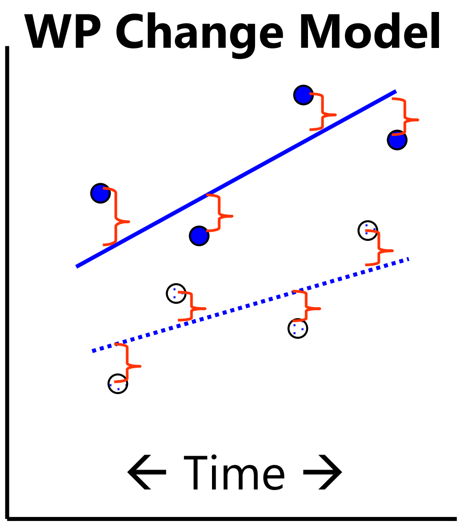
Time-Varying (TV) Predictors in Longitudinal Models of Within-Person Fluctuation

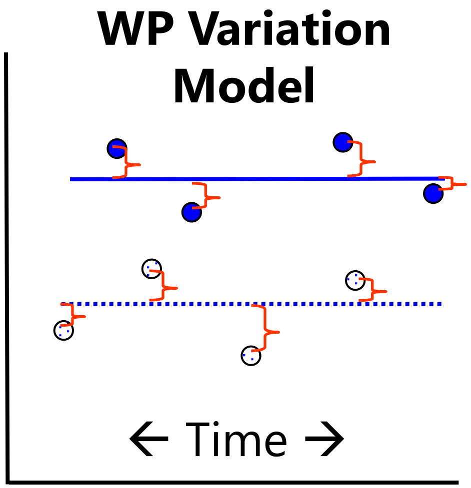
- Topics:
  - Concepts and what NOT to do with level-1 TV predictors
  - Univariate MLM strategies:
    - Person-(group/cluster)-mean-centering (*aka*, variable-centering)
    - Grand-mean-centering (*aka*, constant-centering)
  - Multivariate MLM strategies:
    - Latent centering (*aka*, turn the TV predictor into a TV outcome)
    - Implications for longitudinal (multilevel) mediation

The Joy of Time-Varying (TV) Predictors

- TV predictors predict leftover **Level-1 WP (residual) variation**:



If model for time works, then residuals should look like this →



- Modeling TV predictors (or any level-1 predictor) is complicated because they potentially contain **two different relations with y_{ti}**:
  - Relation of the *level-1 within-person* variation in the predictor x_{ti} with y_{ti}
  - Relation of the *level-2 between-person* variation in the predictor x_{ti} with y_{ti}
  - For now, we are assuming the predictor x_{ti} only **fluctuates** over time…
    - *We will need a **different model** when x_{ti} changes individually over time!*

The Joy of Time-Varying Predictors

- Time-varying (TV) predictors can usually have 2 levels of relations because **they are really 2 predictors in 1 variable**

- Example: Stress measured daily (to be used as predictor)
  - Some days are worse than others:
    - **Level-1 WP variation** *(can be captured using deviation from own mean)*
  - Some people just have more stress than others all the time:
    - **Level-2 BP variation** *(can be captured using person mean over time)*

- Can quantify relative sources of variation with an **ICC**
  - Intraclass Correlation ICC = (BP variance) / (BP variance + WP variance)
  - **ICC < 1?** TV predictor has **WP** variation (so it *could* have a **L1 WP** slope)
  - **ICC > 0?** TV predictor has **BP** variation (so it *could* have a **L2 BP** slope)
    - ICC specifically captures BP mean variation, but change variation is possible, too!

Between-Person vs. Within-Person Slopes

- Between- and within-person slopes could be in SAME direction
  - Time-Varying Stress → Time-Varying Health?
    - **Level-1 WP: People may feel worse than usual when they are currently under more stress than usual (regardless of what "usual" is)**
    - **Level-2 BP: People with more chronic stress than other people may have worse general health than people with less chronic stress**
- Between- and within-person slopes could be in OPPOSITE directions
  - Time-Varying Exercise → Time-Varying Blood pressure?
    - **Level-1 WP: During exercise, blood pressure is higher than during rest**
    - **Level-2 BP: People who exercise more often generally have lower blood pressure than people who are more sedentary**
- L1 within-person and L2 between-person slopes usually differ
  - Why? Because variables have different **meanings** at each level!
  - Why? Because variables have different **scales** at each level!

WAY WRONG: Within-Person Fluctuation Model with x_{ti} **represented at Level 1 Only:** → Its WP and BP Slopes are **Smushed Together**

x_{ti} is centered into TVx_{ti} WITHOUT representation at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{TVx_{ti}}) + \mathbf{e_{ti}}$

$TVx_{ti} = x_{ti} - C_1$ → it still has both Level-2 BP and Level-1 WP variation

Level 2: $\beta_{0i} = \boldsymbol{\gamma_{00}} + \mathbf{U_{0i}}$

$\beta_{1i} = \boldsymbol{\gamma_{10}}$

γ_{10} = *smushed* WP and BP effects

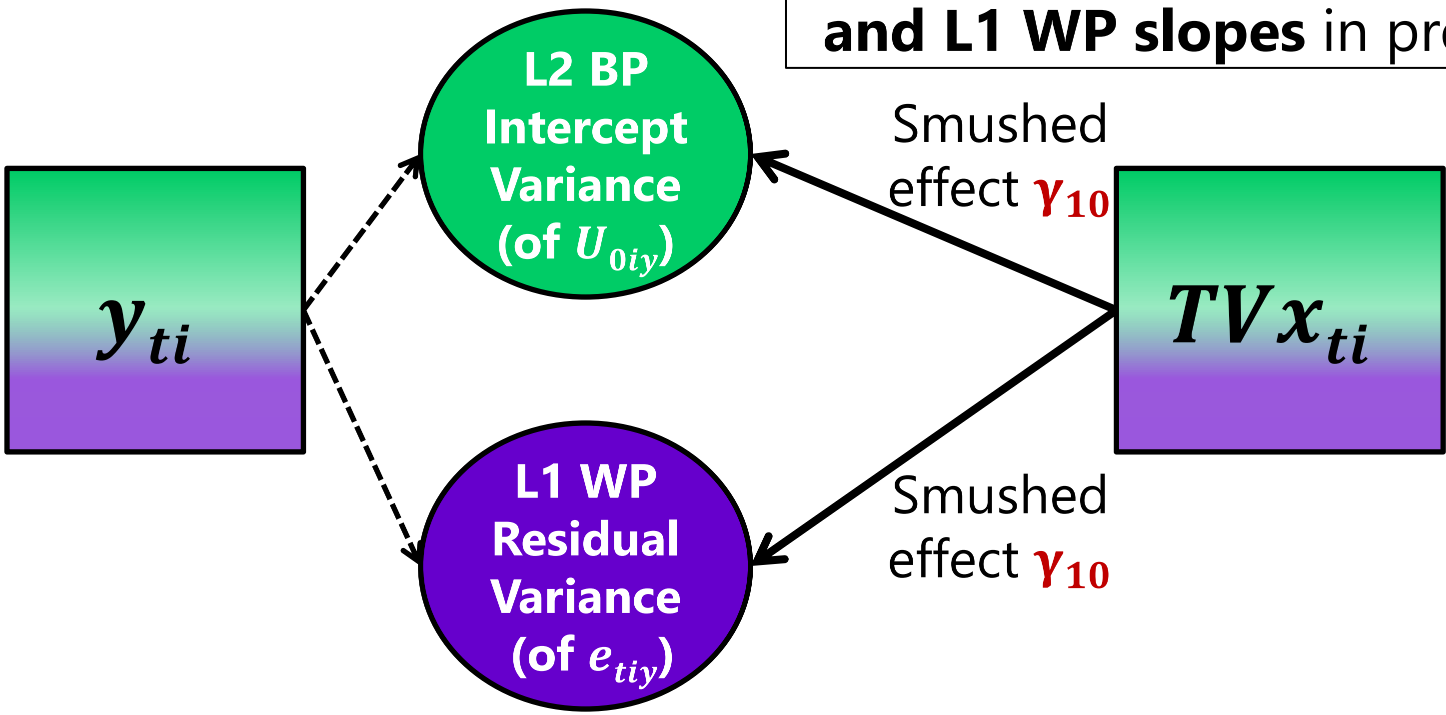
Because TVx_{ti} still contains its original 2 different kinds of variation (BP and WP), its 1 fixed slope has to do the work of 2 predictors!

A *smushed* effect (to me) is also known as a *convergence*, *conflated*, or *composite* effect

Univariate MLM: Adding a Level-1 Predictor Without Level-2 Representation = Smushing

BP and WP variance in the **observed level-1 y_{ti} outcome** is partitioned by the **model** into estimated **variance components**

Observed level-1 TVx_{ti} predictor still has both BP and WP variance. AND given that TVx_{ti} has only **one fixed slope**, it captures a smushed effect that presumes **equal L2 BP and L1 WP slopes** in predicting y_{ti}!



3 Kinds of Fixed Slopes for TV Predictors

- **Is there a Level-1 Within-Person (WP) slope?**
  - When you have a higher x_{ti} predictor value than usual (*at this occasion*), do you also have a higher (or lower) y_{ti} outcome value than usual (*at same or later occasion*)?
  - If so, the **level-1 within-person *part* of the TV predictor** will reduce the level-1 residual variance (σ_e^2) of the TV outcome

- **Is there a Level-2 Between-Person (BP) slope?**
  - Do people with higher x_{ti} predictor values than other people (*on average over time*) also have higher (or lower) y_{ti} outcomes than other people (*on average over time*)?
  - If so, the **level-2 between-person *part* of the TV predictor** will reduce level-2 random intercept variance ($\tau_{U_0}^2$) of the TV outcome

- **Is there a Level-2 Contextual slope: Do the L2 BP and L1 WP slopes differ?**
  - After controlling for the actual value of TV predictor at that occasion, is there still **an incremental contribution** from the **level-2 between-person *part* of the TV predictor** (i.e., does one's general tendency matter beyond current TVP value)?
  - Equivalently, the **Level-2 Contextual slope = L2 BP slope − L1 WP slope**, so the Level-2 Contextual slope directly tests **if a smushed slope is ok (pry not!)**

3 Options to Prevent Smushed Slopes

- Within Univariate MLM framework (predict only one column):
  1. **Person-mean-centering**: manually carve up TV predictor into its level-specific parts using observed variables (1 predictor per level)
     - More generally, this is "**variable-centering**" because you are **subtracting a variable** (e.g., the cluster/group/person mean or person baseline value)
     - Will always yield **level-1 within slopes** and **level-2 between slopes**!
  2. **Grand-mean-centering**: do NOT carve up TV predictor into its level-specific parts, but add level-2 mean to distinguish level-specific slopes
     - More generally, this is "**constant-centering**" because you are **subtracting a constant** but still keeping all levels of variance in level-1 TV predictor
     - Choice of constant is irrelevant (changes where 0 is, not what variance it has)
     - Will always yield **level-1 within slopes** and **level-2 contextual slopes**!
- Within Multivariate MLM framework (via M-SEM or SEM):
  3. **Latent-centering**: Treat the TV predictor as another outcome → let the model carve it up into **level-specific latent variables**
     - Best in theory, but the type of level-2 slope provided (between or contextual) depends on type of model syntax (and the estimator in Mplus)! (Hoffman, 2019)
     - We will forgo this option for now (and will return to it later)

Option 1. Person-Mean-Centering (P-MC)

- In **P-MC**, we turn the TV predictor $\boldsymbol{x_{ti}}$ into **2 observed variables** that directly represent its BP (level-2) and WP (level-1) sources of variation and **include these 2 predictors instead of original $\boldsymbol{x_{ti}}$**:

- **Level-2, BP predictor = person mean of $\boldsymbol{x_{ti}}$**
  - $\mathbf{PMx_i} = \boldsymbol{\bar{x}_i} - \boldsymbol{C_2}$
  - $\mathrm{PMx_i}$ is centered at constant C_2, chosen for meaningful 0 (e.g., sample mean)
  - $\mathrm{PMx_i}$ is positive? Above sample mean → "more than other people"
  - $\mathrm{PMx_i}$ is negative? Below sample mean → "less than other people"

- **Level-1, WP predictor = deviation from person mean of $\boldsymbol{x_{ti}}$**
  - $\mathbf{WPx_{ti}} = \boldsymbol{x_{ti}} - \boldsymbol{\bar{x}_i}$ *(note: uncentered person mean $\boldsymbol{\bar{x}_i}$ is used to center $\boldsymbol{x_{ti}}$)*
  - $\mathrm{WPx_{ti}}$ is NOT centered at a constant – **we subtract a VARIABLE**
  - $\mathrm{WPx_{ti}}$ is positive? Above your own mean → "more than usual"
  - $\mathrm{WPx_{ti}}$ is negative? Below your own mean → "less than usual"

Within-Person Fluctuation Model with **Person-Mean-Centered Level-1 x_{ti}**

→ WP and BP slopes directly as **separate** parameters

x_{ti} is person-mean-centered into WPx_{ti}, with PMx_i at L2:

$WPx_{ti} = x_{ti} - \bar{x}_i$ → **it has only Level-1 WP variation**

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(WPx_{ti}) + e_{ti}$

$PMx_i = \bar{x}_i - C_2$ → **it has only Level-2 BP variation**

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

γ_{10} = L1 WP main effect of having *more x_{ti} than usual*

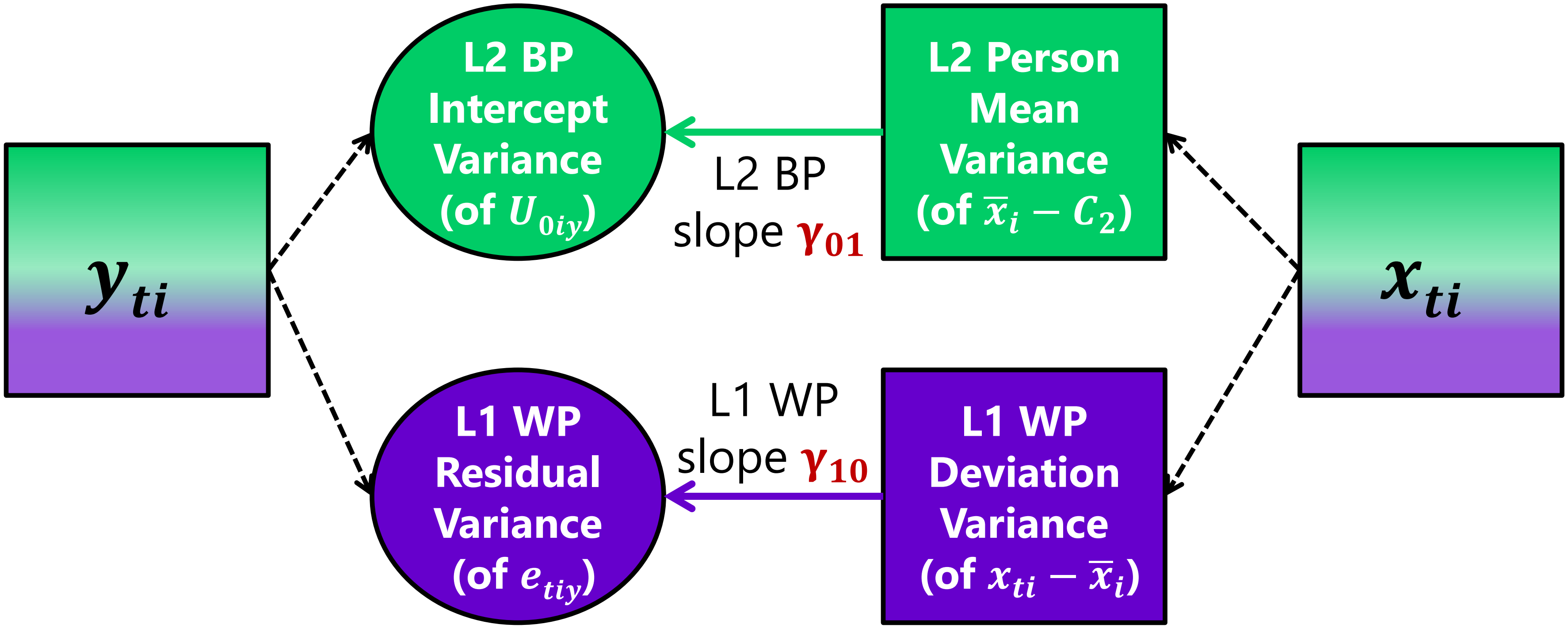
γ_{01} = L2 BP main effect of having *more $\bar{x}_i$ than other people*

Because WPx_{ti} and PMx_i are uncorrelated, each gets the total effect for its level (WP=L1, BP=L2)

Univariate MLM: Variable-Centering*

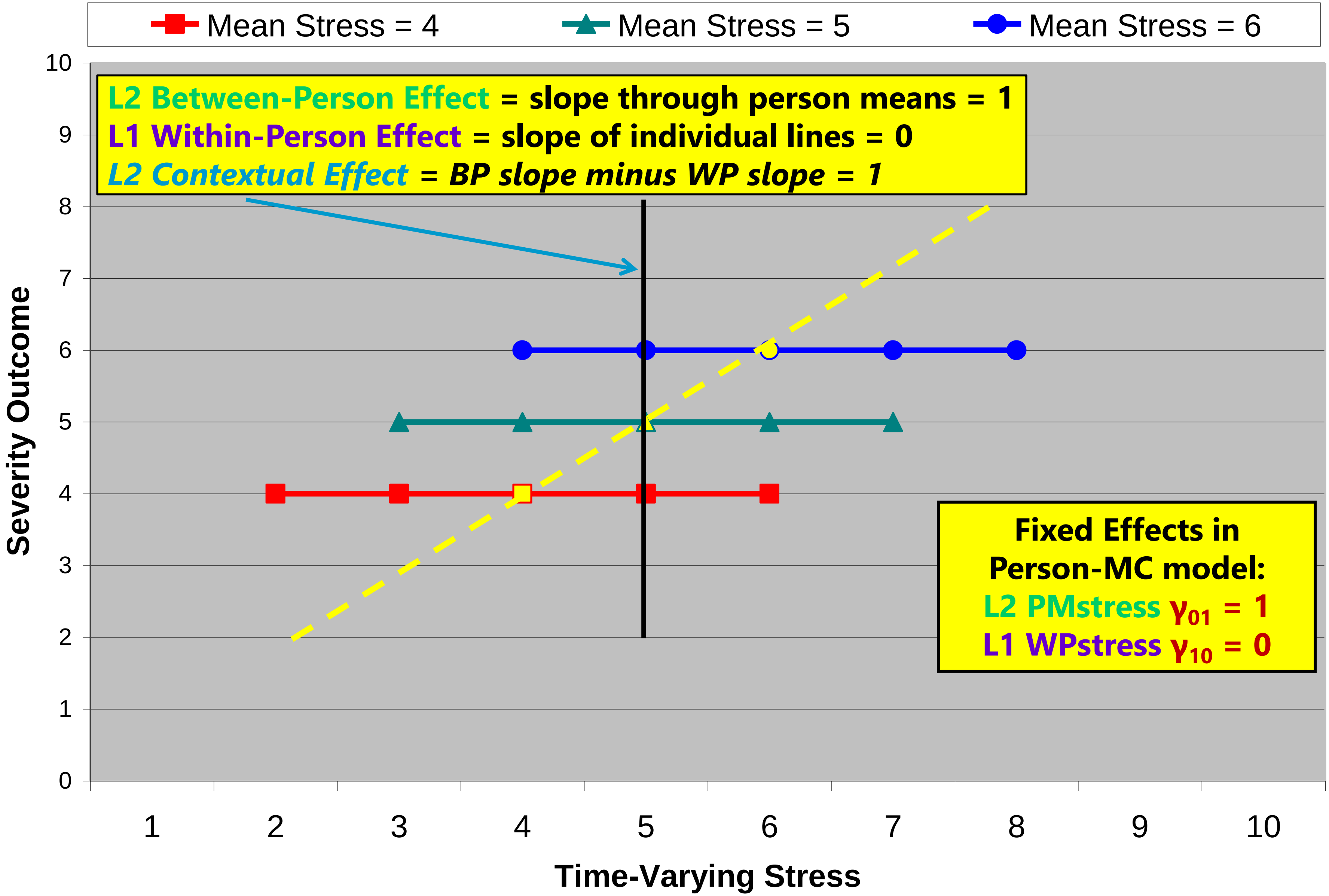
Model-based partitioning of level-1 y_{ti} outcome into level-specific **latent variables**

Manual partitioning of level-1 x_{ti} predictor into level-specific **observed variables**

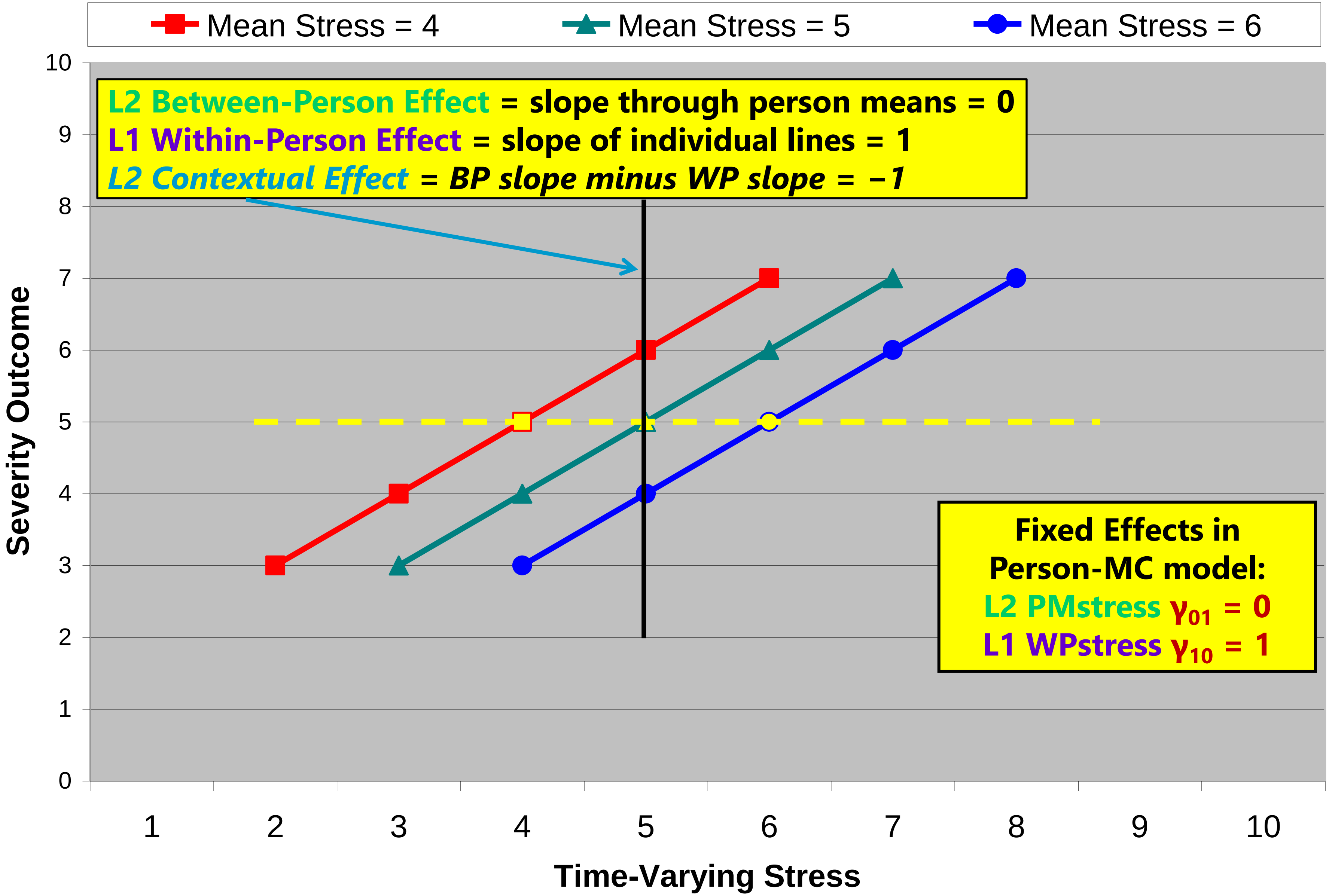


* Known as "person-mean-centering" more generally directly analogous to cluster/group-mean-centering in multilevel models for clustered data)

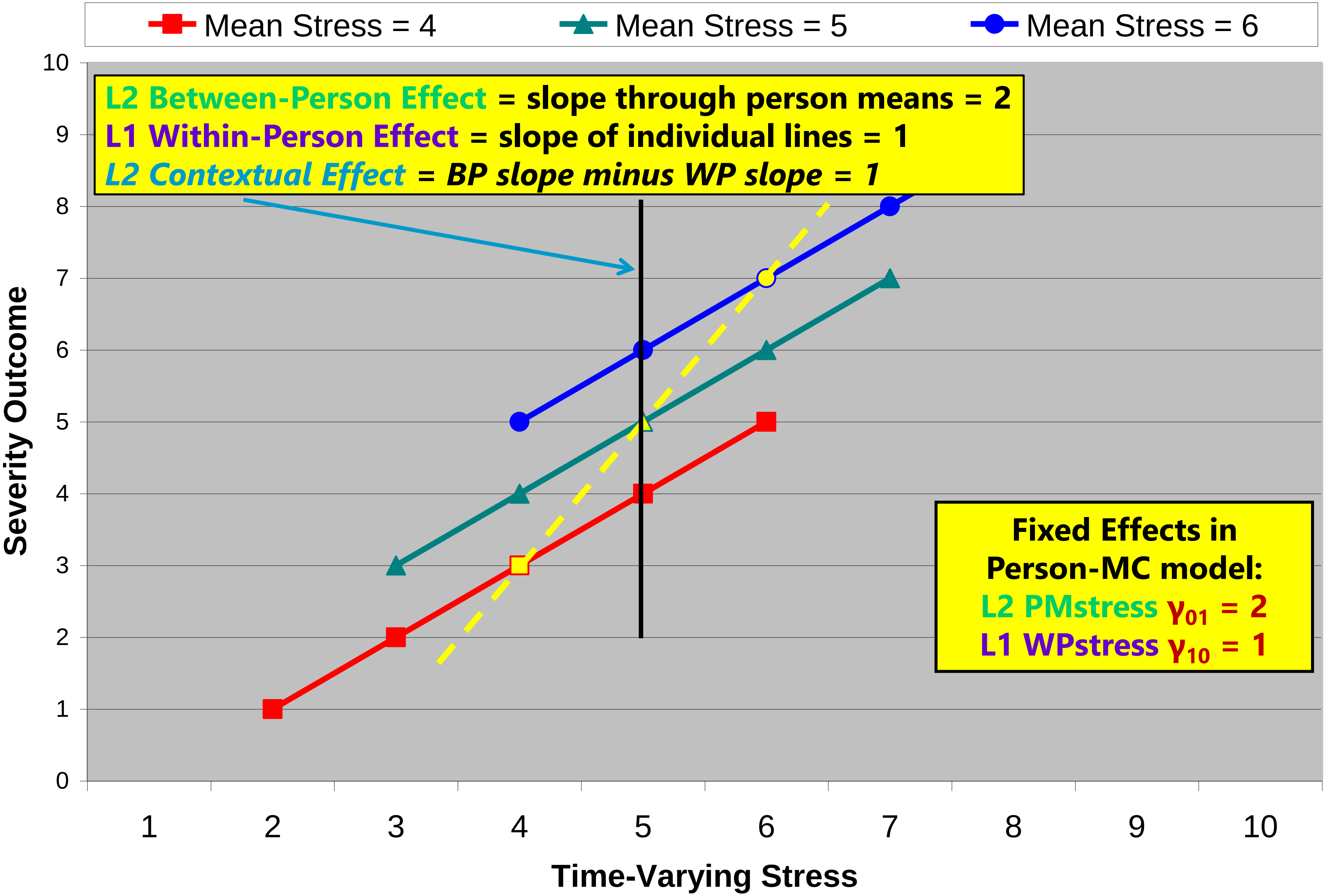
<u>ALL</u> Between-Person Effect, <u>NO</u> Within-Person Effect



NO Between-Person Effect, ALL Within-Person Effect

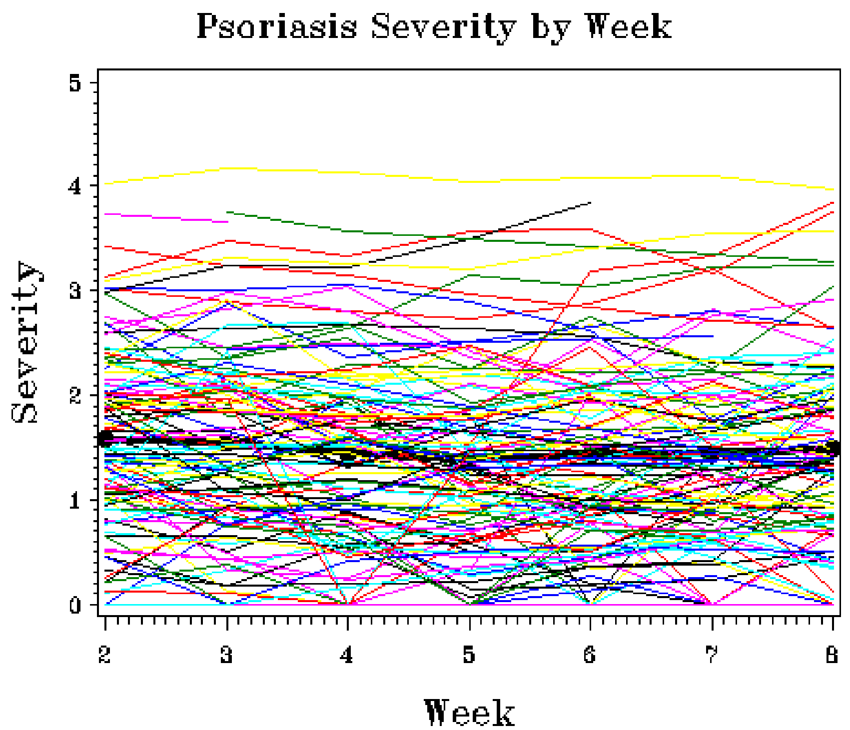


Between-Person Effect > Within-Person Effect



Example: Weekly Stress and Psoriasis

- 127 psoriasis patients, 8 weekly assessments (only last 7 used)
- How does perceived stress predict psoriasis severity? And is there a time lag for these processes to occur?
- No change in treatment → only WP fluctuation over time
- Analysis plan:
  - ICCs for stress and severity—how much variance is at each level?
  - Assess pattern of variance and covariance in severity over time
    - This was [PSQF 6271 Example 4]
  - Evaluate prediction of severity by stress at lag 0 and lag 1 weeks… without smushing!



Example: Weekly Stress and Psoriasis

- Empty means, random intercept model to get ICCs → proportion of total variance due to BP mean differences
  - For each variable: $y_{ti} = \gamma_{00} + U_{0i} + e_{ti}$, $\mathbf{ICC} = \dfrac{\tau^2_{U_0}}{\tau^2_{U_0} + \sigma^2_e} = \dfrac{\mathbf{BP}}{\mathbf{BP+WP}}$
  - Severity outcome: ICC = .83; stress predictor: ICC = .56

- For the severity outcome, the best-fitting unconditional time model for the variance had a level-2 random intercept (in G), along with heterogeneous level-1 residual variances and a Toeplitz (banded) correlation structure up to lag 3 (in R, below)

Estimated R Correlation Matrix for ID 1 → WP residual correlation

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.5115	0.3566	0.1112			
2	0.5115	1.0000	0.5115	0.3566	0.1112		
3	0.3566	0.5115	1.0000	0.5115	0.3566	0.1112	
4	0.1112	0.3566	0.5115	1.0000	0.5115	0.3566	0.1112
5		0.1112	0.3566	0.5115	1.0000	0.5115	0.3566
6			0.1112	0.3566	0.5115	1.0000	0.5115
7				0.1112	0.3566	0.5115	1.0000

Example: Weekly Stress and Psoriasis

Level 1: $\text{severity}_{ti} = \beta_{0i} + \beta_{1i}(\text{WPstressLag0}_{ti}) + \beta_{2i}(\text{WPstressLag1}_{ti}) + e_{ti}$

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{PMstress}_{i}) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\beta_{2i} = \gamma_{20}$

WP effects are fixed (no random slopes) → same for everyone

$\text{WPx}_{ti} = x_{ti} - \bar{x}_i$ → **it has only Level-1 WP variation**

$\text{PMx}_{i} = \bar{x}_i - 2$ → **it has only Level-2 BP variation**

Model for the Means:

- γ_{00} → expected severity for someone with person mean stress = 2, and who had severity = 2 last week and currently
- γ_{01} → BP difference in *average* severity per unit person mean stress
- γ_{10} and γ_{20} → WP change in *current* severity per unit more stress than usual this week (lag 0) and last week (lag 1)

Example: Weekly Stress and Psoriasis

Level 1: $\text{severity}_{ti} = \beta_{0i} + \beta_{1i}(\text{WPstressLag0}_{ti}) + \beta_{2i}(\text{WPstressLag1}_{ti}) + e_{ti}$

Level 2: $\beta_{0i} = 1.96 + 0.48^{*}(\text{PMstress}_{i}) + U_{0i}$

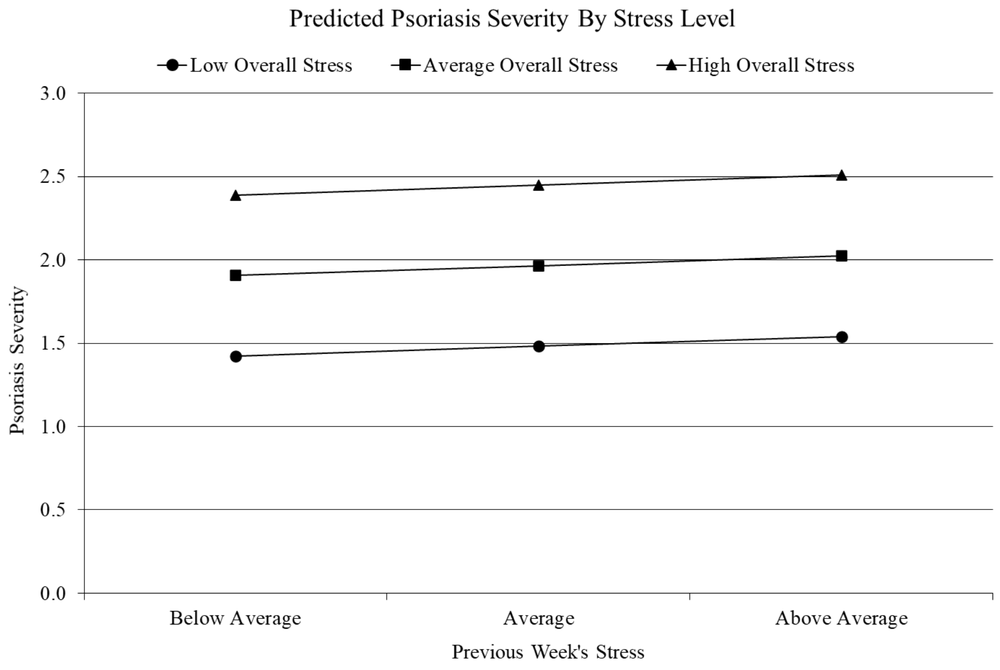
$\beta_{1i} = 0.02$

$\beta_{2i} = 0.06^{*}$

WP effects are fixed (no random slopes) → same for everyone

$\text{WPx}_{ti} = x_{ti} - \bar{x}_i$ → **it has only Level-1 WP variation**

$\text{PMx}_{i} = \bar{x}_i - 2$ → **it has only Level-2 BP variation**



Example: Syntax by Univariate MLM Program (Using Long Data)

SAS:

```
PROC MIXED DATA=work.Example COVTEST METHOD=REML;
     CLASS ID;
     MODEL severity = PMstress WPstressLag0 WPstressLag1 / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT / TYPE=UN SUBJECT=ID;
     REPEATED week / RCORR TYPE=TOEPH(4) SUBJECT=ID;
RUN;
```

R (lmer from lme4 package)—using lmertest package, which does provide correct denominator DF, but custom R matrix structures are not available (might be possible using gls from nlme instead), so RI only here:

```
modelname = lmer(data=Example, REML=TRUE,
                formula=severity~1+PMstress+WPstressLag0+WPstressLag1+(1+|ID))
summary(modelname, ddf="Satterthwaite")
```

STATA—I don't think custom Toeplitz structure with heterogeneous residual variances is possible, so I used RI + a homogeneous residual variance version here:

```
mixed severity c.PMstress c.WPstressLag0 c.WPstressLag1, || ID: , ///
      variance reml covariance(un) residuals(toeplitz3,t(week))   ///
      dfmethod(satterthwaite) dftable(pvalue)
```

SPSS—I don't think custom Toeplitz structure with heterogeneous variances is possible, so RI only here :

```
MIXED severity BY ID WITH PMstress WPstressLag0 WPstressLag1
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV
      /FIXED  = PMstress WPstressLag0 WPstressLag1
      /RANDOM = INTERCEPT | COVTYPE(UN) SUBJECT(ID).
```

3 Kinds of Fixed Slopes for TV Predictors

- **2 kinds of slopes Person-Mean-Centering tells us <u>directly</u>:**

- **Is there a Level-1 Within-Person (WP) slope?**
  - When you have higher predictor values <u>than usual</u> (*at this occasion*), do you also have higher outcomes values <u>than usual</u> (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance (σ_e^2)?
  - **Given directly by fixed slope of $\mathbf{WPx_{ti}}$ regardless of whether $\mathbf{PMx_i}$ is there**
  - Note: L1 slope multiplies the **relative** value of $\boldsymbol{x_{ti}}$, NOT the **original** $\boldsymbol{x_{ti}}$

- **Is there a Level-2 Between-Person (BP) slope?**
  - Do people with higher predictor values <u>than other people</u> (*on average over time*) also have higher outcomes <u>than other people</u> (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random int var ($\tau_{U_0}^2$)?
  - **Given directly by fixed slope of $\mathbf{PMx_i}$ regardless of whether $\mathbf{WPx_{ti}}$ is there**
  - Note: BP slope is NOT controlling for the original value of x_{ti} at each occasion

3rd Kind of Slope for TV Predictors

- **What Person-Mean-Centering DOES NOT tell us directly:**

- **Is there a Level-2 Contextual effect: Do the BP and WP slopes differ?**
  - After controlling for the original value of the TV predictor at that occasion, is there still **an incremental contribution from having a higher person mean** of the TV predictor (i.e., does one's general tendency for the predictor explain more $\tau^2_{U_0}$ above and beyond just the time-specific value of the predictor)?
  - If there is no contextual effect, then the TV predictor's **L2 BP** and **L1 WP** slopes show **convergence**, which means their effects are of equivalent magnitude

- **To answer this question about the Level-2 Contextual effect for the incremental contribution of the person mean, we have two options:**
  - Use Person-MC, and ask for the **contextual slope = between – within** (via SAS ESTIMATE, R contest1D, SPSS TEST, STATA LINCOM, Mplus NEW)
  - Use **"constant-centering"** for time-varying x_{ti} instead: $\mathbf{TVx_{ti}} = \boldsymbol{x_{ti}} - \boldsymbol{C_1}$
    → **centered at CONSTANT C_1, NOT A LEVEL-2 VARIABLE**
    - Which constant only matters for the reference point; it could be the grand mean or any (even 0)

Why the Difference in the Level-2 Slope? Remember Regular Old Regression…

- In this model: $y_i = \beta_0 + \beta_1(x1_i) + \beta_2(x2_i) + e_i$
- If $x1_i$ and $x2_i$ **ARE NOT** correlated:
  - β_1 carries **ALL the relationship** between $x1_i$ and y_i
  - β_2 carries **ALL the relationship** between $x2_i$ and y_i

- If $x1_i$ and $x2_i$ **ARE** correlated:
  - β_1 is **different than** the bivariate relationship between $x1_i$ and y_i
    - "Unique" effect of $x1_i$ *controlling for* $x2_i$ (or *holding* $x2_i$ *constant)*
  - β_2 is **different than** the bivariate relationship between $x2_i$ and y_i
    - "Unique" effect of $x2_i$ *controlling for* $x1_i$ (or *holding* $x1_i$ *constant)*

- Hang onto that idea…

Person-MC vs. Grand-MC: Variable- vs. Constant-Centering for TV Predictors

Level 2		Original	Person-MC Level 1	Grand-MC Level 1
$\bar{x}_i$	$\mathbf{PMx_i} = \bar{x}_i - 5$	x_{ti}	$\mathbf{WPx_{ti}} = x_{ti} - \bar{x}_i$	$\mathbf{TVx_{ti}} = x_{ti} - 5$
3	**−2**	**2**	**−1**	**−3**
3	**−2**	**4**	**1**	**−1**
7	**2**	**6**	**−1**	**1**
7	**2**	**8**	**1**	**3**

Same L2 $\mathbf{PMx_i}$ goes into the model given either way of centering the level-1 variable x_{ti}

In **variable-centering** (P-MC), the level-2 BP mean variation is gone from $\mathbf{WPx_{ti}}$, so it is NOT correlated with $\mathbf{PMx_i}$

In **constant-centering** (G-MC), the level-2 BP mean variation is still inside $\mathbf{TVx_{ti}}$, so it IS STILL CORRELATED with $\mathbf{PMx_i}$

So the effects of $\mathbf{PMx_i}$ and $\mathbf{TVx_{ti}}$ when included together under constant-centering will be different than if either predictor were included by itself…

Within-Person Fluctuation Model with **Constant-Centered Level-1 x_{ti}**

→ Model tests difference of WP vs. BP slopes (it's been fixed!)

x_{ti} is constant-centered into TVx_{ti}, <u>WITH</u> PMx_i at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{TVx_{ti}}) + \mathbf{e_{ti}}$

$TVx_{ti} = x_{ti} - C_1$ → it still has both Level-2 BP and Level-1 WP variation

Level 2: $\beta_{0i} = \boldsymbol{\gamma_{00}} + \boldsymbol{\gamma_{01}}(\mathbf{PMx_i}) + \mathbf{U_{0i}}$

$\beta_{1i} = \boldsymbol{\gamma_{10}}$

$PMx_i = \bar{x}_i - C_2$ → it has only Level-2 BP variation

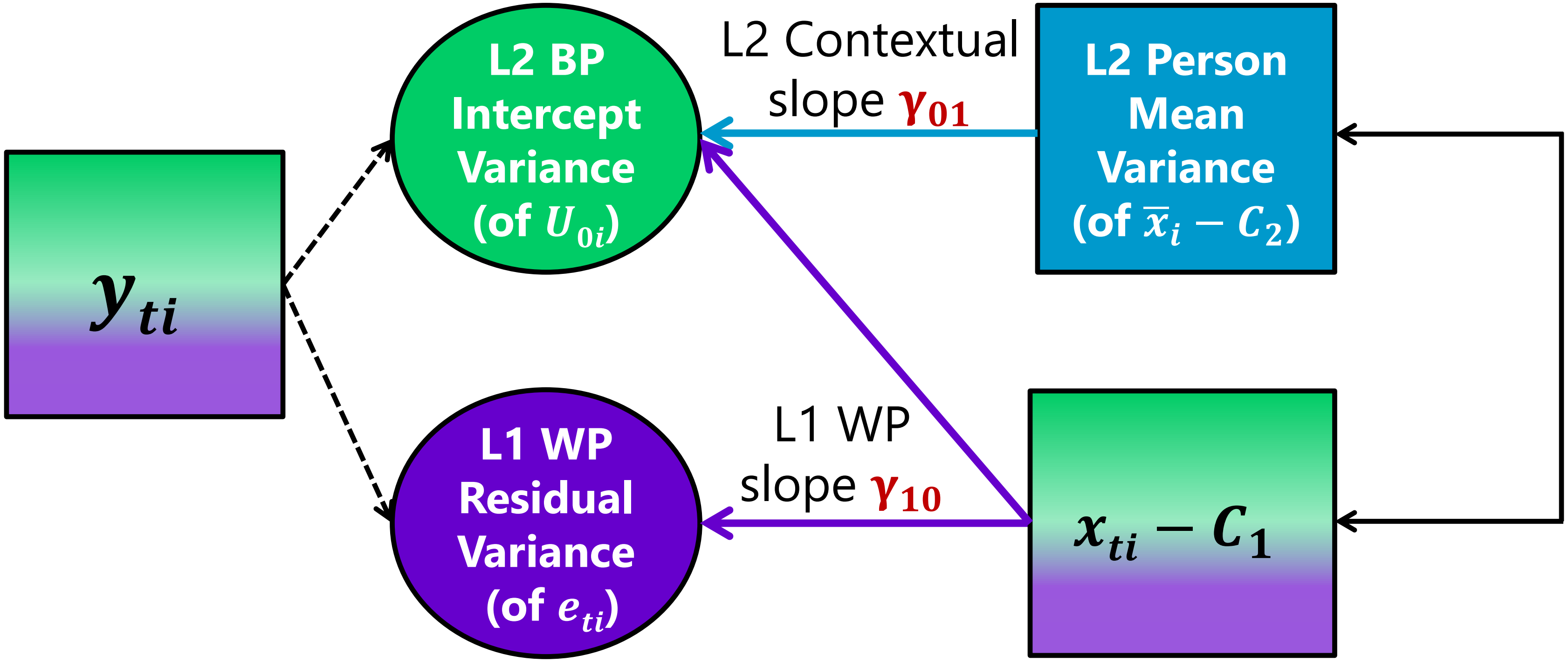
γ_{10} becomes the L1 WP slope → *unique* level-1 effect after controlling for PMx_i

γ_{01} *becomes* the L2 Contextual slope that indicates <u>how the L2 BP effect differs from the L1 WP effect</u>
→ *unique* level-2 slope after controlling for TVx_{ti}
→ does usual level matter beyond current level?

Univariate: Constant-Centering WITH Level-2 Predictor → OK now!

Model-based partitioning of y_{ti} outcome into level-specific **latent variables**

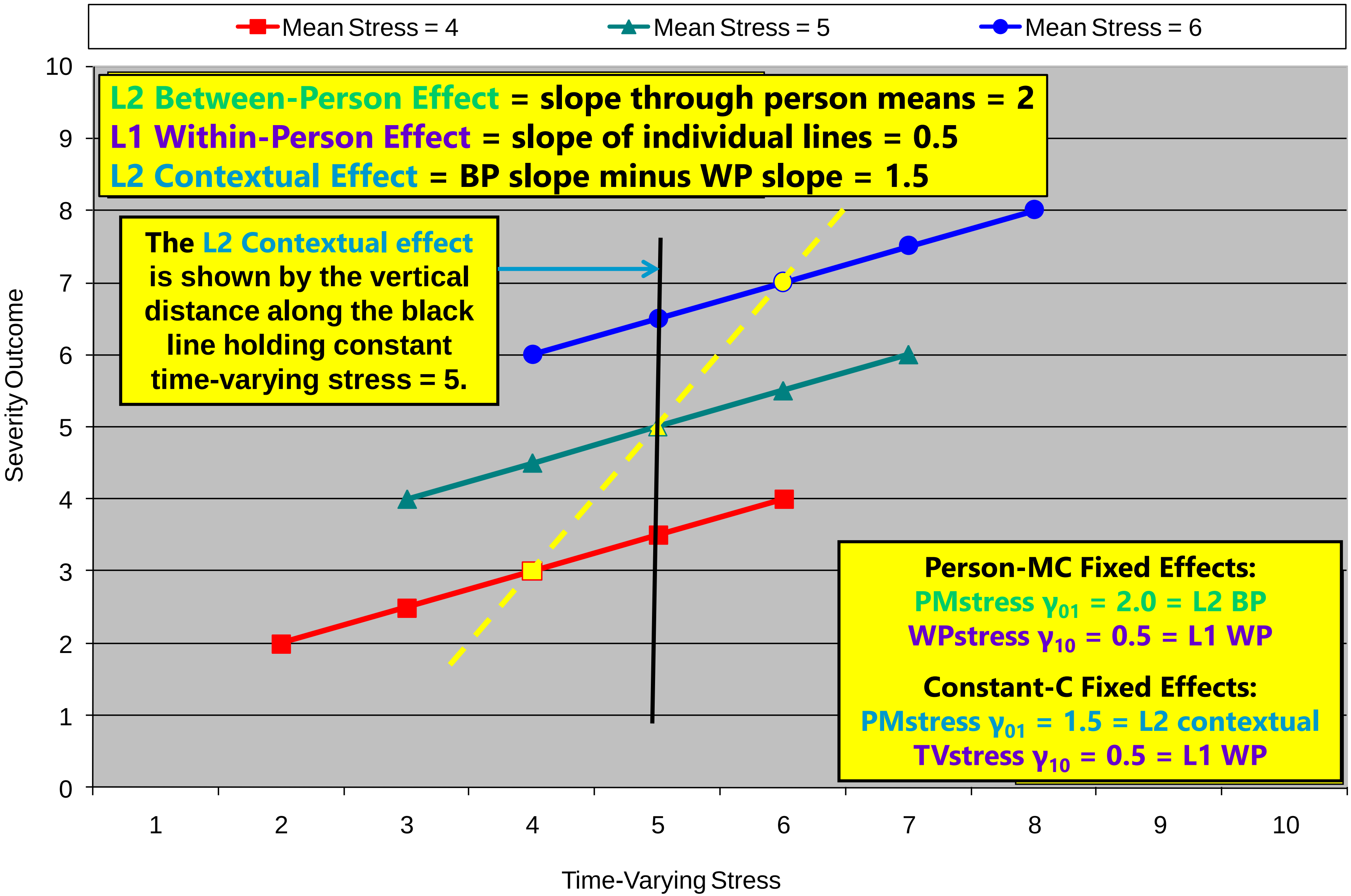
Level-1 x_{ti} is still NOT partitioned, but person mean $\bar{x}_i - C_2$ is added to allow an **incremental L2 effect**



L2 BP slope = **L1 WP slope** + **Level-2 Contextual slope**

Because original x_{ti} still has L2 BP variance, it still carries *some* of the L2 BP effect…

Person-MC vs. Constant-C: Example



Person-MC and Constant-C Models are Equivalent Given Only a **Fixed** Level-1 Main Effect Slope

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

Btw, I am using a centering constant = 0 at both levels to simplify the notation

→ $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$

→ $y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

Composite Model:
← In terms of P-MC
← In terms of Const-C

Constant-C: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

→ $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

Effect	P-MC	Const-C
Intercept	γ_{00}	γ_{00}
L1 WP	γ_{10}	γ_{10}
L2 Context	$\gamma_{01} - \gamma_{10}$	γ_{01}
L2 BP	γ_{01}	$\gamma_{01} + \gamma_{10}$

The Joy of Interactions Involving Time-Varying Predictors

- **<u>Must consider interactions with both its L2 BP and L1 WP parts:</u>**
- Example: Does time-varying stress (x_{ti}) interact with group ($\mathbf{Grp_i}$)?

- <u>Person-Mean-Centering (Variable-Centering):</u>
  - $\mathbf{WPx_{ti}} * \mathbf{Grp_i}$ → Does the **L1 WP** stress slope differ between groups?
  - $\mathbf{PMx_i} * \mathbf{Grp_i}$ → Does the **L2 BP** stress slope differ between groups?
    - Level-2 interaction is not controlling for current levels of stress
    - If forgotten, then $\mathbf{Grp_i}$ moderates the stress effect only at level 1 WP (not L2 BP)

- <u>Constant-Centering:</u>
  - $\mathbf{TVx_{ti}} * \mathbf{Grp_i}$ → Does the **L1 WP** slope effect differ between groups?
  - $\mathbf{PMx_i} * \mathbf{Grp_i}$ → Does the **L2 Contextual** slope effect differ between groups?
    - Incremental L2 stress effects *after controlling for current levels of stress*
    - **If forgotten**, then although the L1 main effect of stress has been unsmushed via the main effect of $\mathbf{PMx_i}$, **the interaction of $\mathbf{TVx_{ti}} * \mathbf{Grp_i}$ is still smushed**

Interactions with Time-Varying Predictors: Example: TV Stress (x_{ti}) by Group (Grp_i)

Btw, I am using a centering constant = 0 at both levels

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(Grp_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
$+ \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + \gamma_{11}(Grp_i)(x_{ti} - PMx_i)$

Constant-C: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(Grp_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + \gamma_{11}(Grp_i)(x_{ti})$

Interactions Involving Time-Varying Predictors Belong at Both Levels of the Model

On the left below → Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti} + \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + \gamma_{11}(Grp_i)(x_{ti} - PMx_i)$$

← Composite model as Person-MC

$$y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti} + \gamma_{02}(Grp_i) + (\gamma_{03} - \gamma_{11})(Grp_i)(PMx_i) + \gamma_{11}(Grp_i)(x_{ti})$$

← Composite model as Constant-C

On the right below → Constant-C: $TVx_{ti} = x_{ti}$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti} + \gamma_{02}(Grp_i) + \gamma_{03}(Grp_i)(PMx_i) + \gamma_{11}(Grp_i)(x_{ti})$$

After adding an interaction for Grp_i with stress at both levels, the Person-MC and Constant-MC models are equivalent

Intercept: $\gamma_{00} = \gamma_{00}$ **BP Slope:** $\gamma_{01} = \gamma_{01} + \gamma_{10}$ **Context:** $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WP Slope: $\gamma_{10} = \gamma_{10}$ **BP*Grp Slope:** $\gamma_{03} = \gamma_{03} + \gamma_{11}$ **Context*Grp:** $\gamma_{03} = \gamma_{03} - \gamma_{11}$

Grp Slope: $\gamma_{20} = \gamma_{20}$ **BP*WP or Context*WP is the same:** $\gamma_{11} = \gamma_{11}$

Intra-Variable* Interactions

- **Still must consider interactions with both its BP and WP parts!**
- Example: Interaction of TV stress (x_{ti}) with person mean stress (PMx_i), such that person mean stress is also a moderator (like $\mathbf{Grp_i}$ before)
- Person-Mean-Centering (Variable-Centering):
  - $\mathbf{WPx_{ti}} * \mathbf{PMx_i}$ → Does the **L1 WP** stress slope differ by overall stress?
  - $\mathbf{PMx_i} * \mathbf{PMx_i}$ → Does the **L2 BP** stress slope differ by overall stress?
    - Level-2 interaction is not controlling for current levels of stress
    - If forgotten, then $\mathbf{PMx_i}$ moderates the stress effect only at level 1 WP (not L2 BP)
- Constant-Centering:
  - $\mathbf{TVx_{ti}} * \mathbf{PMx_i}$ → Does the **L1 WP** stress slope differ by overall stress?
  - $\mathbf{PMx_i} * \mathbf{PMx_i}$ → Does the **L2 Contextual** stress slope differ by overall stress?
    - Incremental BP stress effect *after controlling for current levels of stress*
    - **If forgotten**, then although the L1 main effect of stress has been unsmushed via the main effect of $\mathbf{PMx_i}$, **the interaction of $\mathbf{TVx_{ti}} * \mathbf{PMx_i}$ is still smushed**

* *Btw, we will also see this idea in controlling age slopes for age cohort…*

Intra-Variable Interactions:
Example: TV Stress (x_{ti}) by Person Mean Stress (PMx_i)

Btw, I am using a centering constant = 0 at both levels

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(PMx_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti} - PMx_i)$

Constant-C: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(PMx_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$

Intra-Variable Interactions:
Example: TV Stress (x_{ti}) by Person Mean Stress (PMx_i)

On the left below → Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$$
$$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti} - PMx_i)$$

← **Composite model as Person-MC**

$$y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$$
$$+ (\gamma_{02} - \gamma_{11})(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$$

← **Composite model as Constant-C**

On the right below → Constant-C: $TVx_{ti} = x_{ti}$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$$
$$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$$

After adding an interaction for PMx_i with stress at both levels, the Person-MC and Constant-C models are equivalent

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Intercept: $\gamma_{00} = \gamma_{00}$ **BP Slope:** $\gamma_{01} = \gamma_{01} + \gamma_{10}$ **Contextual:** $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WP Slope: $\gamma_{10} = \gamma_{10}$ **BP² Slope:** $\gamma_{02} = \gamma_{02} + \gamma_{11}$ **Contextual²:** $\gamma_{02} = \gamma_{02} - \gamma_{11}$

BP*WP or Contextual*WP is the same: $\gamma_{11} = \gamma_{11}$

When Person-MC ≠ Constant-Centering: Random Slopes of TV Predictors

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

Variance due to PMx_i is removed from the random slope in Person-MC.

→ $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + U_{1i}(x_{ti} - PMx_i) + e_{ti}$

Constant-C: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

PMx_i variance is still part of the Constant-C random slope → smushed random effect! Thus, the level-1 predictor to be given a random slope should be P-MC to prevent this problem.

→ $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + U_{1i}(x_{ti}) + e_{ti}$

Preventing Smushed (BP=WP) Slopes

- **Fixed side: 2 strategies to prevent smushed slopes**
  - If using variable-centered (P-MC) L1 TVP ($\mathbf{WPx_{ti}}$), it can only have a **L1 WP slope**, and its L2 $\mathbf{PMx_i}$ can only have a **L2 BP slope** (so no problem)
  - If using constant-C L1 TVP ($\mathbf{TVx_{ti}}$), its L1 slope will be smushed (BP=WP) if you don't add its L2 $\mathbf{PMx_i}$ to allow a **L2 contextual slope = BP – WP**

- **Random side: Only 1 strategy is likely possible!** *(see Rights & Sterba, MBR 2023, for details)*
  - If using variable-centered (P-MC) L1 TVP ($\mathbf{WPx_{ti}}$), its L2 random slope variance **only** captures L2 BP differences in its L1 WP slope (so no problem)
    - Creates a pattern of quadratic heterogeneity of variance **across** $\mathbf{WPx_{ti}}$ **ONLY**
  - If using constant-C L1 TVP ($\mathbf{TVx_{ti}}$), its L2 random slope variance **also** creates **intercept heterogeneity of variance** (beyond BP diffs in L1 WP slope)
    - Enforces **SAME** pattern of quadratic heterogeneity of variance across **L1** $\mathbf{WPx_{ti}}$ and **L2** $\mathbf{PMx_i}$
  - If using $\mathbf{TVx_{ti}}$, you need a "contextual" random slope to allow a different pattern of variance heterogeneity across $\mathbf{PMx_i}$ than $\mathbf{WPx_{ti}}$ (for BP – WP)
    - Requires a L2 BP random "slope ?" variance for **L2** $\mathbf{PMx_i}$ – good luck estimating it!

Modeling Time-Varying Categorical Predictors

- Person-MC and Constant-C usually refer to *quantitative* TV predictors, but the need to separate BP and WP effects applies to *categorical* TV predictors too
- Binary level-1 predictors do not lend themselves intuitively to Person-MC
  - e.g., x_{ti} = 0 or 1 per occasion, person mean = .40 across occasions → impossible values (if x_{ti} = 0, then WPx_{ti} = 0 − 0.40 = −0.40; if x_{ti} = 1, then WPx_{ti} = 1 − 0.40 = +0.60)
  - Easier: Leave x_{ti} uncentered in estimating its fixed slope and include person mean as level-2 predictor so that results = Const-C (but still use Person-MC in estimating its random slope)
- For >2 categories, person means of multiple dummy codes may start to break down, but we can think about types of people, and code BP effects accordingly
- Example: Dementia present/not at each time point?
  - **BP effects →** Ever diagnosed with dementia (no, yes) rather than person mean
    - People who will eventually be diagnosed may differ prior to diagnosis (a BP stable effect)
  - **TV effect →** Diagnosed with dementia at each time point (no, yes)?
    - Acute differences of before/after diagnosis logically can only exist in the "ever" people
- Other examples: Mentor status, father absence, type of shift work (AM/PM)

I Prefer Variable-Centering…

- …because constant-centering is much easier to screw up! ☺
- See Table 1 from: Hoffman, L., & Walters, R. W. (2022). Catching up on multilevel modeling. *Annual Review of Psychology, 73*, 629-658.

Table 1 Predictor effect type by model specification

Centering strategy for level-1 predictor (constant-centered level-2 predictor)	Fixed effect type by predictors included		
	Level-1 only	Level-2 only	Both levels
Variable-centered level-1			
Level-1 predictor: $L1x_{wb} = x_{wb} - \bar{x}_b$	Within	(= 0)	Within
Level-2 predictor: $L2x_b = \bar{x}_b - C_2$	(= 0)	Between	Between
Constant-centered level-1			
Level-1 predictor: $L1x_{wb} = x_{wb} - C_1$	Smushed	(= 0)	Within
Level-2 predictor: $L2x_{wb} = \bar{x}_b - C_2$	(= Within)	Between	Contextual

Abbreviations: w, within; b, between; C_1, level-1 centering constant; C_2, level-2 centering constant.
Parentheses indicate assumptions about the fixed slopes of omitted predictors.

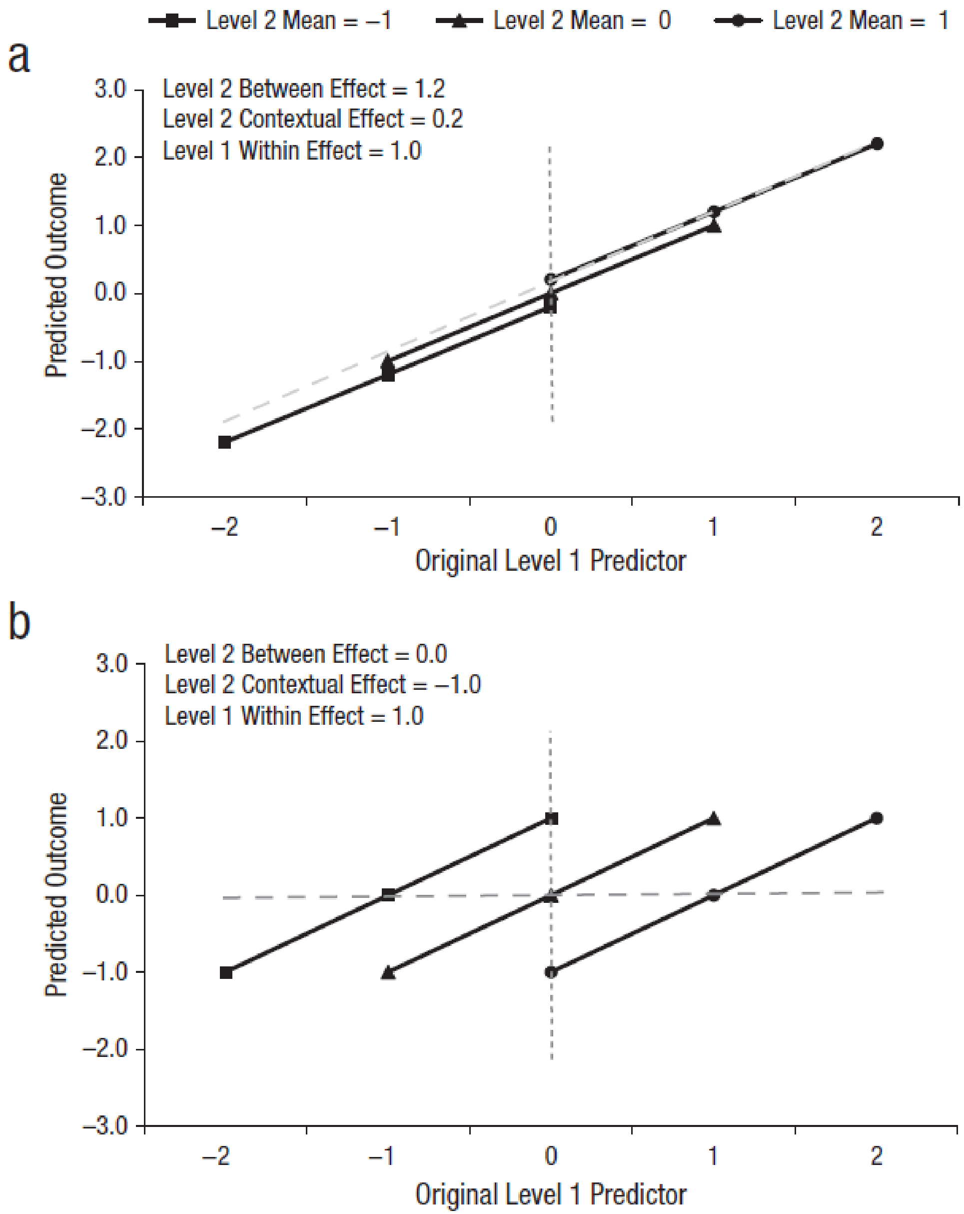
Variance Accounted For By Level-1 Predictors

- **Fixed effects of level-1 TV predictors:**
  - Level-1 WP part of TV predictors (as main effects by themselves or as part of interactions with other TV predictors) reduce Level-1 (WP) residual variance σ_e^2

- **What happens to the level-2 random intercept variance depends on what levels of variance the level-1 TV predictor still has:**
  - If the level-1 TV predictor STILL has level-2 variance (e.g., Grand-MC predictors), then its level-2 part can reduce level-2 random intercept variance $\tau_{U_0}^2$
    - But badly smushed effects could increase level-2 random intercept variance instead!
  - If the level-1 TV predictor DOES NOT have level-2 variance (e.g., Person-MC or latent-centered predictors), then any reduction in the level-1 residual variance σ_e^2 will cause an INCREASE in level-2 random intercept variance $\tau_{U_0}^2$
    - Same thing happens with Grand-MC level-1 predictors, but you don't generally see it
  - It's just an artifact that the estimate of true random intercept variance is:

    True $\tau_{U_0}^2$ = observed $\tau_{U_0}^2 - \frac{\sigma_e^2}{L1n}$ → so if only σ_e^2 decreases, then $\tau_{U_0}^2$ increases

Bonus: Between vs. Contextual Effects



- Image from Hoffman (2019), example using clustered data
- *Top:* Contextual effect is minimal—there is no added benefit to going to a high-SES school when comparing across schools *at same level of student SES*
- *Bottom:* Contextual effect is negative—at the same student SES level, relatively high students from low-SES schools do better than relatively low students from high-SES schools

Summary: Univariate MLM for Specifying Effects of Time-Varying Predictors

- "Univariate" approach to MLM is possible for time-varying predictors that *fluctuate* over time (and lower-level predictors with only mean differences across higher levels in general)

- Level-1 predictor can be created two different ways:
  - Easier to understand is variable-centering: $\mathbf{WPx_{ti}} = x_{ti} - \bar{x}_i$
    - Directly isolates level-1 within variance, so $\mathbf{WPx_{ti}}$ → L1 within effects
  - More common is constant-centering: $\mathbf{TVx_{ti}} = x_{ti} - C_1$
    - Does NOT remove level-2 BP variance, so $\mathbf{TVx_{ti}}$ will have smushed (BP=WP) effects **unless** you add the necessary slopes for its level-2 predictor analog

- Level-2 predictor is always constant-centered: $\mathbf{PMx_i} = \bar{x}_i - C_2$
  - $\mathbf{PMx_i}$ slope is **L2 Between** effect when paired with **L1 $\mathbf{WPx_{ti}}$**
  - $\mathbf{PMx_i}$ slope is **L2 Contextual** effect when paired with **L1 $\mathbf{TVx_{ti}}$**
    - Within + Contextual = Between; Between − Within = Contextual