPE
 Means
    INT                0.000      0.000    999.000    999.000
    SLP               -0.340      0.050     -6.752      0.000 → Total mean decline over time
INTERCEPTS FOR SAME OUTCOME HELD EQUAL OVER TIME (SO CHANGE IS DUE TO FACTORS ONLY!)
Intercepts
    BLOCK1            10.245      0.282     36.364      0.000
    BLOCK2            10.245      0.282     36.364      0.000
    BLOCK3            10.245      0.282     36.364      0.000
    BLOCK4            10.245      0.282     36.364      0.000
    DIGIT1            21.095      0.479     44.045      0.000
    DIGIT2            21.095      0.479     44.045      0.000
    DIGIT3            21.095      0.479     44.045      0.000
    DIGIT4            21.095      0.479     44.045      0.000
    PROSE1             8.423      0.175     48.083      0.000
    PROSE2             8.423      0.175     48.083      0.000
    PROSE3             8.423      0.175     48.083      0.000
    PROSE4             8.423      0.175     48.083      0.000
    T1                 0.000      0.000    999.000    999.000
    T2                 0.000      0.000    999.000    999.000
    T3                 0.000      0.000    999.000    999.000
    T4                -0.145      0.040     -3.638      0.000 → NEW MEAN DEVIATION FOR T4
FACTOR VARIANCES = RANDOM EFFECT VARIANCES (IN G MATRIX)
 Variances
    INT                0.994      0.070     14.106      0.000
    SLP                0.366      0.076      4.837      0.000
OUTCOME "NOT THE FACTOR" LEFTOVER VARIANCES AND RESIDUAL VARIANCE (IN R MATRIX DIAGONAL)
 Residual Variances
    BLOCK1            19.393      1.615     12.005      0.000 = BR1
    BLOCK2            13.651      1.211     11.271      0.000
    BLOCK3            13.651      1.211     11.271      0.000
    BLOCK4            13.651      1.211     11.271      0.000
    DIGIT1            32.163      4.317      7.450      0.000 = DR1
    DIGIT2            23.748      3.110      7.637      0.000
    DIGIT3            23.748      3.110      7.637      0.000
    DIGIT4            23.748      3.110      7.637      0.000
    PROSE1             9.920      0.541     18.334      0.000
    PROSE2             9.920      0.541     18.334      0.000
    PROSE3             9.920      0.541     18.334      0.000
    PROSE4             9.920      0.541     18.334      0.000
    T1                 0.040      0.010      3.915      0.000
    T2                 0.040      0.010      3.915      0.000
    T3                 0.040      0.010      3.915      0.000
    T4                 0.040      0.010      3.915      0.000
```

Comparing model-predicted factor means and variances as given by TECH4 output (at the very end):





Sample results section for these analyses:

The extent of individual differences in change over time (four occasions collected at two-year intervals) in a latent factor of cognitive functioning (with three observed outcomes: block design, digit–symbol substitution, and prose recall) was examined using M*plus* v. 8.8 (Muthén & Muthén, 1998–2017). Robust maximum likelihood (MLR) estimation was used for all analyses; accordingly, nested model comparisons were conducted using the rescaled difference in the model −2LL values with degrees of freedom equal to the difference in the number of model parameters. Prior to examining change in the latent factor over time, partial longitudinal measurement invariance was established by a series of nested models, as described next.

[Table 1 would have the fit of each model, as shown in the excel workbook for this example. Depending on the journal, you may need to add text defining each fit index and what is considered "good fit" for each. You could also make a Table 2 for all the LRTs instead of giving them in the text as I did below.]

Table 1 Model Fit

Model	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	Chi-Square DF	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower CI	RMSEA Higher CI	RMSEA p-value
1. Configural Model	60	27.704	1.0039	30	0.5861	1.000	0.000	0.000	0.027	1.000
2a. Full Metric Invariance	54	41.112	0.9696	36	0.2566	0.999	0.015	0.000	0.033	1.000
2b. Partial Metric (- PL4)	55	31.925	0.9729	35	0.6173	1.000	0.000	0.000	0.025	1.000
3a. Full Scalar Invariance	49	38.075	0.9739	41	0.6014	1.000	0.000	0.000	0.024	1.000
4a. Full Residual Variance	40	74.477	0.9647	50	0.0140	0.993	0.027	0.013	0.040	0.999
4b. Partial Residual Variance (- BR1, -DR1)	42	47.525	0.9672	48	0.4922	1.000	0.000	0.000	0.025	1.000
5a. Latent Basis	36	61.458	0.9715	54	0.2265	0.998	0.015	0.000	0.030	1.000
5b. Revised Latent Basis	37	51.749	0.9748	53	0.5230	1.000	0.000	0.000	0.024	1.000

First, a configural invariance model was specified in which four correlated factors (i.e., one factor for each occasion) were estimated simultaneously; all factor means were fixed to 0 and all factor variances were fixed to 1 for identification. Residual covariances for the same outcome across the four occasions were also estimated. As shown in Table 1, the configural invariance model had excellent fit by every index, indicating that the 12 outcome means, variances, and covariances were well recreated by the model.

Equality of the unstandardized factor loadings across occasions was then examined in a metric invariance model. The factor variance was fixed to 1 at occasion 1 for identification but was freely estimated at occasions 2, 3, and 4. The factor means were all fixed to 0 for identification. All factor loadings were constrained equal across occasions, but all outcome intercepts and residual variances varied over time. Factor covariances and outcome residual covariances were estimated as described previously. Although the metric invariance model had excellent global fit, it fit significantly worse than the configural invariance model $-2\Delta LL(6) = 15.09$, $p = .020$. Modification indices suggested that the loading of prose recall at occasion 4 was a significant source of local misfit and should be freed. After doing so, the partial metric invariance model had excellent fit (as shown in Table 1) that was not significantly worse than the configural invariance model, $-2\Delta LL(5) = 4.13$, $p = .531$. The fact that partial metric invariance (i.e., "weak invariance") held indicates that the same latent factor was being measured at each occasion, or that the outcomes were related to their latent factor equivalently over time (except for prose recall, which was slightly more related to its factor at occasion 4 than at occasions 1, 2, or 3).

Equality of the unstandardized outcome intercepts across occasions was then examined in a scalar invariance model. The factor mean and variance at occasion 1 were fixed to 0 and 1, respectively, for identification, but the factor mean and variance were then estimated at occasions 2, 3, and 4. All factor loadings (except for prose recall at occasion 4) and all outcome intercepts were constrained equal across occasions; all outcome residual variances still differed over time. Factor covariances and residual covariances were estimated as described previously. The scalar invariance model had excellent fit (as shown in Table 1) that was not significantly worse than the partial metric invariance model, $-2\Delta LL(6) = 6.14$, $p = .407$. The fact that full scalar invariance (i.e., "strong invariance") held indicates that all occasions have the same expected response for each outcome at the same absolute level of the latent factor, or that the observed difference in the outcome means across occasions 1–4 was due to factor mean differences only.

Equality of the unstandardized outcome residual variances across occasions was then examined in a residual variance invariance model. As in the scalar invariance model, the factor mean and variance were fixed to 0 and 1, respectively, at occasion 1 for identification, but the factor mean and variance were estimated at occasions 2, 3, and 4. All factor loadings (except for prose recall at occasion 4), all outcome intercepts, and all outcome residual variances were constrained to be equal over time. Factor covariances and outcome residual covariances were estimated as described previously. Although the residual variance invariance model had excellent global fit, it fit significantly worse than the scalar invariance model, $-2\Delta LL(9) = 37.68$, $p < .001$. Modification indices suggested that the residual variances of block design and digit–symbol substitution at occasion 1 were the largest sources of misfit and should be freed. After doing so, the partial residual variance invariance model had excellent fit (as shown in Table 1) that was not significantly worse than the scalar invariance model, $-2\Delta LL(7) = 9.58$, $p = .214$. The fact that partial residual variance invariance (i.e., "strict invariance") held indicates that the amount of outcome variance not accounted for by the latent factor was the same across time (except for block design and digit–symbol substitution, for which there was more residual variance at occasion 1).

In the final invariance model, the factor means showed increasing decline over time, while the factor variances showed increasing individual differences over time. The factors were highly correlated across occasions ($r \approx .8$ to .9). The extent to which two higher-order factors—for an intercept and latent basis change—could recreate the lower-order factor means, variances, and covariances was then examined. To create a meaningful scale by which to identify the model, the factor loading for block design was fixed to 5.972, its value from the last invariance model in which the occasion 1 factor variance was fixed to 1. Consequently, the total SD will be ≈ 1 for occasion 1, setting the scale of the latent outcome to be predicted. All lower-order factor variances were estimated but constrained equal over time so that any heterogeneity of variance over time in the lower-order factors would be captured by the higher-order factor for latent basis change. Likewise, all lower-order factor covariances were fixed to 0 so that all factor correlation over time would be captured by the estimated variance of the higher-order factors for intercept and latent basis change (and their estimated covariance). All lower-order factor intercepts and the mean of the higher-order intercept factor were fixed to 0 for identification given the estimation of the outcome intercepts. All residual covariances for the same outcome over time were estimated as in previous models. Finally, the latent basis factor loadings were fixed to 0 and 1 at occasions 1 and 4, respectively, with estimated factor loadings at occasions 2 and 3. Consequently, the higher-order intercept factor will capture the expected latent factor at occasion 1, and the mean of the higher-order latent basis change factor will capture the amount of overall change in the latent factor across the four occasions.

Although the latent basis change model had excellent fit, it fit significantly worse than the last invariance model, $-2\Delta LL(6) = 13.66$, $p = .034$. Modification indices suggested that the occasion 4 factor intercept was the largest source of misfit and should be freed. After doing so, the latent basis change invariance model had excellent fit (as shown in Table 1) that was not significantly worse than the last invariance model, $-2\Delta LL(5) = 4.27$, $p = .511$. Figure 1 displays the predicted lower-order factor means and variances for each occasion. There was a significant average decline of 0.340 (as given by the mean of the higher-order factor for latent basis change), 32.9% and 75.2% of which happened by occasions 2 and 3, respectively. The occasion 4 intercept (capturing its deviation from the predicted trajectory) was significantly negative (–0.145). Wald tests* indicated significant individual differences in the predicted latent outcome at occasion 1 and in its subsequent decline, as captured by the variances of the higher-order intercept and latent basis change factors, respectively.

** Yes, I know that Wald tests should not be used for testing the significance of variances, but this is very commonly done in the SEM world. In this case, the likelihood ratio tests would have agreed, and so I didn't report those additional model comparisons.*