

# Considerations in Selecting Amongst Alternative Metrics of Time

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# Goals of Longitudinal\* Modeling

- 5 rationales of longitudinal research
  - Baltes & Nesselroade, 1979
    - Chapter 1: *Longitudinal Research in the Study of Behavior and Development*
- 7 levels of longitudinal analysis
  - Hofer & Sliwinski, 2006
    - Chapter 2: *Handbook of the Psychology of Aging (6<sup>th</sup> edition)*
- 7+ steps in longitudinal modeling
  - Singer & Willett, 2003
    - Chapter 4: *Applied Longitudinal Data Analysis*

\*Applicable to both the MLM and SEM analytic frameworks

# Steps in Longitudinal Analysis

1. Decompose **BP and WP variation**—Intraclass Correlation
  - **ICC** = proportion of outcome variance that is *constant* over time, and that results from *cross-sectional* differences
3. Describe pattern of average change over time (**fixed effects**) and individual differences therein (**random effects**)
  - Piecewise slopes models—Phases of discontinuous change
  - Polynomial models—Approximate nonlinear continuous change
  - Truly nonlinear models—Exponential or logistic change
  - Latent basis models—Estimate shape of nonlinear change

# Steps in Longitudinal Analysis

4. Predict **inter-individual differences** in change
  - *Why do people need their own intercepts and slopes?*
5. Predict **intra-individual variation** from predicted change
  - *Why are you off your line today (time-specific influences)?*
6. Examine **multivariate relationships**
  - *Between-person correlations among intercepts and slopes*
  - *Within-person covariation of residuals (or lead-lag associations)*
7. Examine other sources of underlying **heterogeneity**
  - *Mixture models for discrete types of individual differences*
  - *Predict individual differences in within-person variability*

# Road Map

- Steps in longitudinal analysis
- **The missing step #2**
- Example: Alternative metrics of “time”
- What about just time?
- What else contributes to “time”?

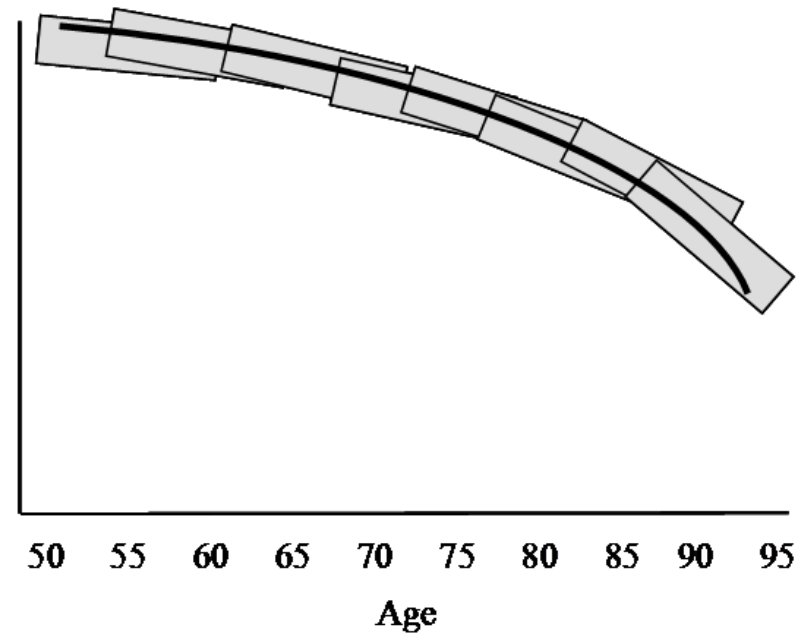
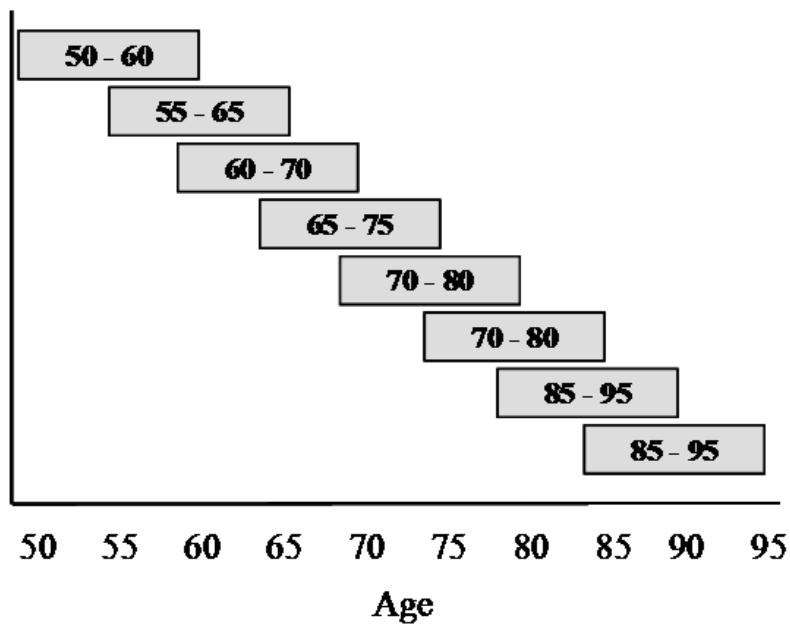
# The Missing Step 2

- **Summary across steps:** The goal of creating statistical models of change is to describe the overall pattern of and predict individual differences in **change over time**.
- These models employ an often unrecognized assumption **that we know exactly what “time” should be**.
- So the missing Step 2 is a pre-cursor to every other step in longitudinal analysis, and involves 2 related concerns:
  - **What should “time” be?**
  - **How should “time” be modeled when people differ in “time”?**
  - Concerns apply specifically to *accelerated longitudinal designs*

# Accelerated Longitudinal Designs

*Want to do a longitudinal study but just don't have the time?*

**Accelerate:** Model trajectories over a wider span of time than would be possible using only the observed longitudinal info...



# The Missing Step 2

- **First: What should “time” be?**
  - Which **metric of time** best matches the **causal process** thought to be responsible for observed change?
  - Do **alternative metrics of time** for **multiple processes** create different pictures of change and individual differences therein?
  - Relevant for aligning different persons onto same time trajectory, but this distinction is **not relevant within persons**
- **Second: What do we do when people differ in “time”?**
  - **How should “time” be modeled in accelerated designs?**
  - When people begin a study at different points in time, how should we distinguish effects of **between-person differences** in time from effects of **within-person changes** in time?



# Road Map

- Steps in longitudinal analysis
- The missing step #2
- **Example: Alternative metrics of “time”**
- What about just time?
- What else contributes to “time”?

# Example Data: *Octogenarian (Twin) Study of Aging*

- **173 persons (65% women)**

- Measured up to **5 occasions** over 8 years
- **Known** dates of birth and death
- **Estimated** dates of dementia diagnosis (91 Alz., 50 Vas., 32 Mixed)

- **Baseline occasion “time” variability:**

- 79 to 100 years of age (M = 84, SD = 3)
- -16 to 0 years from death (M = -6, SD = 4)
- -12 to 18 years from diagnosis (M = 0, SD = 5)

Correlation	Age	Death
<b>Death</b>	.23	
<b>Dementia</b>	.17	.52

- **Cognition outcomes (each T-scored):**

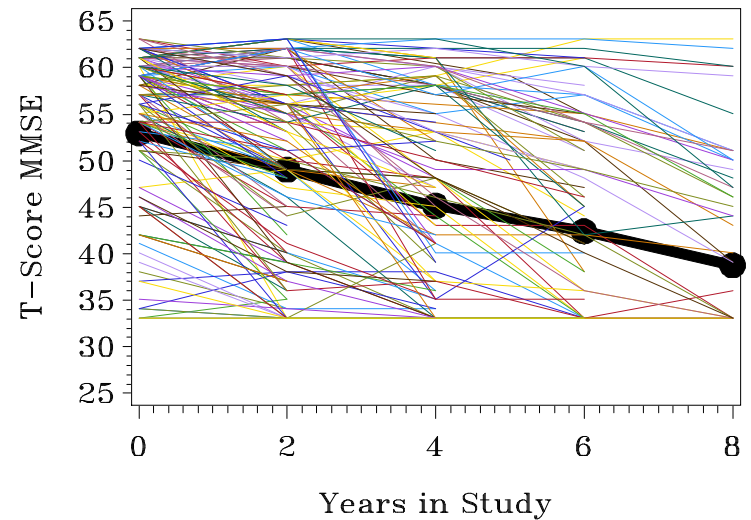
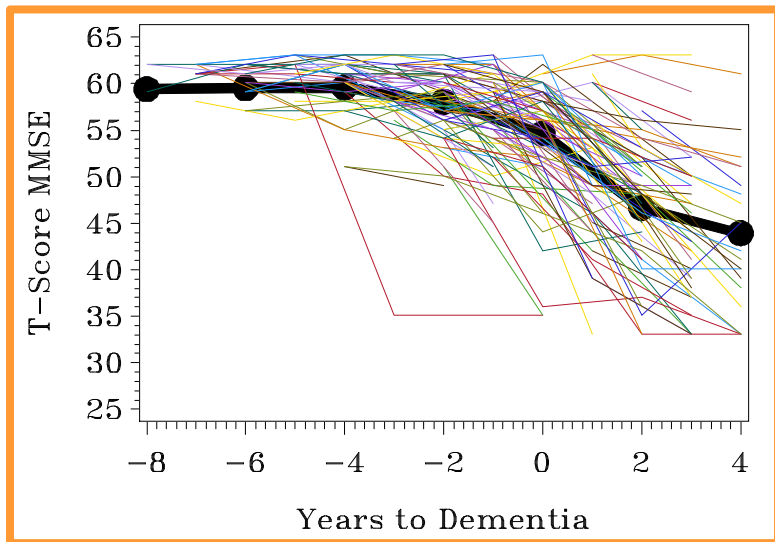
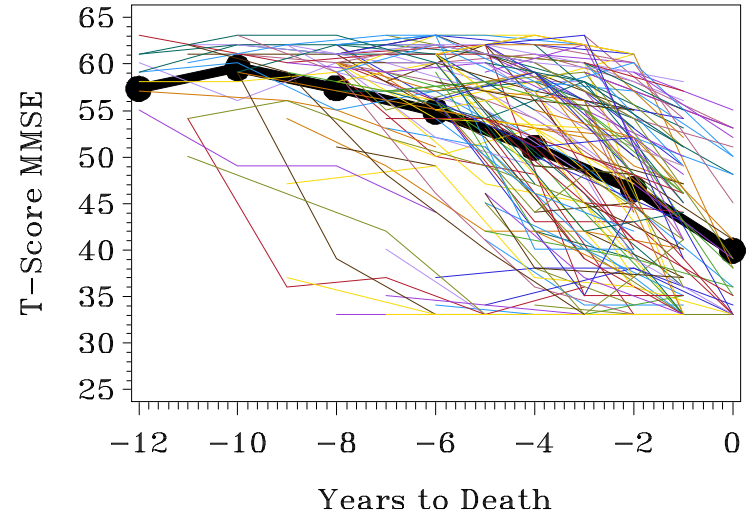
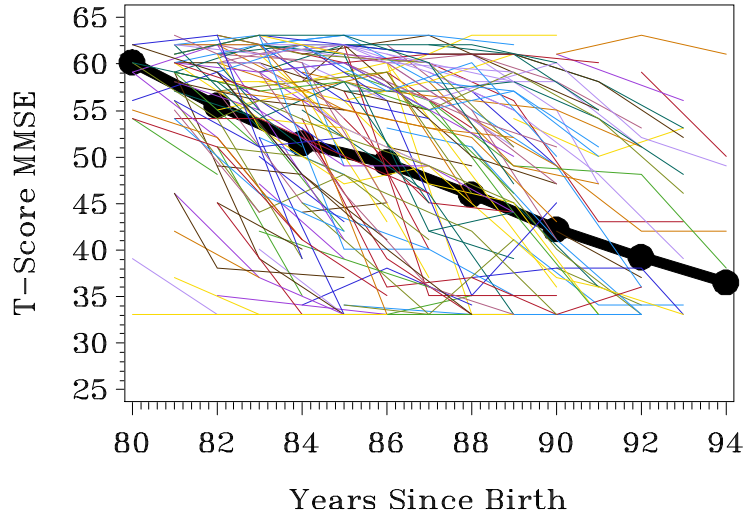
- General: Mini-Mental Status Exam
- Memory: Object Recall
- Spatial Reasoning: Block Design

#Persons per #Occasions				
1	2	3	4	5
28	37	36	36	35
29	31	39	29	18
37	32	31	22	19

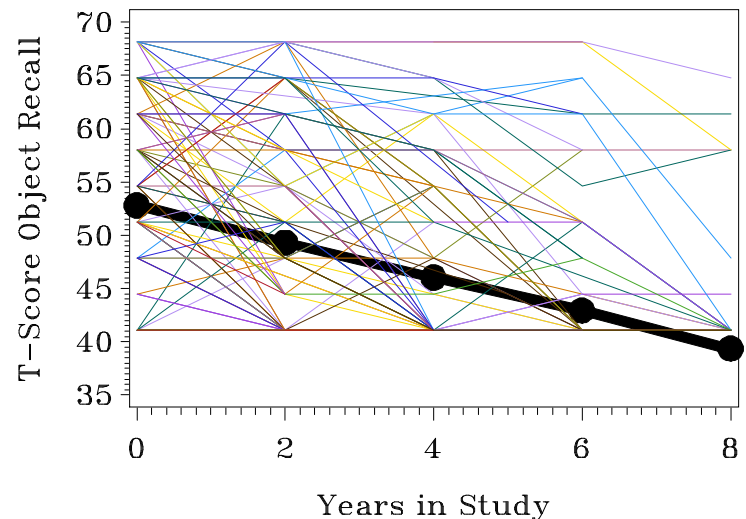
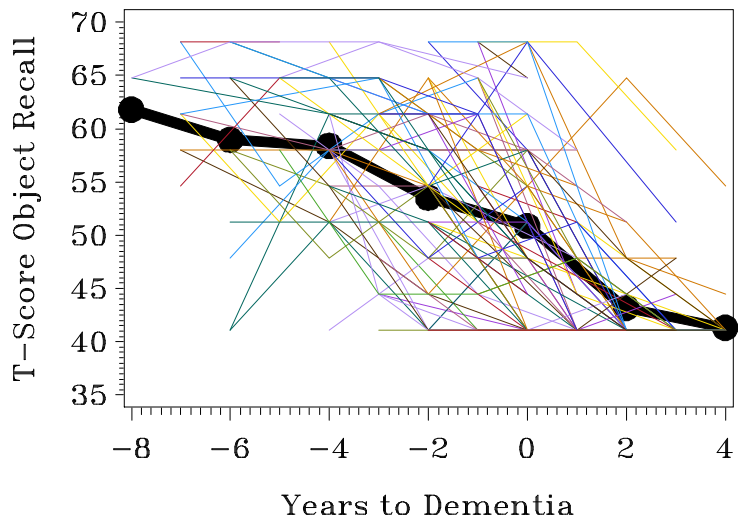
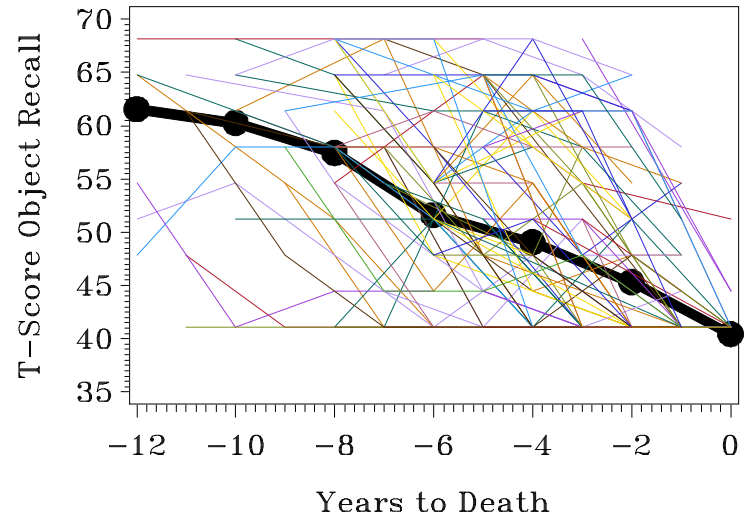
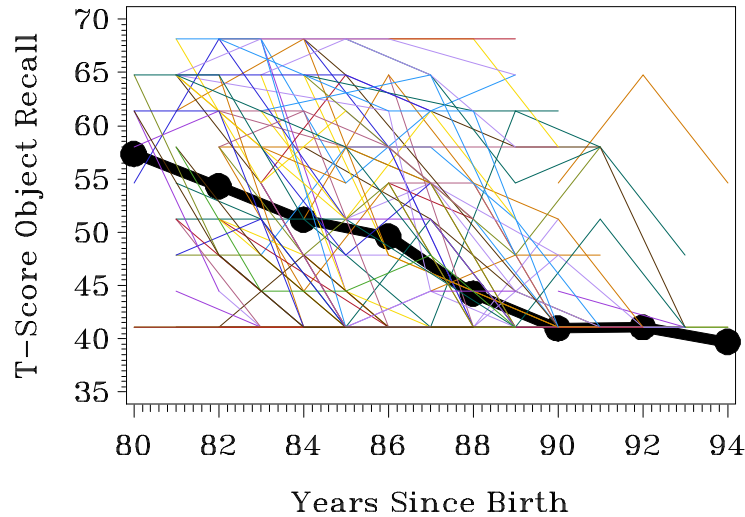
# Alternative Metrics of Time (and ICC)

- Chronological Age as Time (47% BP)
  - Individual differences are organized around the mean level for a given **distance from birth** (84 years) and change with distance from birth
- Years to Death as Time (24% BP)
  - Individual differences are organized around the mean level for a given **distance from death** (-7 years) and change with distance from death
- Years to Dementia Diagnosis as Time (70% BP)
  - Individual differences are organized around the mean level for a given **distance from diagnosis** (=0) and change with distance from diagnosis
- Years in Study as Time (0% BP)
  - Individual differences are organized around the mean level for a given **distance from baseline** (=0) and change with distance from baseline

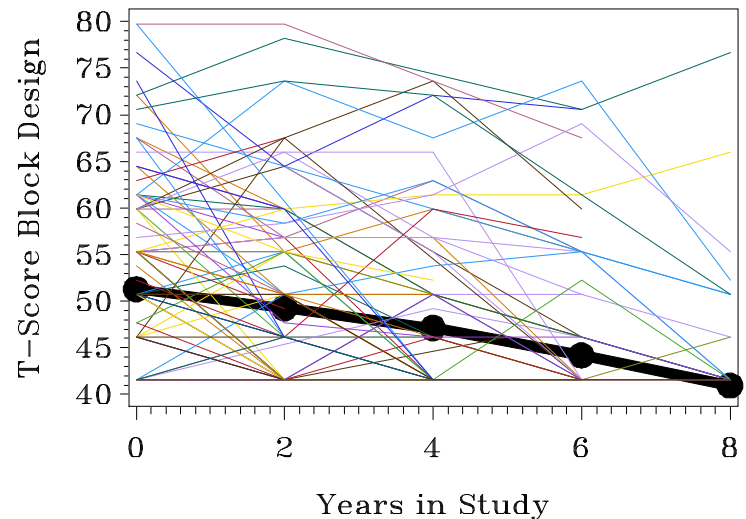
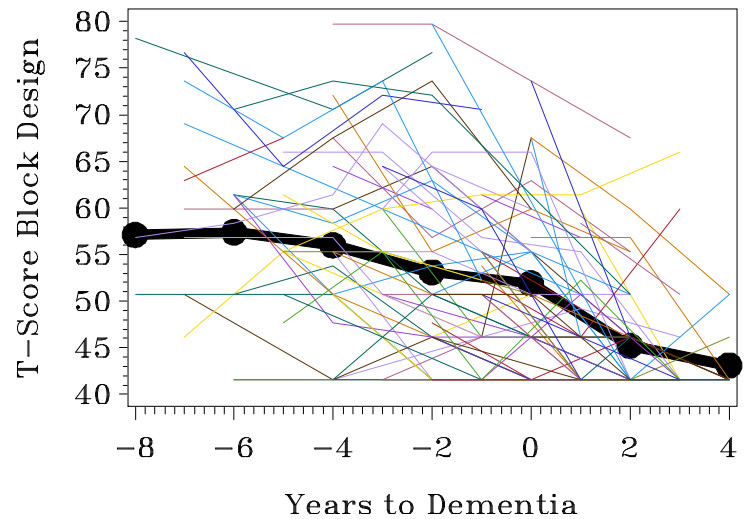
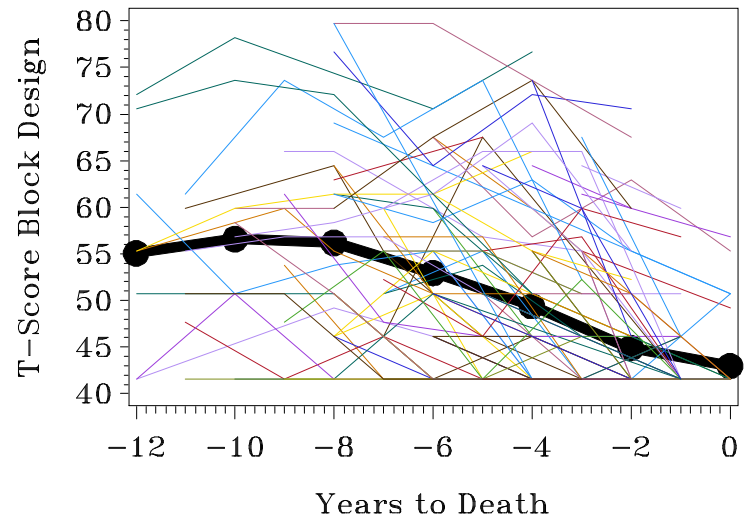
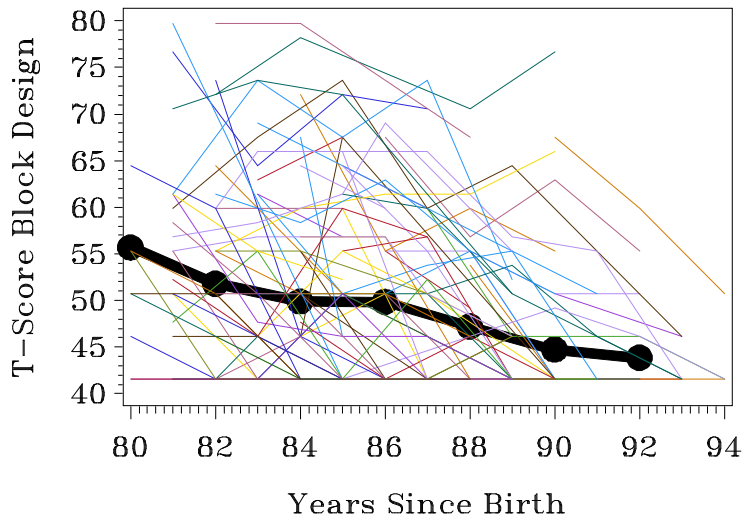
# General Cognition: MMSE



# Memory: Object Recall



# Spatial Reasoning: Block Design



# First Option: Age-as-Time

Level-1:  $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Age}_{ti} - 84) + \beta_{2i}(\text{Age}_{ti} - 84)^2 + e_{ti}$

Level-2 Equations → **Fixed** and **Random** Effects:

$\beta_{0i} =$   
↑  
**Intercept**  
for person  $i$

$Y_{00}$   
↑  
**Fixed Intercept**  
(mean)

+

$U_{0i}$   
↑  
**Random Intercept**  
Deviation

→ predicted  $Y$  when age=84

$\beta_{1i} =$   
↑  
**Linear Slope**  
for person  $i$

$Y_{10}$   
↑  
**Fixed Linear Slope**  
(mean)

+

$U_{1i}$   
↑  
**Random Linear Slope**  
Deviation

→ rate of  $\Delta$  when age=84

$\beta_{2i} =$   
↑  
**Quad Slope**  
for person  $i$

$Y_{20}$   
↑  
**Fixed Quad Slope**  
(mean)

+

$U_{2i}$   
↑  
**Random Quad Slope**  
Deviation

→  $1/2$  rate of  $\Delta$  in  $\Delta$  per year

# First Option: Age-as-Time

- If people differ in initial age, measuring change as a function of age requires assuming **age convergence**:
  - Younger people and older people differ *only* by age
  - Effects of between-person, **cross-sectional age differences** are equivalent to effects of within-person, **longitudinal age changes**
- Age convergence is not likely to hold when:
  - Initial **age range** is large (47% of age is BP here)
  - **Cohort** differences and **selection** effects are large
- Is exactly the same problem as not fully separating the BP and WP effects of **ANY** time-varying predictor
  - *i.e., conflated, convergence, composite, or smushed effect*



# Examining Age Convergence Effects

Can use a variant of **grand-mean-centering** to test equivalence of BP and WP age effects empirically

Level-1 **Age-Based** Change:

$$y_{ti} = \beta_{0i} + \beta_{1i}(\text{Age}_{ti} - 84) + \beta_{2i}(\text{Age}_{ti} - 84)^2 + e_{ti}$$

Level-2 Equations:

$$\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{AgeT1}_i - 84) + U_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{AgeT1}_i - 84) + U_{1i}$$

$$\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{AgeT1}_i - 84) + U_{2i}$$

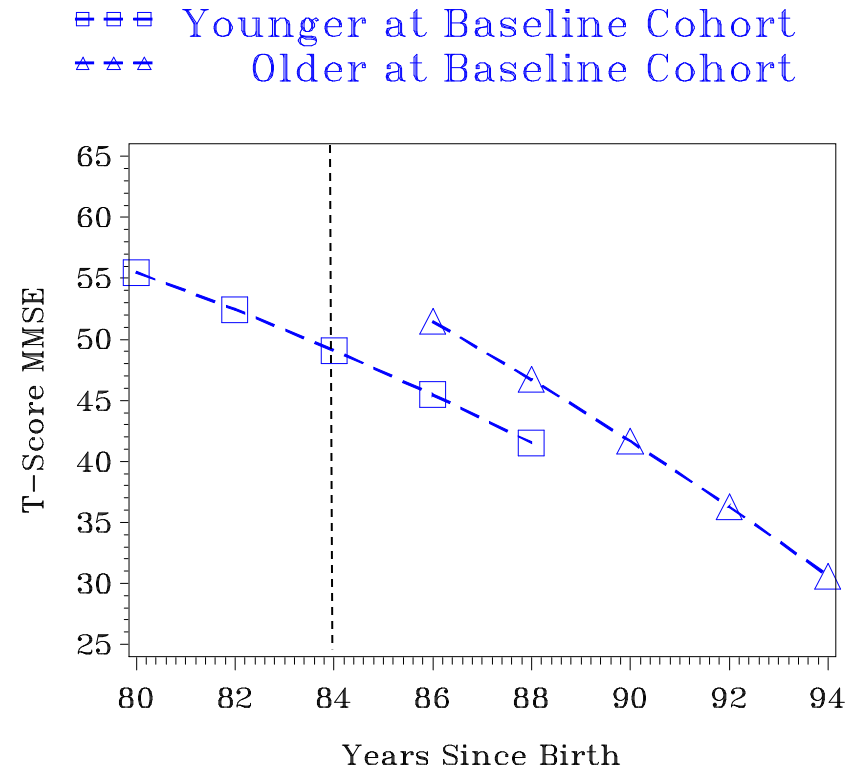
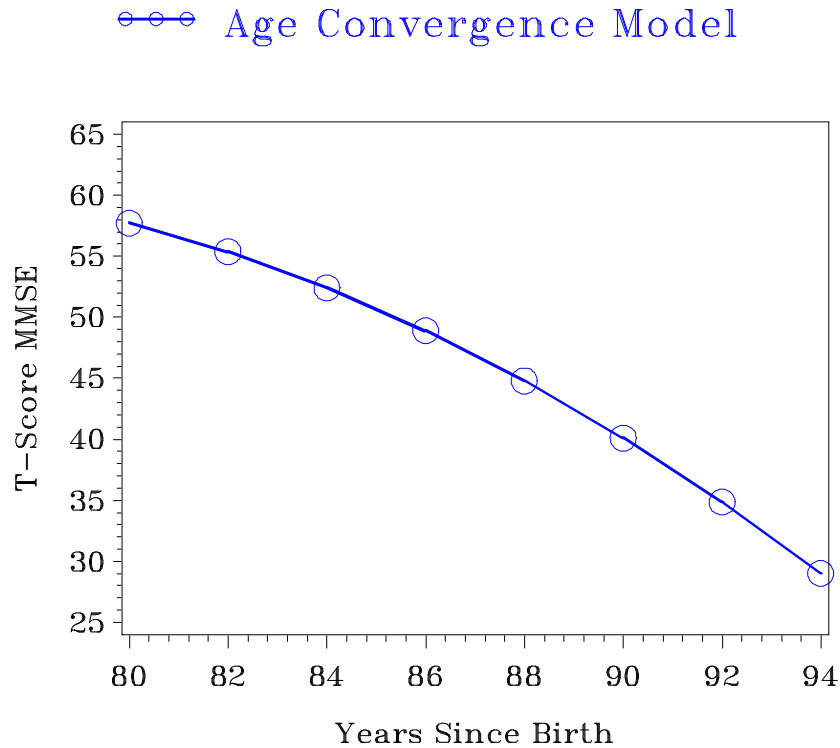
**AgeT1** → Incremental effect of cross-sectional age (**cohort**)

Use **age at time 1** (or birth year) instead of mean age to lessen bias from attrition-related missing data

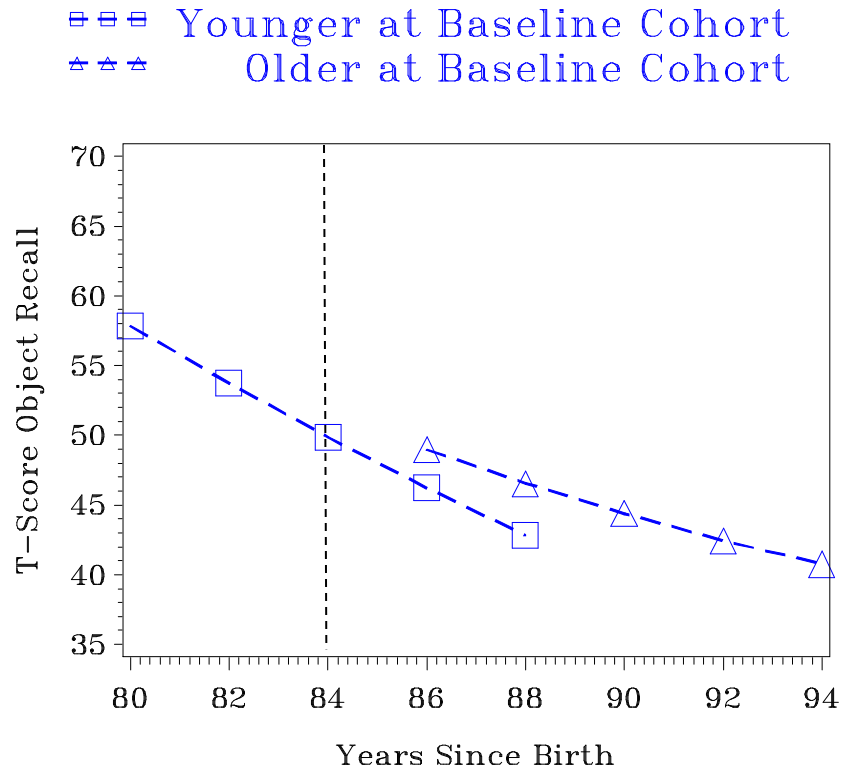
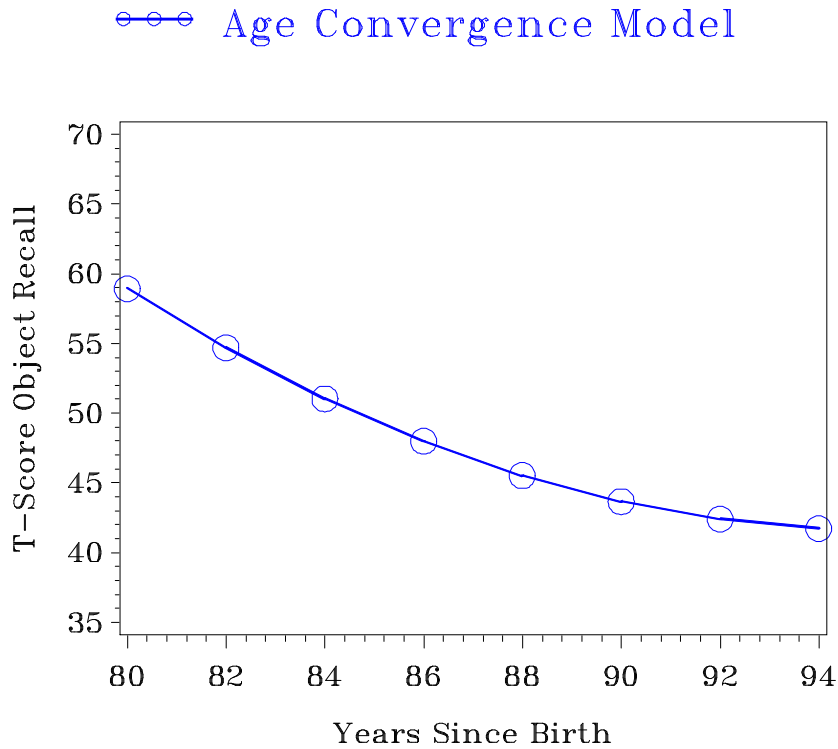
**Significance → Nonconvergence**  
It matters **WHEN** you were 84

Persons create **contextual effects** (2023 update—captured only for the fixed age effects here!)

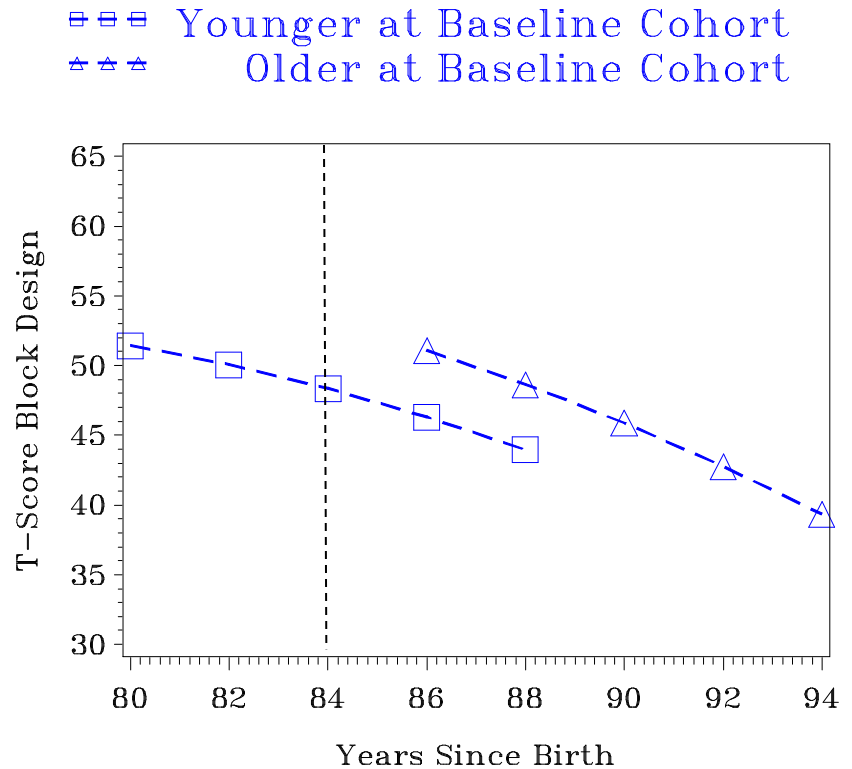
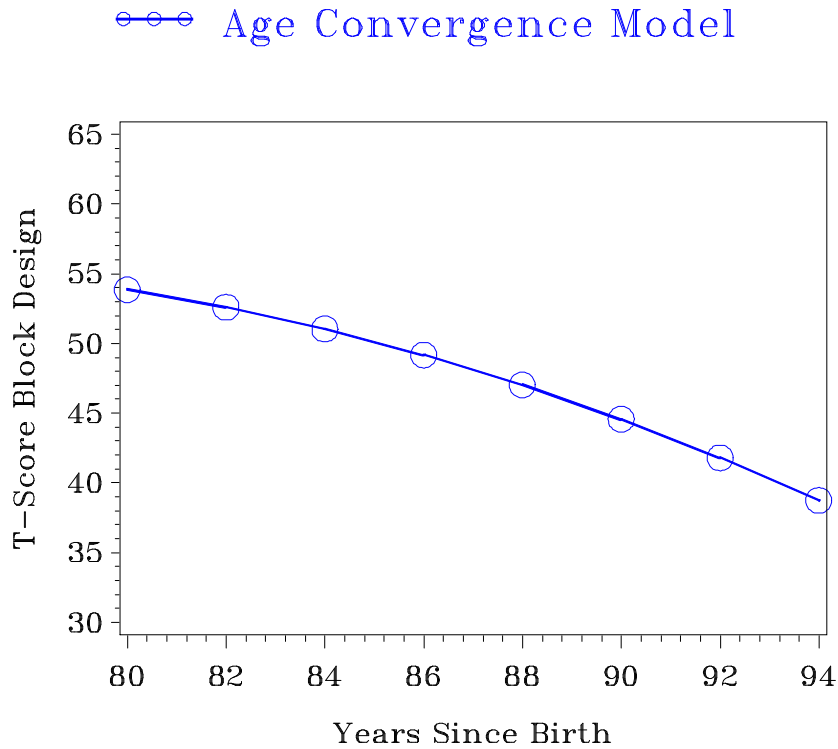
# Age-Based Models: MMSE



# Age-Based Models: Object Recall



# Age-Based Models: Spatial Reasoning



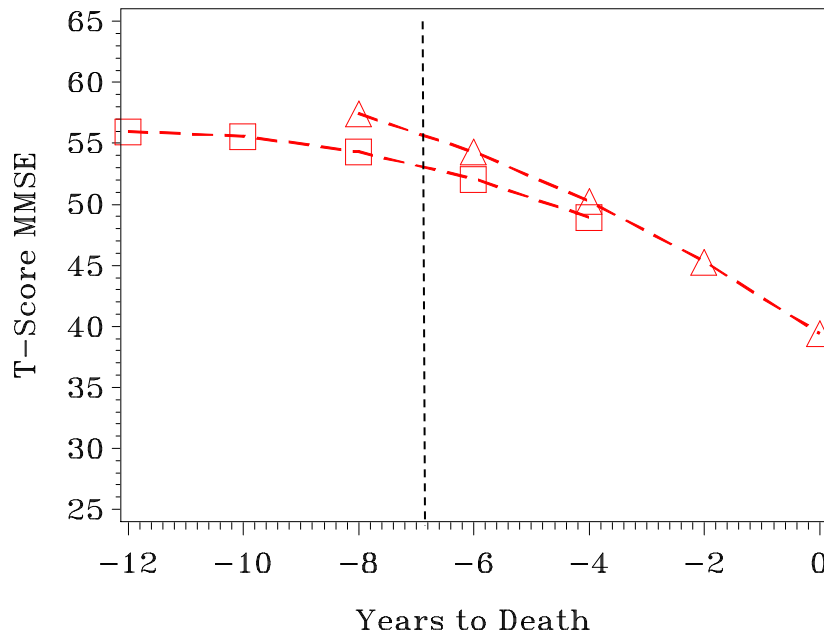
# So if age is just a time-varying predictor...

- Because **years to death** and **years to diagnosis** also have BP variation (24%, 70%), the same concerns about **testing convergence** apply to them, too
- **Years to death**
  - Level 1:  $YTdeath_{ti} + 7$
  - Level 2:  $YTdeathT1_i + 7$
- **Years to diagnosis**
  - Level 1:  $YTdem_{ti} - 0$
  - Level 2:  $YTdemT1_i - 0$
- If the level-2 effects in these models are significant, then:
  - **Years to death**: it matters **WHEN** you were 7 years from death
  - **Years to diagnosis**: it matters **WHEN** you were at diagnosis

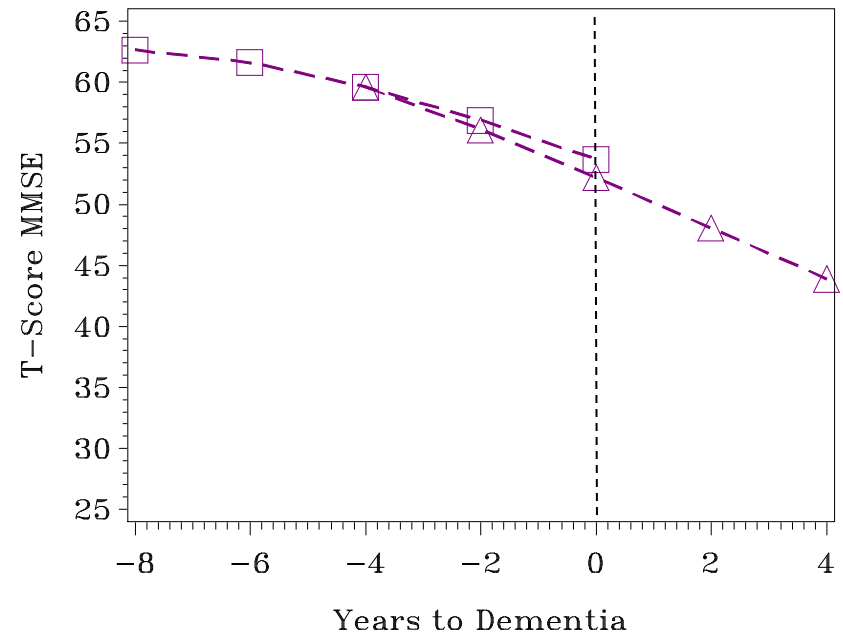
**WHEN = cohort difference**

# Death-Based and Dementia-Based Models: MMSE

□ □ □ Further from Death Cohort  
△ △ △ Closer to Death Cohort

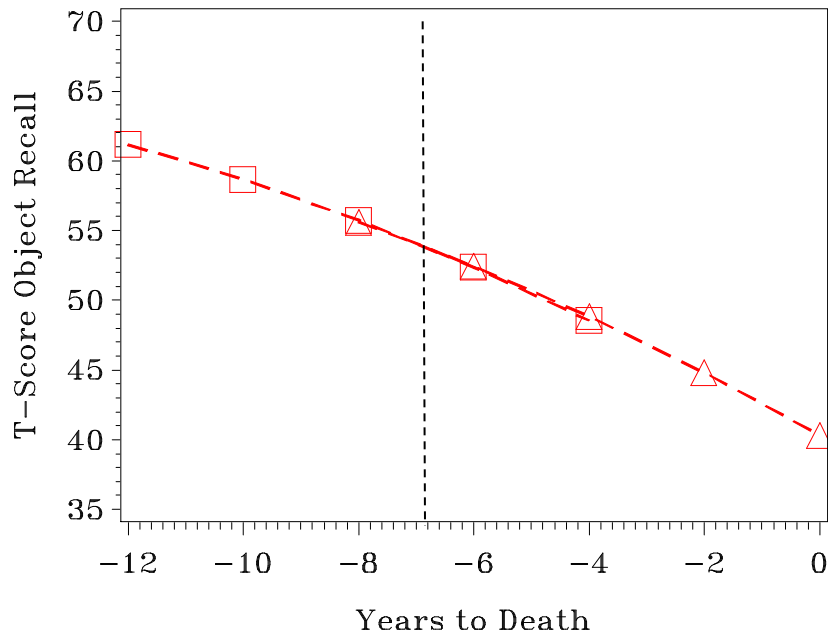


□ □ □ Further from Diagnosis Cohort  
△ △ △ Closer to Diagnosis Cohort

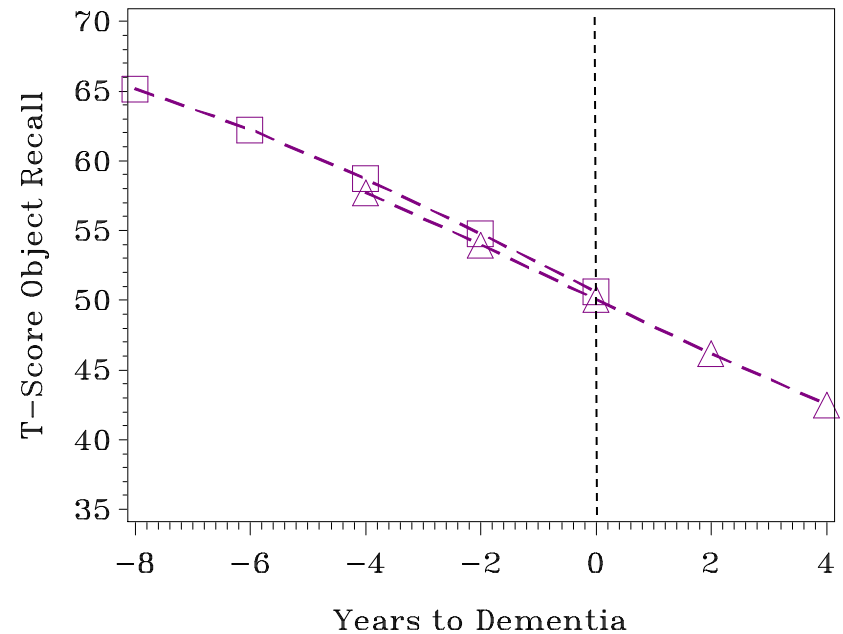


# Death-Based and Dementia-Based Models: Object Recall

□ □ □ Further from Death Cohort  
△ △ △ Closer to Death Cohort

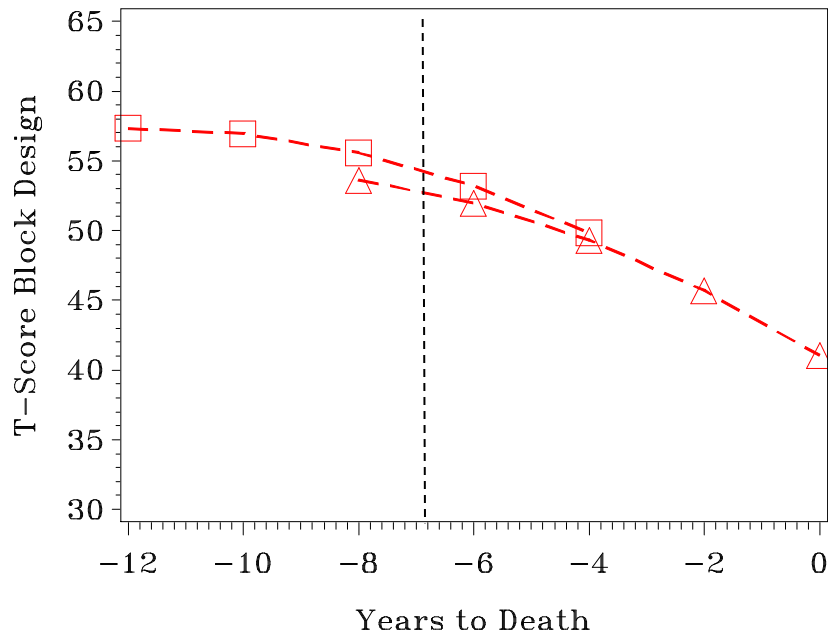


□ □ □ Further from Diagnosis Cohort  
△ △ △ Closer to Diagnosis Cohort

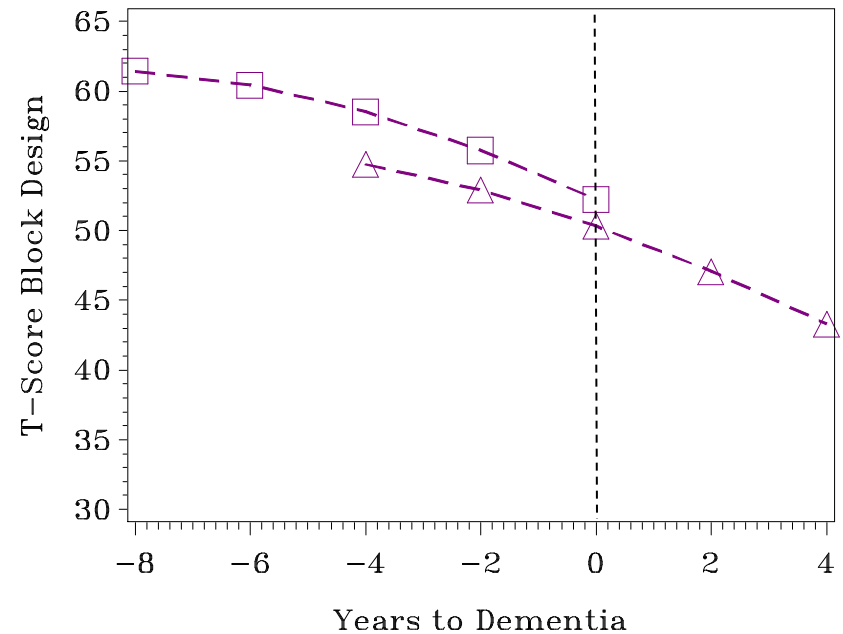


# Death-Based and Dementia-Based Models: Spatial Reasoning

□ □ □ Further from Death Cohort  
△ △ △ Closer to Death Cohort



□ □ □ Further from Diagnosis Cohort  
△ △ △ Closer to Diagnosis Cohort

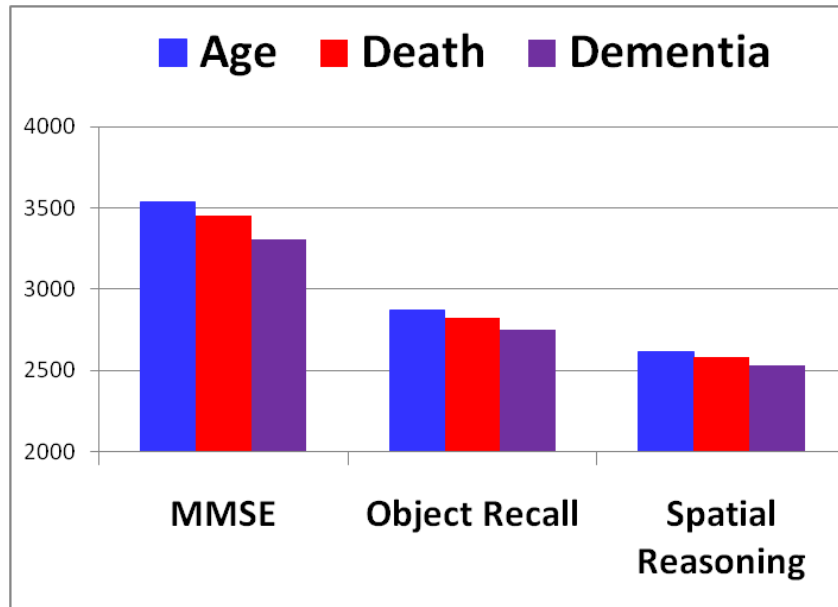




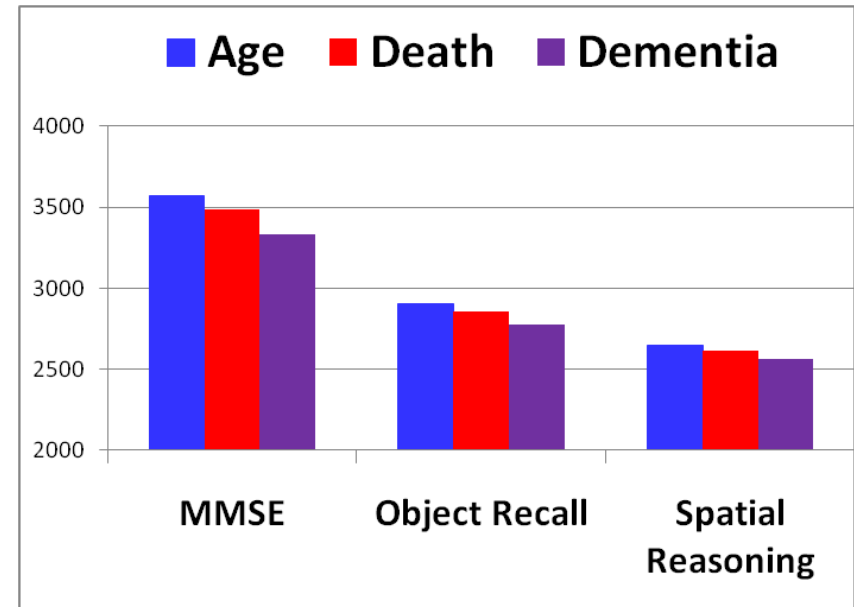
# Comparing Models by Fit...

The fit of these alternative metrics of time to the data can be compared using their **information criteria**...

## ML AIC



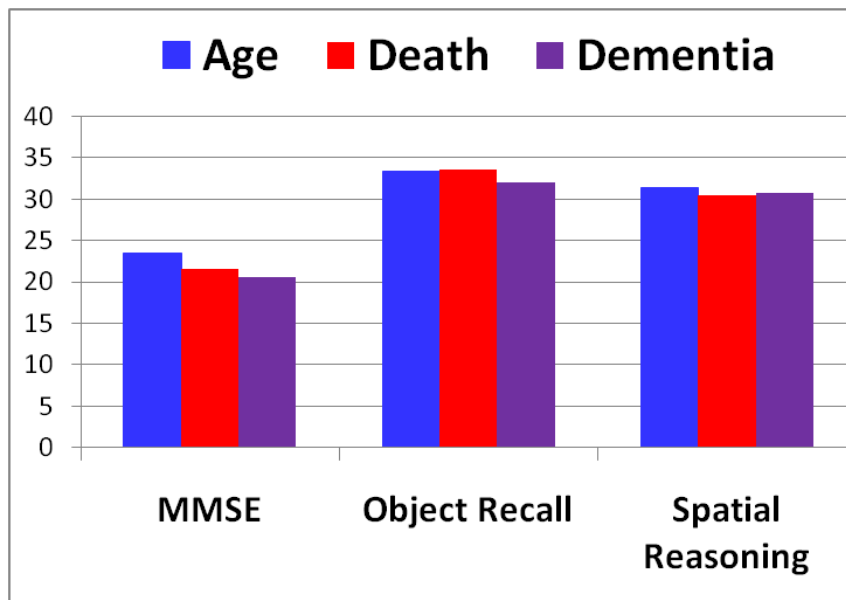
## ML BIC



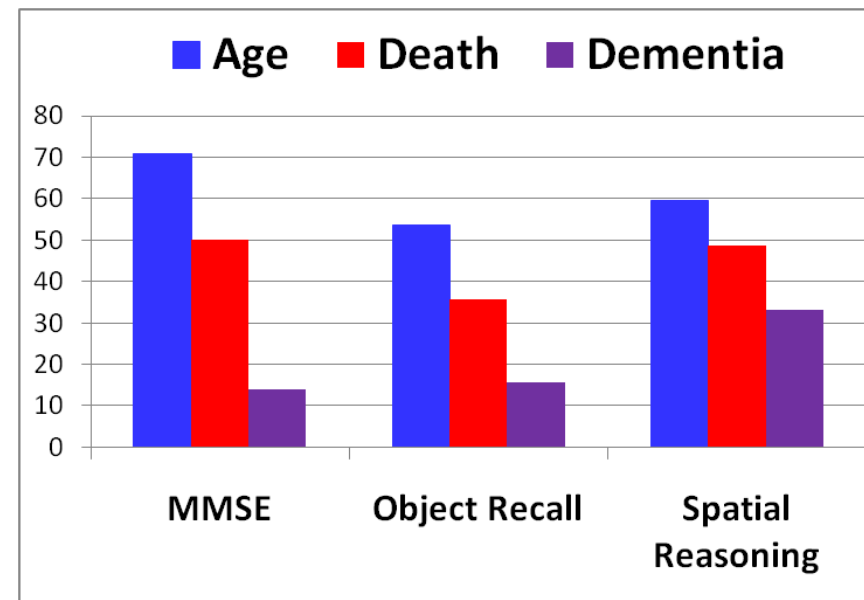
# Comparing Models by Variances...

The fit of these alternative metrics of time to the data can also be compared using their **variance** components...

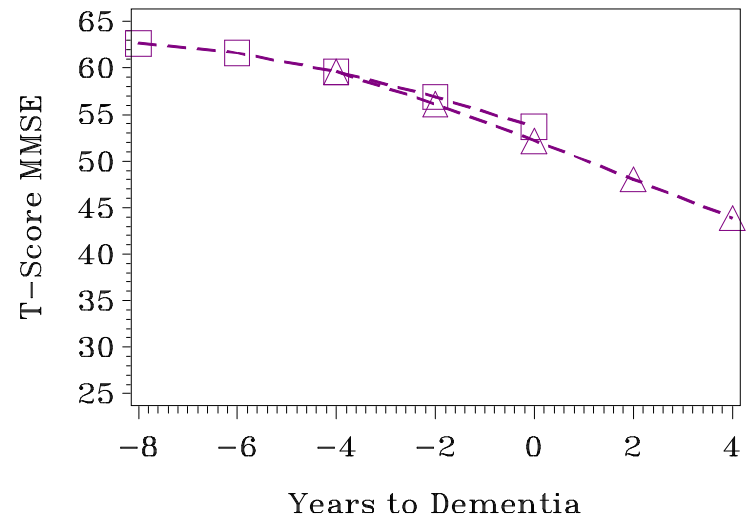
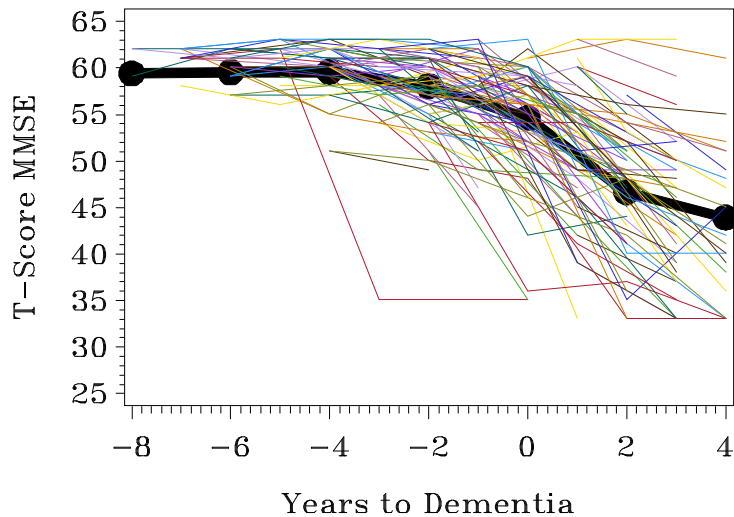
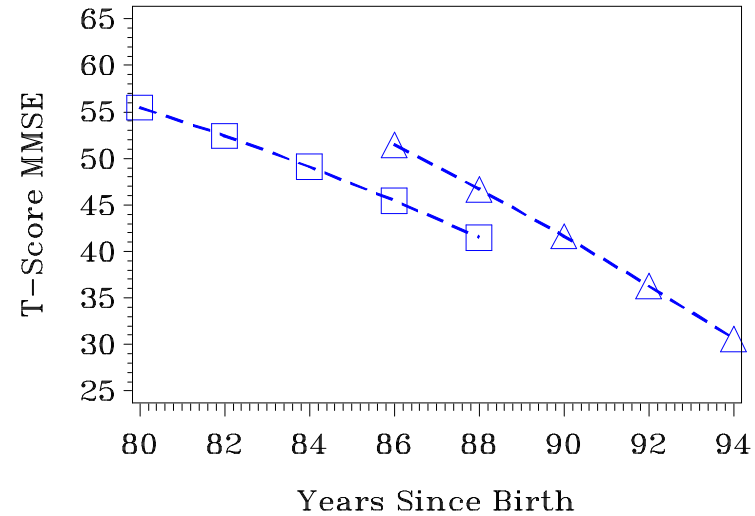
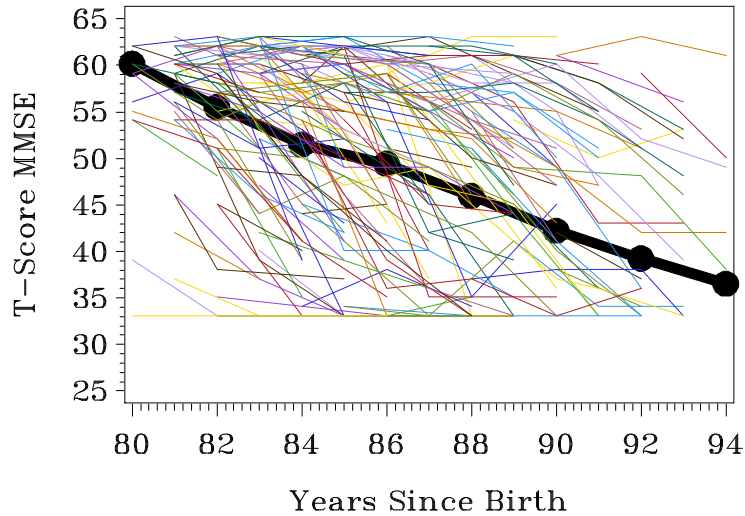
## Residual Variance



## Intercept Variance



# Comparing Models By Data...



# Road Map

- Steps in longitudinal analysis
- The missing step #2
- Example: Alternative metrics of “time”
- **What about just time?**
- What else contributes to “time”?

# What about just time as “Time”?

- When the accelerated time metrics do not show convergence of their BP and WP time effects, an alternative model specification may be more useful
- **Time-in-study models** separate BP and WP effects
  - Accelerated time (age, death...) model → Grand-mean-centering
  - Time-in-study model → **Person/group-mean-centering**
- Time-in-study models **can be made equivalent** to models with accelerated time metric in their fixed effects, but not in their random effects (as shown shortly)

# Just Time as "Time"

Original Age
Age <sub>ti</sub>
80
82
84
80
82
84
84
86
88
84
86
88

# Just Time as "Time"

BP Age	Original Age	WP Age
$\text{AgeT1}_i$	$\text{Age}_{ti}$	$\text{Age}_{ti} - \text{AgeT1}_i$
80	80	0
80	82	2
80	84	4
80	80	0
80	82	2
80	84	4
84	84	0
84	86	2
84	88	4
84	84	0
84	86	2
84	88	4

# Just Time as "Time"

BP Age	Original Age	WP Age	Original YTD
$\text{AgeT1}_i$	$\text{Age}_{ti}$	$\text{Age}_{ti} - \text{AgeT1}_i$	$\text{YTDdeath}_{ti}$
80	80	0	-12
80	82	2	-10
80	84	4	-8
80	80	0	-8
80	82	2	-6
80	84	4	-4
84	84	0	-12
84	86	2	-10
84	88	4	-8
84	84	0	-8
84	86	2	-6
84	88	4	-4



# Just Time as "Time"

BP Age	Original Age	WP Age	BP Years to Death	Original YTD	WP Years to Death
$AgeT1_i$	$Age_{ti}$	$Age_{ti} - AgeT1_i$	$YTdeathT1_i$	$YTdeath_{ti}$	$YTdeath_{ti} - YTdeathT1_i$
80	80	0	-12	-12	0
80	82	2	-12	-10	2
80	84	4	-12	-8	4
80	80	0	-8	-8	0
80	82	2	-8	-6	2
80	84	4	-8	-4	4
84	84	0	-12	-12	0
84	86	2	-12	-10	2
84	88	4	-12	-8	4
84	84	0	-8	-8	0
84	86	2	-8	-6	2
84	88	4	-8	-4	4

# Just Time as "Time"

Time:	BP Age	Original Age	WP Age	BP Years to Death	Original YTD	WP Years to Death
Years in Study	$Age_{T1_i}$	$Age_{t_i}$	$Age_{t_i} - Age_{T1_i}$	$YTdeath_{T1_i}$	$YTdeath_{t_i}$	$YTdeath_{t_i} - YTdeath_{T1_i}$
0	80	80	0	-12	-12	0
2	80	82	2	-12	-10	2
4	80	84	4	-12	-8	4
0	80	80	0	-8	-8	0
2	80	82	2	-8	-6	2
4	80	84	4	-8	-4	4
0	84	84	0	-12	-12	0
2	84	86	2	-12	-10	2
4	84	88	4	-12	-8	4
0	84	84	0	-8	-8	0
2	84	86	2	-8	-6	2
4	84	88	4	-8	-4	4

# Model Variants Using Age

## Level-1 Age-Based (Grand-MC):

$$y_{ti} = \beta_{0i} + \beta_{1i}(\text{Age}_{ti} - 84) + e_{ti}$$

## Level-1 Time-Based (Person-MC):

$$y_{ti} = \beta_{0i} + \beta_{1i}(\text{Age}_{ti} - \text{AgeT1}_i) + e_{ti}$$

## Same Level-2 Equations:

$$\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{AgeT1}_i - 84) + U_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{AgeT1}_i - 84) + U_{1i}$$

Same pattern would result in any other accelerated time metric (such as years to death)

## Level-2 AgeT1 effects:

**Age-Based: Incremental** effect of cross-sectional age (**contextual** age cohort effect)

**Time-Based: Total** effect of cross-sectional age (**between-person** age effect)

# Effect of Age Cohort on Intercept (Fixed Level-1 Linear Age Slope)

## Time-in-Study $\approx$ Person-MC:

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}(\text{Age}_{ti} - \text{AgeT1}_i) + e_{ti}$$

$$\text{Level 2: } \beta_{0i} = \gamma_{00} + \gamma_{01}(\text{AT1}_i) + U_{0i}$$

$$\beta_{1i} = \gamma_{10}$$

$$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(\text{AT1}_i) + \gamma_{10}(\text{Age}_{ti} - \text{AT1}_i) + U_{0i} + e_{ti}$$

← In terms of **Time**

$$\rightarrow y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(\text{AT1}_i) + \gamma_{10}(\text{Age}_{ti}) + U_{0i} + e_{ti}$$

← In terms of **Age**

## Age-Based $\approx$ Grand-MC:

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}(\text{Age}_{ti}) + e_{ti}$$

$$\text{Level 2: } \beta_{0i} = \gamma_{00} + \gamma_{01}(\text{AT1}_i) + U_{0i}$$

$$\beta_{1i} = \gamma_{10}$$

$$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(\text{AT1}_i) + \gamma_{10}(\text{Age}_{ti}) + U_{0i} + e_{ti}$$

Term	Time	Age
Intercept	$\gamma_{00}$	$\gamma_{00}$
WP Effect	$\gamma_{10}$	$\gamma_{10}$
Contextual	$\gamma_{01} - \gamma_{10}$	$\gamma_{01}$
BP Effect	$\gamma_{01}$	$\gamma_{01} + \gamma_{10}$

# Effect of Age Cohort on Level-1 Age Slope (Fixed Level-1 Linear Age Slope)

On left below: Time-in-Study  $\approx$  Person-MC:

$$\text{Time as Time: } y_{ti} = \gamma_{00} + \gamma_{10}(\text{Age}_{ti} - \text{AT1}_i) + \gamma_{11}(\text{Age}_{ti} - \text{AT1}_i)(\text{AT1}_i) + \gamma_{01}(\text{AT1}_i) + \gamma_{02}(\text{AT1}_i^2) + U_{0i} + e_{ti}$$

$$\text{Time as Age: } y_{ti} = \gamma_{00} + \gamma_{10}(\text{Age}_{ti}) + \gamma_{11}(\text{Age}_{ti})(\text{AT1}_i) + (\gamma_{01} - \gamma_{10})(\text{AT1}_i) + (\gamma_{02} - \gamma_{11})(\text{AT1}_i^2) + U_{0i} + e_{ti}$$

On right below: Age-Based  $\approx$  Grand-MC:

$$y_{ti} = \gamma_{00} + \gamma_{10}(\text{Age}_{ti}) + \gamma_{11}(\text{Age}_{ti})(\text{AT1}_i) + \gamma_{01}(\text{AT1}_i) + \gamma_{02}(\text{AT1}_i)^2 + U_{0i} + e_{ti}$$

Adding AgeT1<sup>2</sup>  
creates equivalence

Intercept:  $\gamma_{00} = \gamma_{00}$       BP Effect:  $\gamma_{01} = \gamma_{01} + \gamma_{10}$       Cohort:  $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WP Effect:  $\gamma_{10} = \gamma_{10}$       BP<sup>2</sup> Effect:  $\gamma_{02} = \gamma_{02} + \gamma_{11}$       Cohort<sup>2</sup>:  $\gamma_{02} = \gamma_{02} - \gamma_{11}$

BP\*WP or Cohort\*WP is the same:  $\gamma_{11}$

# Add Quadratic Level-1 Age Slope (Fixed Level-1 Age Slopes)

On left below: Time-in-Study  $\approx$  Person-MC:

$$\text{Time as Time: } y_{ti} = \gamma_{00} + \gamma_{10}(\text{Age}_{ti} - \text{AT1}_i) + \gamma_{20}(\text{Age}_{ti} - \text{AT1}_i)^2 \\ + \gamma_{11}(\text{Age}_{ti} - \text{AT1}_i)(\text{AT1}_i) + \gamma_{01}(\text{AT1}_i) + \gamma_{02}(\text{AT1}_i^2) + U_{0i} + e_{ti}$$

$$\text{Time as Age: } y_{ti} = \gamma_{00} + \gamma_{10}(\text{Age}_{ti}) + \gamma_{20}(\text{Age}_{ti}^2) + (\gamma_{11} - 2\gamma_{20})(\text{Age}_{ti})(\text{AT1}_i) \\ + (\gamma_{01} - \gamma_{10})(\text{AT1}_i) + (\gamma_{02} + \gamma_{20} - \gamma_{11})(\text{AT1}_i^2) + U_{0i} + e_{ti}$$

On right below: Age-Based  $\approx$  Grand-MC:

$$y_{ti} = \gamma_{00} + \gamma_{10}(\text{Age}_{ti}) + \gamma_{20}(\text{Age}_{ti})^2 + \gamma_{11}(\text{Age}_{ti})(\text{AT1}_i) \\ + \gamma_{01}(\text{AT1}_i) + \gamma_{02}(\text{AT1}_i)^2 + U_{0i} + e_{ti}$$

Intercept:  $\gamma_{00} = \gamma_{00}$

BP Effect:  $\gamma_{01} = \gamma_{01} + \gamma_{10}$

Cohort:  $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WP Effect:  $\gamma_{10} = \gamma_{10}$

BP<sup>2</sup> Effect:  $\gamma_{02} = \gamma_{02} + \gamma_{11} + \gamma_{20}$

Cohort<sup>2</sup>:  $\gamma_{02} = \gamma_{02} - \gamma_{11} + \gamma_{20}$

WP<sup>2</sup> Effect:  $\gamma_{20} = \gamma_{20}$

BP\*WP:  $\gamma_{11} = \gamma_{11} + 2\gamma_{20}$

Cohort\*WP:  $\gamma_{11} = \gamma_{11} - 2\gamma_{20}$

# Time-in-Study Models so far...

- Specify WP change using only longitudinal information
- Are equivalent within persons across accelerated time metrics
- Because unique information from the alternative time metrics is really only available BP, it only shows up in the level-2 model
- Can (usually) be made equivalent in their fixed effects to models based in alternative accelerated time metrics
- **So why make a distinction? Different random effects...**

# Random Slopes Across Models

## Time-in-Study $\approx$ Person-MC:

$$\text{as Time: } y_{ti} = \gamma_{00} + \gamma_{10}(\text{Age}_{ti} - \text{AT1}_i) + \gamma_{01}(\text{AT1}_i) \\ + U_{0i} + U_{1i}(\text{Age}_{ti} - \text{AT1}_i) + e_{ti}$$

$$\text{as Age: } y_{ti} = \gamma_{00} + \gamma_{10}(\text{Age}_{ti}) + (\gamma_{01} - \gamma_{10})(\text{AT1}_i) \\ + U_{0i} + \boxed{U_{1i}(\text{Age}_{ti}) - U_{1i}(\text{AT1}_i)} + e_{ti}$$

## Age-Based $\approx$ Grand-MC:

$$y_{ti} = \gamma_{00} + \gamma_{10}(\text{Age}_{ti}) + \gamma_{01}(\text{AT1}_i) \\ + U_{0i} + \boxed{U_{1i}(\text{Age}_{ti})} + e_{ti}$$

**So which do we choose?**

Variance due to  $\text{AT1}_i$  is still part of the random slope in the age-based model. So the time-based and age-based models cannot be made equivalent in terms of random effects variances.



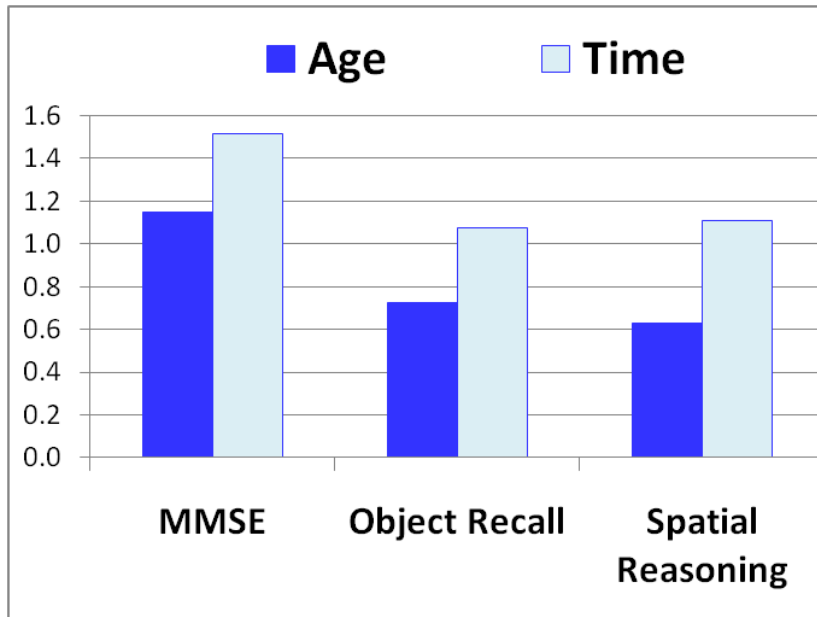
# Random Slopes Across Models

- **Random intercepts** mean different things under each model:
  - **Time: Person-MC** → Individual differences at **time=0** (that everyone has)
  - **Age: Grand-MC** → Individual differences at **age=0** (that not everyone has)
- **Differential shrinkage of the random intercepts** results from differential reliability of the intercept data across models:
  - **Person-MC** → Won't affect shrinkage of slopes unless highly correlated
  - **Grand-MC** → Will affect shrinkage of slopes due to forced extrapolation
- As a result, the **random slope variance may be smaller** under **grand-MC (age, death...)** than under **person-MC (time)**
  - Problem worsens with greater BP variation in time (more extrapolation)
  - Anecdotal example of downward bias using clustered data was presented in Raudenbush & Bryk (2002; chapter 6), but what about in these data?
  - **2023 update: The age-as-time model assumes no contextual effect for the random age slope, whereas the time model does not—stay tuned!**

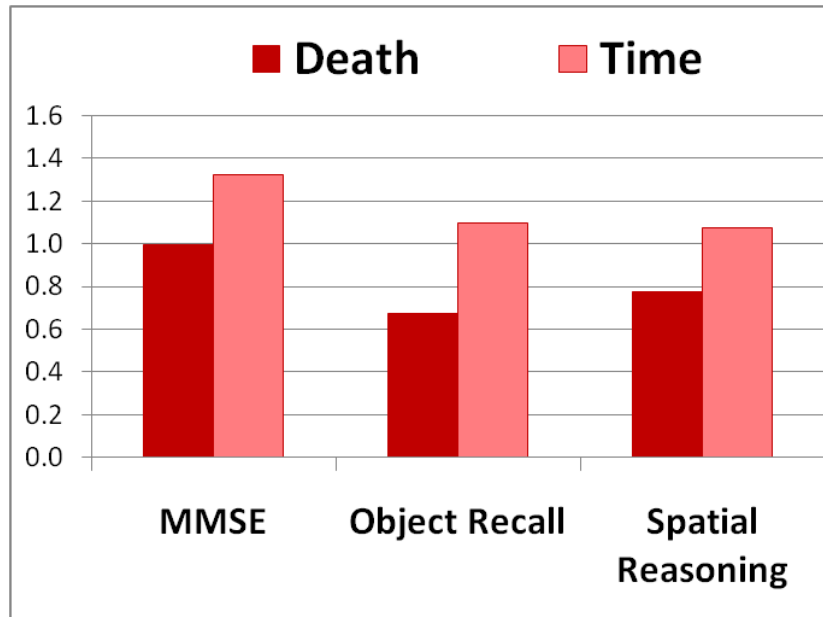
# Slope Variance in Example Models

- Slope variance estimate was indeed **33-77% larger** in the time-based model versions across outcomes...

Years-Since-Birth (47% BP)

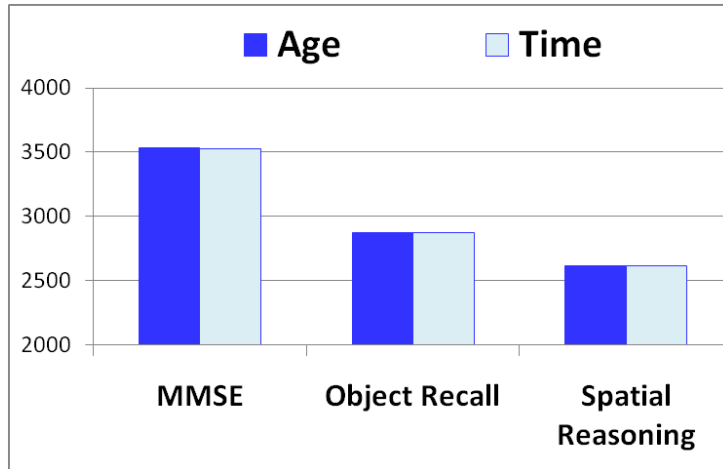


Years-to-Death (24% BP)

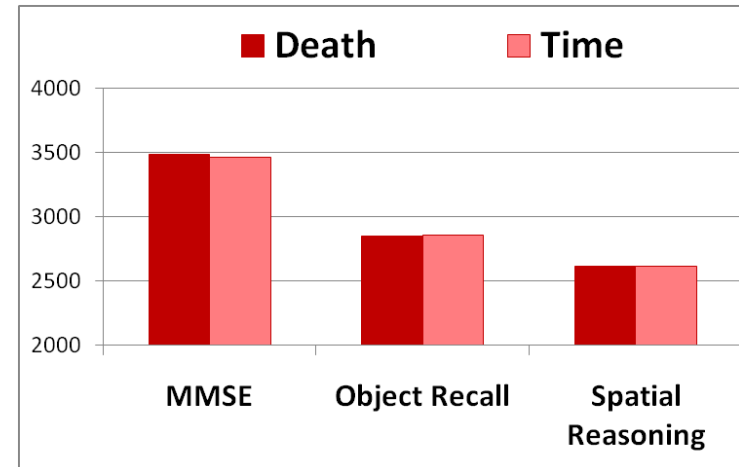
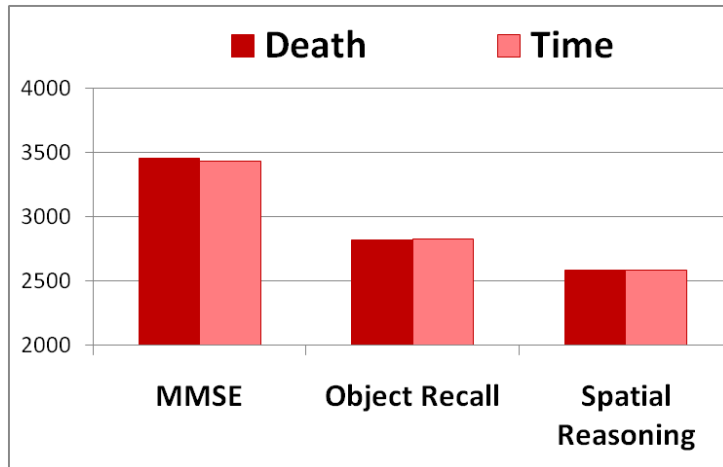
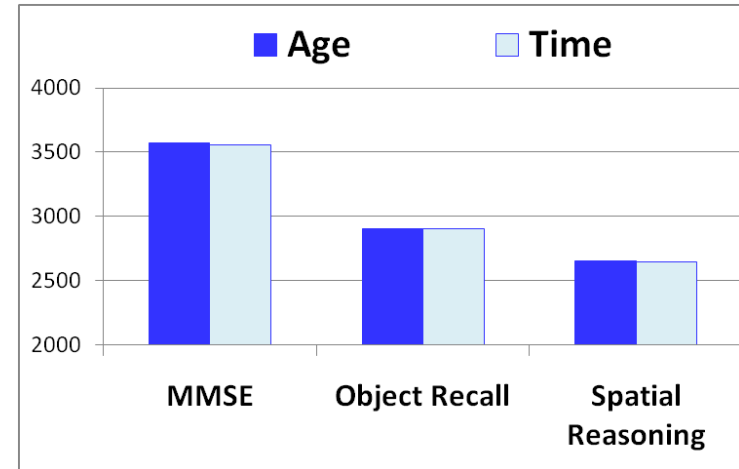


# ... Although model fit was the same

## ML AIC

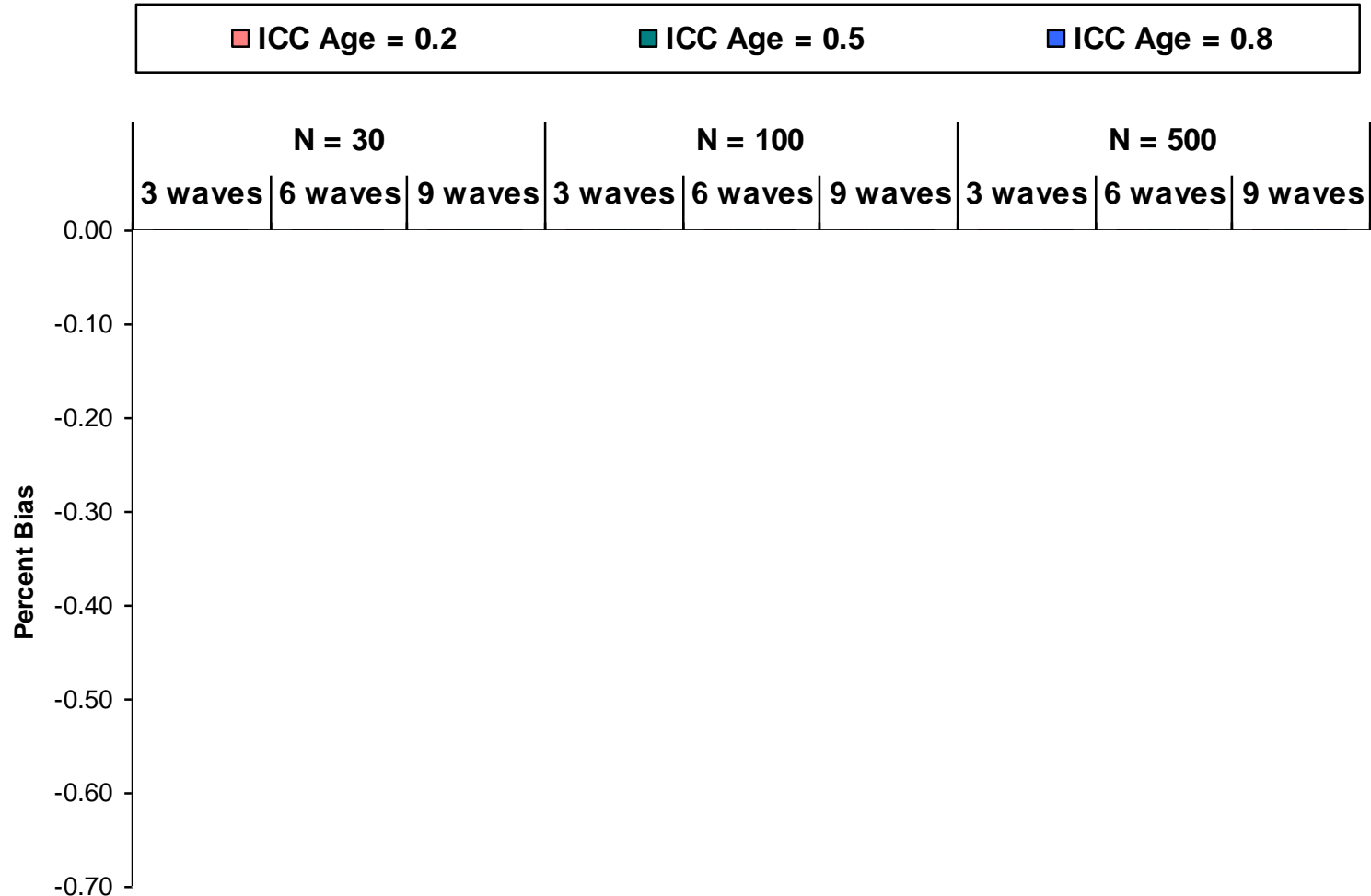


## ML BIC



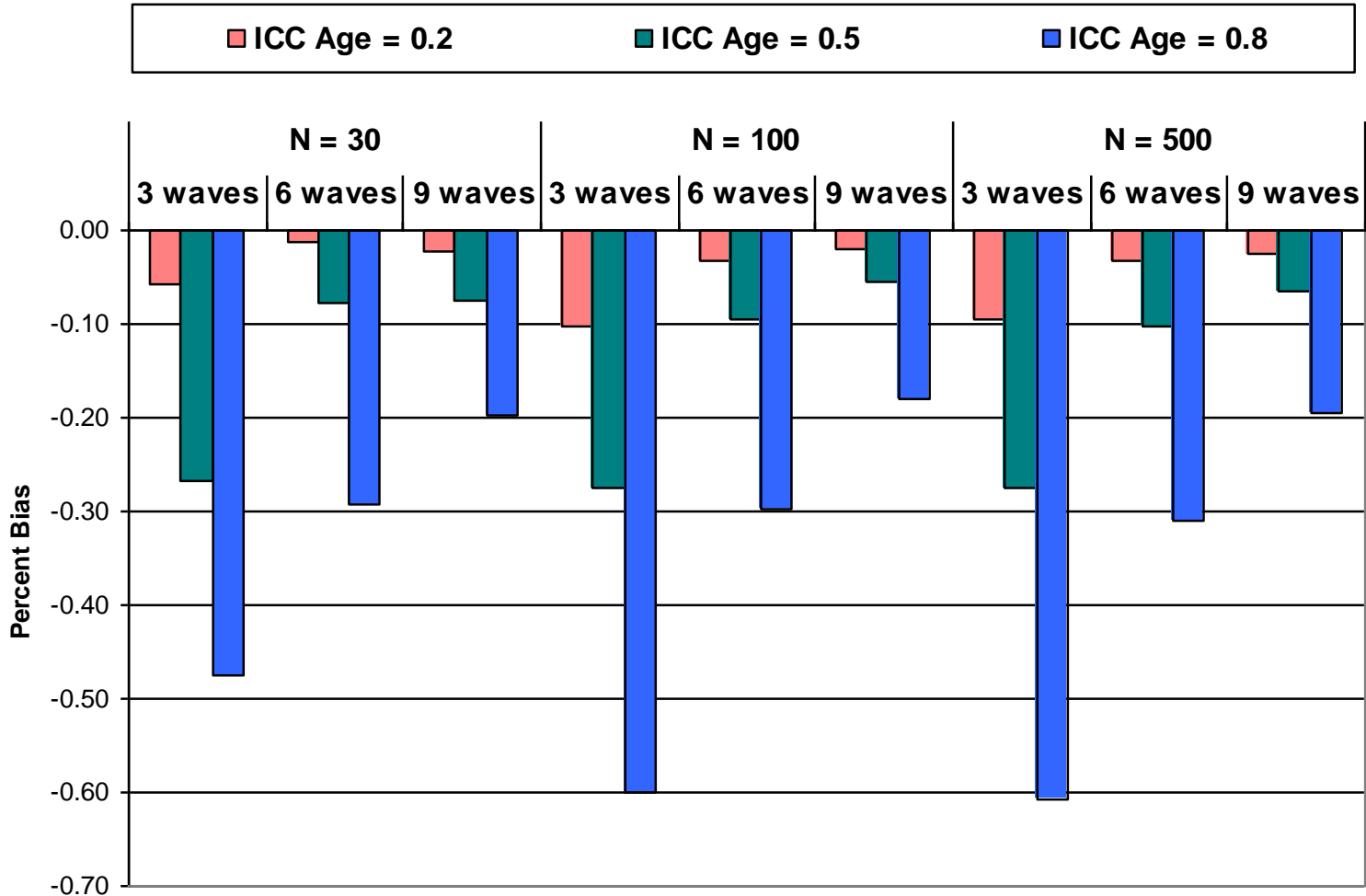
# 2008 Simulation Study Results (Generated by Time, Analyzed by Age)

## Percent Bias in Random Slope Variance



# 2008 Simulation Study Results (Generated by Time, Analyzed by Age)

Percent Bias in Random Slope Variance



# And so the winner is... Time?

- Although seemingly the most non-informative choice, simply tracking **change as a function of study duration**:
  - Represents **WP changes** as directly and parsimoniously as possible
  - Seems to recover **random slope variance** better in accelerated designs **(2023 update—because it doesn't assume no contextual effect!)**
  - Permits inclusion of persons who have not experienced events in an alternative time metric (e.g., death, dementia diagnosis)
- Time-in-study models make no assumptions about processes causing change, so these become **testable hypotheses**
  - Do persons who are older start lower and decline faster?
    - *Age main effect, Age\*Time interaction*
  - After considering mortality, do older persons *still* decline faster?
    - *Competing YTdeath\*Time and Age\*Time interactions*

# Road Map

- Steps in longitudinal analysis
- The missing step #2
- Example: Alternative metrics of “time”
- What about just time?
- **What else contributes to “time”?**

# What about retest effects?

- Are estimates of age-related change too small without controlling for **practice effects** due to repeated testing?

- Can **time-in-study** index **retest** in age-based models?

$$y_{ti} = \gamma_{00} + \gamma_{10}(\text{Age}_{ti}) + \gamma_{20}(\text{Age}_{ti})^2 \dots$$

$$+ e_{ti} + U_{0i} + \gamma_{1i}(\text{Age}_{ti}) + \dots$$

Individual change due to age

$$+ \gamma_{30}(\text{Retest}_{ti})$$

$$+ \gamma_{40}(\text{Retest}_{ti})(\text{Age}_{ti}) \dots$$

Retest = Time = Difference due to which occasion of measurement

- But not including age cohort assumes **age convergence**...  
What if age cohort (AT1) and retest effects are BOTH included?
  - 2011 simulation results: missing cohort effects will masquerade as retest effects in the opposite direction—they are **confounded by design**



# Conclusions

- When time has both BP and WP variation, one should always **carefully consider what “time” could and should be**
  - Otherwise, aggregate trends may not actually describe any individuals
  - Individual differences can be created artificially through the mis-alignment of different persons onto a single “time” trajectory
- **Multiple processes** may be at work simultaneously, but they have to be **observed independently** to be distinguishable
  - Age vs. Mortality: can be distinguished if not everyone dies at same age
  - But if **aging and retest occur simultaneously within-persons**, retest effects cannot be distinguished from effects of aging and age cohort
  - **Age/Cohort/Time in design** → **Age/Cohort/Retest in models**
- Considering the effects of time is an important precursor to making informed use of advances in longitudinal models...

# Thank you for your time...

- **Questions or comments? Please email me!**
- **Slides available at for download [on this page](#)**
- **Relevant Works:**
  - Hoffman, L., Hofer, S. M., & Sliwinski, M. J. (2011). On the confounds among retest gains and age-cohort differences in the estimation of within-person change in longitudinal studies: A simulation study. *Psychology and Aging*, 26(4), 778–791.
  - Hoffman, L. (2012). *Considering alternative metrics of time: Does anybody really know what “time” is?* In J. Harring & G. Hancock (Eds.), *Advances in Longitudinal Methods in the Social and Behavioral Sciences* (pp. 255–287). Information Age Publishing.
  - Hoffman, L., & Templin, J. L. (April, 2008). *The impact of alternative specifications of time on examining individual differences in change.* Poster presented at the Cognitive Aging Conference, Atlanta, GA.