Example 6: Three-Level Models for Longitudinal Twin Data (Time within Twin within Pair)
(complete syntax and output is available electronically for STATA, R, and SAS for all models and for two models in Mplus; SPSS is also in my 2013 University of Kentucky Workshop on this page)

The data for this example come from the longitudinal Octogenarian Twin Study of Aging. The current models include 1,622 observations from 340 incomplete same-sex twin pairs (615 individuals) initially age 79–100 years measured for up to four occasions approximately every two years, over six possible years. We will be examining change over time in a measure of cognition (information test). These data are already stacked into "long form" such that one row contains the data for one occasion for one person. The ID variables PairID and TwinID index which twin pair and which twin (1 or 2), respectively. The very last model (which converged in SAS only) also examines the extent of heritability (i.e., differences between MZ and DZ twins) in the random intercepts and random linear change over time.

Based on the sampling design in which twin pairs began the study as close in time as possible (and as supported by the analyses that follow), we create two age-related predictors. First, level-1 (time-varying) age will be time-in-study (0=wave1) to capture the longitudinal effect of age. Second, level-3 (between-pair) age will be the pair's mean wave1 age centered at 85 years to capture the cross-sectional effect of age. We also include pair zygosity (0=MZ, 1=DZ).

STATA Data Import and Manipulation:

```
// Import Example6 long data
clear // clear memory in case a dataset is already open
import excel "AdvLong_Example6.xlsx", firstrow case(preserve) clear
// Sort by PairID TwinID Wave
sort PairID TwinID Wave

// Create pair-level (between-family) age at wave1
egen PairAge = mean(agew1), by(PairID)
// Center pair-level (between-family) age at 85 for use at level 3
gen PairAge85 = PairAge - 85
label variable PairAge85 "PairAge85: Pair Mean Age at Wave1 (0=85)"
// Within-person center age for use at level-1 (VARIABLE CENTERING)
gen time = age - agew1
label variable time "time: Time since Wave1 (0= Age at Wave1)"
// Quadratic time to use as random slope
gen timesq = time*time
label variable timesq "timesq: Squared Time since Wave1 (0= Age at Wave1)"

// Center zygosity so 0=MZ, 1=DZ, will be treated as numeric
gen IsDZ = zygosity-1
label variable IsDZ "IsDV: Zygosity (0=MZ, 1=DZ)"

// Select only cases with complete data per occasion
egen nummiss = rowmiss(info time PairAge IsDZ)
drop if nummiss>0

// Remove last occasion that is mostly missing data
drop if Wave==5
```

R Data Import and Manipulation (after loading packages *readxl*, *expss*, *TeachingDemos*, *nlme*, *lme4*, *lmerTest*, and *performance*):

```
# Import Example 6 stacked data
Example6 = read_excel(paste0(filesave,filename))
# Convert to data frame as data frame without labels to use for analysis
Example6 = as.data.frame(Example6)
# Sort by PairID, TwinID, and Wave
sort_asc(data=Example6, PairID, TwinID, Wave)

# Create pair-level (between-family) age at wave1
# Uses function from above to add pair means (to same data here)
Example6 = addUnitMeans(data=Example6, unitVariable="PairID",
                        meanVariables=c("agew1"), newNames=c("PairAge"))
```

```
# Center pair-level (between-family) age at 85 for use at level 3
Example6$PairAge85 = Example6$PairAge - 85
# Within-person center age for use at level-1 (VARIABLE CENTERING)
Example6$time = Example6$age - Example6$agew1
Example6$timesq = Example6$time * Example6$time
# Center zygosity so 0=MZ, 1=DZ, will be treated as numeric
Example6$IsDZ = Example6$zygosity - 1

# Select only cases with complete data per occasion
Example6 = Example6[complete.cases(Example6[ , c("info","time","PairAge","IsDZ")]),]

# Remove last occasion that is mostly missing data
Example6 = subset(x=Example6, Example6$Wave<5)
```

Model 1a: Empty Means, Two-Level Model for Cognition Outcome (*t* = time, *I* = individual)

Level 1: $\text{Info}_{ti} = \beta_{0i} + e_{ti}$

Level 2: $\beta_{0i} = \gamma_{00} + U_{0i}$

This model has two variance components: level-1 residual and level-2 random intercept. It assumes that all people are independent (i.e., it does not account for twin pair membership).

```
display "STATA Model 1a: Empty Means, Two-Level Model for Cognition Outcome"
mixed info ,  || Case:  ,    /// Level2+3 (Case = unique person ID)
              reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2    // Print -2LL for model
estat icc                     // Intraclass correlation
estimates store TwoLevel      // Save LL for LRT
```

```
print("R Model 1a: Empty Means, Two-Level Model for Cognition Outcome")
Model1a = lmer(data=Example6, REML=TRUE,
               formula=info~1+(1|Case))   # Level2+3 (Case = Unique PersonID)
llikAIC(Model1a); summary(Model1a); icc(Model1a)
print("LRT for removing random intercept"); ranova(Model1a, reduce.term=TRUE)
```

Model 1a R output:

```
'log Lik.' -5694.7379 (df=3) → -2LL for model

Random effects:
 Groups   Name        Variance Std.Dev.
 Case     (Intercept) 136.551  11.6855  → Twin+Pair Variance
 Residual              23.915   4.8903  → Time-specific Variance
Number of obs: 1622, groups:  Case, 615

Fixed effects:
             Estimate Std. Error        df t value  Pr(>|t|)
(Intercept)  25.54684    0.49108 604.81092  52.022 < 2.2e-16

# Intraclass Correlation Coefficient
    Adjusted ICC: 0.851
  Unadjusted ICC: 0.851

ANOVA-like table for random-effects: Single term deletions
           npar   logLik     AIC     LRT Df Pr(>Chisq)
<none>        3 -5694.74 11395.5
(1 | Case)    2 -6361.47 12726.9 1333.46  1 < 2.22e-16
```

Calculate the ICC for the proportion of between-person variation in Info:

$$\text{ICC} = \frac{136.551}{136.551 + 23.915} = .851$$

The ranova LRT below tells us that the random intercept variance is significantly greater than 0, and thus so is the ICC for the correlation of occasions within persons (and pairs).

Model 1b: Empty Means, Three-Level Model for Information Test Outcome (add *c* = cluster pair)

Level 1: $\text{Info}_{tic} = \beta_{0ic} + e_{tic}$

Level 2: $\beta_{0ic} = \delta_{00c} + U_{0ic}$

Level 3: $\delta_{00c} = \gamma_{000} + V_{00c}$

This model now has 3 variance components: level-1 residual, level-2 twin random intercept, and level-3 pair random intercept. It now allows a correlation between people from the same twin pair.

```
display "STATA Model 1b: Empty Means, Three-Level Model for Cognition Outcome"
mixed info ,  || PairID: ,   ///
              || TwinID: , reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2    // Print -2LL for model
estat icc                     // Intraclass correlations
estimates store ThreeLevel    // Save LL for LRT
lrtest ThreeLevel TwoLevel    // Compare three-level empty to two-level empty
```

TwinID is sufficient for level 2 here because STATA assumes any random effects written after the first level are nested within it.

```
print("R Model 1b: Empty Means, Three-Level Model for Cognition Outcome")
Model1b = lmer(data=Example6, REML=TRUE,
               formula=info~1+(1|PairID)+(1|PairID:TwinID)) # L3 Pairs + L2 Twins
llikAIC(Model1b); summary(Model1b); icc(Model1b)
print("LRT for removing random intercepts"); ranova(Model1b, reduce.term=TRUE)
```

Model 1b R output:

```
'log Lik.' -5639.0516 (df=4) → -2LL for model

Random effects:
 Groups        Name        Variance Std.Dev.
 PairID:TwinID (Intercept) 49.941   7.0669 → L2 Within-pair twin random intercept variance
 PairID        (Intercept) 87.313   9.3442 → L3 Between-pair random intercept variance
 Residual                  23.967   4.8956 → L1 Time-specific residual variance
Number of obs: 1622, groups:  PairID:TwinID, 615; PairID, 340

Fixed effects:
             Estimate Std. Error        df t value  Pr(>|t|)
(Intercept)  25.22030    0.60174 330.46972  41.913 < 2.2e-16

# Intraclass Correlation Coefficient → Not helpful for understanding level 3!
    Adjusted ICC: 0.851
  Unadjusted ICC: 0.851

ANOVA-like table for random-effects: Single term deletions
                    npar   logLik     AIC     LRT Df Pr(>Chisq)
<none>                 4 -5639.05 11286.1
(1 | PairID)           3 -5694.74 11395.5 111.373  1 < 2.22e-16 → We need L3 Pair random intercept
(1 | PairID:TwinID)    3 -5841.99 11690.0 405.885  1 < 2.22e-16 → We need L2 Twin random intercept
```

Proportion variance at each level:
Total = 87.313 + 49.941 + 23.967 = 161.221
Level 3 (pair) = 87.313 / 161.221 = .542
Level 2 (person) = 49.941 / 161.221 = .310
Level 1 (time) = 23.967 / 161.221 = .149

ICC_{L2} for time within person and pair (proportion between persons) = (87.313 + 49.941) / (161.221) = .851 (~same as before!)

ICC_{L3} for person within pair = 87.313 / (87.313 + 49.941) = .636
This ICC = .636 is significantly > 0 via −2ΔLL for 3- vs. 2-level.

Translation: Of the total outcome variation, 85.1% is between persons (cross-sectional) and 14.9% is within persons (longitudinal). Of the 85.1% between-person variance, 63.6% is between pairs (L3).

Model 1b STATA output for ICC for comparison:

```
Intraclass correlation

------------------------------------------------------------------------------
                       Level |        ICC   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
                      PairID |   .5415762   .0379215      .4668787    .6144506 → Prop var at L3
               TwinID|PairID |   .8513409   .0109349      .8286109    .8715238 → Prop BP var (L2+L3)
------------------------------------------------------------------------------
```

Btw, what Mplus MLM or M-SEM calls "ICC" is just the proportion of variance at that level. In these data, the average correlation of occasions from the same *person* is .851 (ICC_{L2}), and the average correlation of occasions from the same pair is .542 (ICC_{L3B}). The expected correlation of twins from the same pair is .636 (ICC_{L3}).

Now let's do the same empty means, three-level model for our time-varying predictor of age:

```
display "STATA Age Model: Empty Means, Three-Level Model for Age Predictor"
mixed age ,   || PairID: ,    /// Level 3
              || TwinID: ,    /// Level 2
                 reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2     // Print -2LL for model
estat icc                      // Intraclass correlations

print("R Age Model: Empty Means, Three-Level Model for Age Predictor")
EmptyAge = lmer(data=Example6, REML=TRUE,
                formula=age~1+(1|PairID)+(1|PairID:TwinID)) # L3 Pairs + L2 Twins
llikAIC(EmptyAge); summary(EmptyAge); icc(EmptyAge)
```

Age model R output:

```
Random effects:
 Groups        Name        Variance Std.Dev.
 PairID:TwinID (Intercept) 0.0000   0.0000 → L2 Within-pair twin random intercept variance = 0%
 PairID        (Intercept) 6.8525   2.6177 → L3 Between-pair random intercept variance = 63.6%
 Residual                  5.2498   2.2912 → L1 Time-specific residual variance = 36.4%
Number of obs: 1622, groups:  PairID:TwinID, 615; PairID, 340

Fixed effects:
             Estimate Std. Error        df t value  Pr(>|t|)
(Intercept)  85.52829    0.15563 316.01516  549.55 < 2.2e-16
optimizer (nloptwrap) convergence code: 0 (OK)
boundary (singular) fit: see help('isSingular')

> icc(EmptyAge)
[1] NA
```

Because there is ~no age variance at level 2, age will be a predictor at levels 1 and 3 only.

R is unhappy that one of the variances is 0.

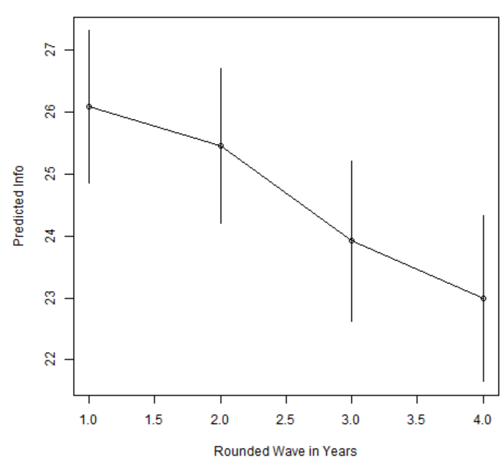
Model 1c: Saturated Means for Wave, Random Intercepts at Levels 2 and 3

```
display "STATA Model 1c: Saturated Wave Means, Three-Level Model for Cognition Outcome"
mixed info i.Wave,  || PairID: ,    /// Level 3
                    || TwinID: ,    /// Level 2
            reml dfmethod(satterthwaite) dftable(pvalue) nolog
margins i.Wave                        // Print saturated means by wave
marginsplot, name(sat_means, replace) // Plot  saturated means by wave
graph export "STATA Saturated Means by Wave.png", replace

SatWave = lmer(data=Example6, REML=TRUE,      # L3 Pairs + L2 Twins
               formula=info~0+as.factor(Wave)+(1|PairID)+(1|PairID:TwinID))
summary(SatWave)
```

Model 1c relevant R output (see code online for making the plot):

```
Fixed effects:
                 Estimate Std. Error        df t value  Pr(>|t|)
as.factor(Wave)1 26.08814    0.62473 371.45745  41.759 < 2.2e-16
as.factor(Wave)2 25.45963    0.63843 402.92593  39.879 < 2.2e-16
as.factor(Wave)3 23.91720    0.65748 449.79047  36.377 < 2.2e-16
as.factor(Wave)4 22.98772    0.68088 510.76205  33.762 < 2.2e-16
```



This pattern of average change looks like it might need a fixed quadratic effect of time, so let's start there.

Model 2a: Fixed Quadratic Time, Random Intercepts at Level 2 (Twin) and Level 3 (Pair)

Level 1: $\text{Info}_{tic} = \beta_{0ic} + \beta_{1ic}\left(\text{Age}_{tic} - \text{Agew1}_{ic}\right) + \beta_{2ic}\left(\text{Age}_{tic} - \text{Agew1}_{ic}\right)^2 + e_{tic}$

Level 2:

Intercept: $\beta_{0ic} = \delta_{00c} + U_{0ic}$

Linear Time: $\beta_{1ic} = \delta_{10c}$

Quadratic Time: $\beta_{2ic} = \delta_{20c}$

Level 3:

Intercept: $\delta_{00c} = \gamma_{000} + V_{00c}$

Linear Time: $\delta_{10c} = \gamma_{100}$

Quadratic Time: $\delta_{20c} = \gamma_{200}$

```
display "STATA Model 2a: Fixed Quadratic, Random Intercepts for L2 Twin and L3 Pair"
mixed info c.time c.time#c.time ,  ///
           || PairID: ,            /// Level 3
           || TwinID: ,            /// Level 2
             reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2          // Print -2LL for model
estimates store RI2RI3              // Save LL for LRT

print("R Model 2a: Fixed Quadratic, Random Intercepts for L2 Twin and L3 Pair")
Model2a = lmer(data=Example6, REML=TRUE,      # L3 Pairs + L2 Twins
               formula=info~1+time+I(time^2)+(1|PairID)+(1|PairID:TwinID))
llikAIC(Model2a); summary(Model2a)
```

Model 2a R output:

```
'log Lik.' -5605.7863 (df=6) → -2LL for model

Random effects:
 Groups        Name        Variance Std.Dev.
 PairID:TwinID (Intercept) 52.933   7.2755
 PairID        (Intercept) 88.048   9.3834
 Residual                  21.970   4.6872
Number of obs: 1622, groups:  PairID:TwinID, 615; PairID, 340

Fixed effects:
              Estimate  Std. Error          df t value Pr(>|t|)
(Intercept)   26.121219   0.623284  368.862025 41.9090  < 2e-16
time          -0.321639   0.183380 1039.759391 -1.7539  0.07973
I(time^2)     -0.036726   0.030766 1026.701485 -1.1937  0.23287 → Not significant for now….
```

PsuedoR2 (% Reduction) for work.CovEmpty vs. work.CovFQuad (from SAS)

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	PseudoR2
work.CovEmpty	UN(1,1)	PairID	87.2970	9.9794	8.75	<.0001	.
work.CovEmpty	UN(1,1)	PairID*TwinID	49.9360	5.3371	9.36	<.0001	.
work.CovEmpty	Residual		23.9684	1.0735	22.33	<.0001	.
work.CovFQuad	UN(1,1)	PairID	88.0484	10.1556	8.67	<.0001	-0.008607
work.CovFQuad	UN(1,1)	PairID*TwinID	52.9334	5.5159	9.60	<.0001	-0.060025
work.CovFQuad	Residual		21.9701	0.9854	22.30	<.0001	0.083373

The level-1 fixed linear and quadratic effects of time explained 8.33% of the level-1 residual variance. The level-2 twin intercept variance consequently increased.

Model 2b: Fixed Quadratic Time, Random Linear Time Slope over Level-2 Twins

Level 1: $\text{Info}_{tic} = \beta_{0ic} + \beta_{1ic}\left(\text{Age}_{tic} - \text{Agew1}_{ic}\right) + \beta_{2ic}\left(\text{Age}_{tic} - \text{Agew1}_{ic}\right)^2 + e_{tic}$

Level 2:

- Intercept: $\beta_{0ic} = \delta_{00c} + U_{0ic}$
- Linear Time: $\beta_{1ic} = \delta_{10c} + U_{1ic}$ ←
- Quadratic Time: $\beta_{2ic} = \delta_{20c}$

Level 3:

- Intercept: $\delta_{00c} = \gamma_{000} + V_{00c}$
- Linear Time: $\delta_{10c} = \gamma_{100}$
- Quadratic Time: $\delta_{20c} = \gamma_{200}$

```
display "STATA Model 2b: Add Random Linear Time over L2 Twins"
mixed info c.time c.time#c.time ,                  ///
      || PairID:     ,                             /// Level 3
      || TwinID: time, covariance(unstructured)    /// Level 2
         reml dfmethod(satterthwaite) dftable(pvalue) nolog
estat recovariance, relevel(TwinID) correlation  // L2 GCORR matrix
display "-2LL = " e(ll)*-2   // Print -2LL for model
estimates store RL2RI3       // Save LL for LRT
lrtest RL2RI3 RI2RI3         // Test random linear time over L2 twins
```

```
print("R Model 2b: Add Random Linear Time over L2 Twins")
Model2b = lmer(data=Example6, REML=TRUE,       # L3 Pairs + L2 Twins
               formula=info~1+time+I(time^2)+(1|PairID)+(1+time|PairID:TwinID))
llikAIC(Model2b); summary(Model2b)
print("Test random linear time over L2 twins"); ranova(Model2b, reduce.term=TRUE)
```

Model 2b R output:

```
'log Lik.' -5537.5275 (df=8) → -2LL for model

Random effects:
 Groups        Name        Variance Std.Dev. Corr
 PairID:TwinID (Intercept) 47.6642  6.9039
               time         1.5662  1.2515   0.193
 PairID        (Intercept) 85.7647  9.2609
 Residual                  13.5083  3.6754
Number of obs: 1622, groups:  PairID:TwinID, 615; PairID, 340

Fixed effects:
              Estimate Std. Error         df t value  Pr(>|t|)
(Intercept)  26.179916   0.599151 337.732520 43.6950 < 2.2e-16
time         -0.314727   0.158329 929.250375 -1.9878  0.047126
I(time^2)    -0.070752   0.025712 722.248544 -2.7518  0.006076 → Now significant!

ANOVA-like table for random-effects: Single term deletions
                                   npar   logLik     AIC     LRT Df Pr(>Chisq)
<none>                                8 -5537.53 11091.1
(1 | PairID)                          7 -5593.72 11201.5 112.391  1 < 2.22e-16
time in (1 + time | PairID:TwinID)    6 -5605.79 11223.6 136.518  2 < 2.22e-16 → Keep random L2 slope
```

Model 2c: Fixed Quadratic, Random Linear Slope over Level-2 Twins AND Level-3 Pairs

Level 1: $\text{Info}_{tic} = \beta_{0ic} + \beta_{1ic}\left(\text{Age}_{tic} - \text{Agew1}_{ic}\right) + \beta_{2ic}\left(\text{Age}_{tic} - \text{Agew1}_{ic}\right)^2 + e_{tic}$

Level 2:
- Intercept: $\beta_{0ic} = \delta_{00c} + U_{0ic}$
- Linear Time: $\beta_{1ic} = \delta_{10c} + U_{1ic}$
- Quadratic Time: $\beta_{2ic} = \delta_{20c}$

Level 3:
- Intercept: $\delta_{00c} = \gamma_{000} + V_{00c}$
- Linear Time: $\delta_{10c} = \gamma_{100} + V_{10c}$ ←
- Quadratic Time: $\delta_{20c} = \gamma_{200}$

```
display "STATA Model 2c: Add Random Linear Time over L3 Pairs"
mixed info c.time c.time#c.time ,                    ///
      || PairID: time, covariance(unstructured)  /// Level 3
      || TwinID: time, covariance(unstructured)  /// Level 2
         reml dfmethod(satterthwaite) dftable(pvalue) nolog
estat recovariance, relevel(PairID) correlation    // L3 GCORR matrix
estat recovariance, relevel(TwinID) correlation    // L2 GCORR matrix
display "-2LL = " e(ll)*-2    // Print -2LL for model
estimates store RL2RL3        // Save LL for LRT
lrtest RL2RL3 RL2RI3          // Test random linear time over L3 pairs

print("R Model 2c: Add Random Linear Time over L3 Pairs")
Model2c = lmer(data=Example6, REML=TRUE,       # L3 Pairs + L2 Twins
               formula=info~1+time+I(time^2)+(1+time|PairID)+(1+time|PairID:TwinID))
llikAIC(Model2c); summary(Model2c)
print("Test random linear time over L3 pairs"); ranova(Model2c, reduce.term=TRUE)
```

Model 2c R output:

```
'log Lik.' -5537.3821 (df=10) → -2LL for model

Random effects:
 Groups        Name        Variance Std.Dev. Corr
 PairID:TwinID (Intercept) 47.79812 6.91362        → L2 Within-pair twin random intercept variance
               time         1.45339 1.20556  0.187 → L2 Within-pair twin random linear change variance
 PairID        (Intercept) 85.49100 9.24613        → L3 Between-pair random intercept variance
               time         0.10657 0.32644  0.081 → L2 Between-pair random linear change variance
 Residual                  13.52493 3.67763        → L1 Time-specific residual variance
Number of obs: 1622, groups:  PairID:TwinID, 615; PairID, 340

Fixed effects:
              Estimate Std. Error         df t value  Pr(>|t|)
(Intercept)  26.180962   0.598674 335.522601 43.7316 < 2.2e-16
time         -0.318129   0.158850 859.761958 -2.0027  0.045522
I(time^2)    -0.070551   0.025725 721.009942 -2.7425  0.006249

ANOVA-like table for random-effects: Single term deletions

                                   npar   logLik     AIC     LRT Df Pr(>Chisq)
<none>                               10 -5537.38 11094.8
time in (1 + time | PairID)           8 -5537.53 11091.1  0.2908  2    0.86468 → Don't need L3 slope
time in (1 + time | PairID:TwinID)    8 -5571.78 11159.6 68.7930  2 1.1529e-15 → DO need L2 slope
```

ICC_{L3} for correlation of twins within pairs for random intercept and linear change:
For Intercept = 85.491 / (85.491 + 47.798) = .641
For Linear Time = 0.107 / (0.107 + 1.453) = .068 (≈ 0 because LRT is not significant)

Translation: Of the total between-person *intercept* variance, 64.1% is between pairs, and of the total between-person *linear change* variance, 6.8% is between pairs.

I then tested random quadratic time slopes at the twin and pair levels, but neither was significant. Given our interest in examining heritability of intercept and time slopes, we will retain the nonsignificant random linear time slope at level 3 (pairs) for now. So we continue by adding level-3 baseline age as a predictor of intercept and linear slope differences.

Model 3a: Add Baseline Age as a Predictor of Pair Intercept and Change

Level 1: $\text{Info}_{tic} = \beta_{0ic} + \beta_{1ic}\left(\text{Age}_{tic} - \text{Agew1}_{ic}\right) + \beta_{2ic}\left(\text{Age}_{tic} - \text{Agew1}_{ic}\right)^2 + e_{tic}$

Level 2:

Intercept: $\beta_{0ic} = \delta_{00c} + U_{0ic}$

Linear Time: $\beta_{1ic} = \delta_{10c} + U_{1ic}$

Quadratic Time: $\beta_{2ic} = \delta_{20c}$

Level 3:

Intercept: $\delta_{00c} = \gamma_{000} + \gamma_{001}\left(\text{PairAgew1}_c - 85\right) + V_{00c}$

Linear Time: $\delta_{10c} = \gamma_{100} + \gamma_{101}\left(\text{PairAgew1}_c - 85\right) + V_{10c}$

Quadratic Time: $\delta_{20c} = \gamma_{200} + \gamma_{201}\left(\text{PairAgew1}_c - 85\right)$

```
display "STATA Model 3a: Add Baseline Age as Predictor of Pair Intercept and Change"
mixed info c.time c.time#c.time c.PairAge85 c.time#c.PairAge85 c.time#c.time#c.PairAge85, ///
      || PairID: time, covariance(unstructured)  /// Level 3
      || TwinID: time, covariance(unstructured)  /// Level 2
         reml dfmethod(satterthwaite) dftable(pvalue) nolog
estat recovariance, relevel(PairID) correlation   // L3 GCORR matrix
estat recovariance, relevel(TwinID) correlation   // L2 GCORR matrix
display "-2LL = " e(ll)*-2   // Print -2LL for model
// Trajectory diffs by age
test (c.PairAge85=0)(c.time#c.PairAge85=0)(c.time#c.time#c.PairAge85=0), small

print("R Model 3a: Add Baseline Age as Predictor of Pair Intercept and Change")
Model3a = lmer(data=Example6, REML=TRUE,       # L3 Pairs + L2 Twins
               formula=info~1+time+I(time^2)+PairAge85 +time:PairAge85 +I(time^2):PairAge85
                            +(1+time|PairID)+(1+time|PairID:TwinID))
llikAIC(Model3a); summary(Model3a)
print("Trajectory diffs by age"); contestMD(Model3a, ddf="Satterthwaite",
      L=rbind(c(0,0,0,1,0,0),c(0,0,0,0,1,0),c(0,0,0,0,0,1)))
```

Model 3a R output:

```
'log Lik.' -5531.5228 (df=13) → -2LL for model

Random effects:
 Groups        Name        Variance  Std.Dev. Corr
 PairID:TwinID (Intercept) 47.534348 6.89452
               time         1.448965 1.20373  0.203
 PairID        (Intercept) 78.720564 8.87246
               time         0.064989 0.25493  -0.008
 Residual                  13.616752 3.69009
Number of obs: 1622, groups:  PairID:TwinID, 615; PairID, 340

Fixed effects:
                     Estimate  Std. Error          df t value     Pr(>|t|)
(Intercept)        24.8632200   0.6478194 346.7525832 38.3799   < 2.2e-16
time               -0.3379124   0.2000953 899.4674070 -1.6888    0.091612
I(time^2)          -0.0899402   0.0342763 799.3406201 -2.6240    0.008857
PairAge85          -0.8755224   0.1873270 355.5670724 -4.6738 0.000004204
time:PairAge85     -0.0138649   0.0595930 905.4710760 -0.2327    0.816079
I(time^2):PairAge85 -0.0084818  0.0101454 796.8858178 -0.8360    0.403388

[1] "Trajectory diffs by age"
     Sum Sq   Mean Sq NumDF     DenDF   F value          Pr(>F)
1 360.72052 120.24017     3 428.25751 8.8303125 0.000010843518
```

PsuedoR2 (% Reduction) for work.CovRL2RL3 vs. work.CovAge (from SAS)

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	PseudoR2
work.CovRL2RL3	UN(1,1)	PairID	85.4911	9.8263	8.70	<.0001	.
work.CovRL2RL3	UN(2,2)	PairID	0.1066	0.2203	0.48	0.3143	.
work.CovRL2RL3	UN(1,1)	PairID*TwinID	47.7968	5.2453	9.11	<.0001	.
work.CovRL2RL3	UN(2,2)	PairID*TwinID	1.4534	0.3050	4.77	<.0001	.
work.CovRL2RL3	Residual		13.5251	0.8191	16.51	<.0001	.
work.CovAge	UN(1,1)	PairID	78.7210	9.2932	8.47	<.0001	0.07919
work.CovAge	UN(2,2)	PairID	0.06503	0.2187	0.30	0.3831	0.38971
work.CovAge	UN(1,1)	PairID*TwinID	47.5312	5.2127	9.12	<.0001	0.00556
work.CovAge	UN(2,2)	PairID*TwinID	1.4490	0.3048	4.75	<.0001	0.00300
work.CovAge	Residual		13.6169	0.8274	16.46	<.0001	-0.00678

The level-3 main effect of age and its interaction with time explained 7.9% and 39.0% of the level-3 pair intercept and time slope variance, respectively.

I also tried quadratic effects of age in predicting the intercept and linear time slope, but neither was significant.

Model 3b: Add Zygosity as a Predictor of Pair Intercept and Change

Level 1: $\text{Info}_{tic} = \beta_{0ic} + \beta_{1ic}\left(\text{Age}_{tic} - \text{Agew1}_{ic}\right) + \beta_{2ic}\left(\text{Age}_{tic} - \text{Agew1}_{ic}\right)^2 + e_{tic}$

Level 2:

Intercept: $\beta_{0ic} = \delta_{00c} + U_{0ic}$

Linear Time: $\beta_{1ic} = \delta_{10c} + U_{1ic}$

Quadratic Time: $\beta_{2ic} = \delta_{20c}$

Level 3:

Intercept: $\delta_{00c} = \gamma_{000} + \gamma_{001}\left(\text{PairAgew1}_c - 85\right) + \gamma_{002}\left(\text{MZvDZ}_c\right) + \gamma_{003}\left(\text{PairAgew1}_c - 85\right)\left(\text{MZvDZ}_c\right) + V_{00c}$

Linear Time: $\delta_{10c} = \gamma_{100} + \gamma_{101}\left(\text{PairAgew1}_c - 85\right) + \gamma_{102}\left(\text{MZvDZ}_c\right) + \gamma_{103}\left(\text{PairAgew1}_c - 85\right)\left(\text{MZvDZ}_c\right) + V_{10c}$

Quadratic Time: $\delta_{20c} = \gamma_{200} + \gamma_{201}\left(\text{PairAgew1}_c - 85\right) + \gamma_{202}\left(\text{MZvDZ}_c\right) + \gamma_{103}\left(\text{PairAgew1}_c - 85\right)\left(\text{MZvDZ}_c\right)$

```
display "STATA Model 3b: Add Zygosity as Predictor of Pair Intercept and Change"
mixed info c.time c.time#c.time c.PairAge85 c.time#c.PairAge85 c.time#c.time#c.PairAge85  ///
           c.IsDZ c.time#c.IsDZ c.time#c.time#c.IsDZ c.PairAge85#c.IsDZ  ///
           c.time#c.PairAge85#c.IsDZ c.time#c.time#c.PairAge85#c.IsDZ,   ///
       || PairID: time, covariance(unstructured)  /// Level 3
       || TwinID: time, covariance(unstructured)  /// Level 2
          reml dfmethod(satterthwaite) dftable(pvalue) nolog
estat recovariance, relevel(PairID) correlation   // L3 GCORR matrix
estat recovariance, relevel(TwinID) correlation   // L2 GCORR matrix
// Trajectory diffs by zygosity
test (c.IsDZ=0)(c.time#c.IsDZ=0)(c.time#c.time#c.IsDZ=0)(c.PairAge85#c.IsDZ=0) ///
     (c.time#c.PairAge85#c.IsDZ=0)(c.time#c.time#c.PairAge85#c.IsDZ=0), small
display "-2LL = " e(ll)*-2   // Print -2LL for model
estimates store FitZyg       // Save LL for LRT
       test (c.zyg=0) (c.zyg#c.time=0) (c.zyg#c.BFage85=0) (c.zyg#c.time#c.BFage85=0)
       estimates store Fit_Zyg

print("R Model 3b: Add Zygosity as Predictor of Pair Intercept and Change")
Model3b = lmer(data=Example6, REML=TRUE,      # L3 Pairs + L2 Twins
               formula=info~1+time+I(time^2)+PairAge85+IsDZ
               +time:PairAge85 +I(time^2):PairAge85 +time:IsDZ +I(time^2):IsDZ
               +PairAge85:IsDZ +time:PairAge85:IsDZ +I(time^2):PairAge85:IsDZ
                +(1+time|PairID)+(1+time|PairID:TwinID))
llikAIC(Model3b); summary(Model3b)
print("Trajectory diffs by zygosity"); contestMD(Model3b, ddf="Satterthwaite",
      L=rbind(c(0,0,0,0,1,0,0,0,0,0,0,0),c(0,0,0,0,0,0,0,1,0,0,0,0),
              c(0,0,0,0,0,0,0,0,1,0,0,0),c(0,0,0,0,0,0,0,0,0,1,0,0),
              c(0,0,0,0,0,0,0,0,0,0,1,0),c(0,0,0,0,0,0,0,0,0,0,0,1)))
```

Model 3b R output:

```
'log Lik.' -5529.9695 (df=19) → -2LL for model

Random effects:
 Groups        Name        Variance  Std.Dev. Corr
 PairID:TwinID (Intercept) 47.641188 6.90226
               time         1.449558 1.20398  0.213
 PairID        (Intercept) 76.956400 8.77248
               time         0.072504 0.26927  0.065
 Residual                  13.484959 3.67219
Number of obs: 1622, groups:  PairID:TwinID, 615; PairID, 340

Fixed effects:
                            Estimate  Std. Error          df t value  Pr(>|t|)
(Intercept)               26.3616884   0.9792924 330.9032049 26.9191 < 2.2e-16
time                      -0.6099965   0.2947881 869.7535973 -2.0693  0.038815
I(time^2)                 -0.0264300   0.0486937 746.0684582 -0.5428  0.587443
PairAge85                 -1.0396957   0.2830501 333.1337988 -3.6732  0.000279
IsDZ                      -2.6116731   1.2982367 340.6047293 -2.0117  0.045038
time:PairAge85             0.0583338   0.0887729 869.9437974  0.6571  0.511283
I(time^2):PairAge85       -0.0077679   0.0145517 731.5462891 -0.5338  0.593631
time:IsDZ                  0.5338225   0.4008775 894.3100103  1.3316  0.183320
I(time^2):IsDZ            -0.1299474   0.0683962 794.9019491 -1.8999  0.057805
PairAge85:IsDZ             0.2847112   0.3754082 347.8419516  0.7584  0.448722
time:PairAge85:IsDZ       -0.1167975   0.1196067 898.7822427 -0.9765  0.329073
I(time^2):PairAge85:IsDZ  -0.0052820   0.0202670 790.5274022 -0.2606  0.794451

[1] "Trajectory diffs by zygosity"
     Sum Sq  Mean Sq NumDF     DenDF   F value       Pr(>F)
1 227.03814 37.83969     6 413.12946 2.8060663 0.010949239
```

PsuedoR2 (% Reduction) for work.CovAge vs. work.CovZyg (from SAS)

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	PseudoR2
work.CovAge	UN(1,1)	PairID	78.7210	9.2932	8.47	<.0001	.
work.CovAge	UN(2,2)	PairID	0.06503	0.2187	0.30	0.3831	.
work.CovAge	UN(1,1)	PairID*TwinID	47.5312	5.2127	9.12	<.0001	.
work.CovAge	UN(2,2)	PairID*TwinID	1.4490	0.3048	4.75	<.0001	.
work.CovAge	Residual		13.6169	0.8274	16.46	<.0001	.
work.CovZyg	UN(1,1)	PairID	76.9636	9.2115	8.36	<.0001	0.02232
work.CovZyg	UN(2,2)	PairID	0.07240	0.2177	0.33	0.3697	-0.11334
work.CovZyg	UN(1,1)	PairID*TwinID	47.6390	5.2167	9.13	<.0001	-0.00227
work.CovZyg	UN(2,2)	PairID*TwinID	1.4495	0.3028	4.79	<.0001	-0.00037
work.CovZyg	Residual		13.4851	0.8198	16.45	<.0001	0.00968

The level-3 main effect of zygosity explained another 2.61% of the level-3 pair intercept variance, but zygosity by time actually increased the level-3 pair slope variance instead.

Model 3c: Add Heterogeneous Variances by Zygosity (to quantify heritability)

Note: STATA and R would not provide results, so only those from SAS are shown.

```
TITLE "SAS Model 3c: Add Heterogeneous G and R matrices by Zygosity";
PROC MIXED DATA=work.Example6 NOCLPRINT COVTEST NAMELEN=100 IC METHOD=REML;
     CLASS PairID TwinID zyg;
     MODEL info = time time*time PairAge85 time*PairAge85
                  IsDZ time*IsDZ time*time*IsDZ
                  PairAge85*IsDZ time*PairAge85*IsDZ time*time*PairAge85*IsDZ
                  / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT time / TYPE=UN SUBJECT=PairID        GROUP=zyg; * Level 3;
     RANDOM INTERCEPT time / TYPE=UN SUBJECT=PairID*TwinID GROUP=zyg; * Level 2;
     REPEATED / GROUP=zyg; * Level 1 residual variance;
     ODS OUTPUT InfoCrit=work.FitHet CovParms=work.CovHet; * Save for LRT, pseudo-R2;
RUN; TITLE;
* Test het variances by zygosity;
%FitTest(FitFewer=work.FitZyg, FitMore=work.FitHet);
```

Model 3c SAS output:

Covariance Parameter Estimates							
Cov Parm	Subject	Group	Estimate	Standard Error	Z Value	Pr Z	
UN(1,1)	PairID	zyg DZ	54.9920	11.7725	4.67	<.0001	L3 Pair random int variance for DZ
UN(2,1)	PairID	zyg DZ	-0.4277	1.3152	-0.33	0.7451	
UN(2,2)	PairID	zyg DZ	0	.	.	.	L3 Pair random time variance for DZ
UN(1,1)	PairID	zyg MZ	106.06	15.1028	7.02	<.0001	L3 Pair random int variance for MZ
UN(2,1)	PairID	zyg MZ	0.8988	1.7049	0.53	0.5981	
UN(2,2)	PairID	zyg MZ	0.6328	0.3596	1.76	0.0392	L3 Pair random time variance for MZ
UN(1,1)	PairID*TwinID	zyg DZ	70.5171	9.5142	7.41	<.0001	L2 Twin random int variance for DZ
UN(2,1)	PairID*TwinID	zyg DZ	2.5277	1.3452	1.88	0.0602	
UN(2,2)	PairID*TwinID	zyg DZ	1.2073	0.2529	4.77	<.0001	L2 Twin random time variance for DZ
UN(1,1)	PairID*TwinID	zyg MZ	18.7130	4.0828	4.58	<.0001	L2 Twin random int variance for MZ
UN(2,1)	PairID*TwinID	zyg MZ	0.5448	1.0423	0.52	0.6012	
UN(2,2)	PairID*TwinID	zyg MZ	1.3248	0.4041	3.28	0.0005	L2 Twin random time variance for MZ
Residual		zyg DZ	13.8241	1.1221	12.32	<.0001	L1 residual variance for DZ
Residual		zyg MZ	12.9887	1.1698	11.10	<.0001	L1 residual variance for MZ

Information Criteria						
Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
11011.0	13	11037.0	11037.2	11056.8	11086.8	11099.8

Solution for Fixed Effects					
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	26.2264	1.0304	140	25.45	<.0001
time	-0.6226	0.2572	373	-2.42	0.0160
time*time	-0.01593	0.03719	315	-0.43	0.6687
PairAge85	-1.0355	0.2966	139	-3.49	0.0006
time*PairAge85	0.03584	0.05154	102	0.70	0.4884
IsDZ	-2.4425	1.3185	294	-1.85	0.0650
time*IsDZ	0.5283	0.3736	870	1.41	0.1576
time*time*IsDZ	-0.1338	0.06074	784	-2.20	0.0279
PairAge85*IsDZ	0.2789	0.3810	297	0.73	0.4648
time*PairAge85*IsDZ	-0.09702	0.09513	546	-1.02	0.3082
time*time*PairAge85*IsDZ	-0.01264	0.01410	474	-0.90	0.3703

Is the heterogeneous variance model a better fit?
Yes, $-2\Delta LL(7) = 43.66$, $p < .001$
(note SAS didn't count the 0, but I will in the results)

Likelihood Ratio Test for work.FitZyg vs. work.FitHet

Name	Neg2LogLike	Parms	AIC	BIC	DevDiff	DFdiff	Pvalue
work.FitZyg	11059.9	7	11073.9	11100.7	.	.	.
work.FitHet	11011.0	13	11037.0	11086.8	48.9280	6	7.7076E-9

Heritability (A or H^2), or the contribution of genetics, can be found as twice the difference of the intraclass correlation (ICC) between MZ and DZ twins. **Common environment** (C^2) can be found as the difference between the ICC for MZ twins and the heritability estimate (usually constrained to be ≥ 0), and the **unique environment** (E^2) can be found as the remainder (i.e., 1 – [heritability + common environment]). Applying these calculations to our results reveals evidence for heritability in both the intercept and the linear time slope, but with much greater uncertainty in the latter (given that variance components should not be negative in this partitioning strategy).

SAS:	Intercept			Linear Time Slope		
Intercept	DZ	MZ	HCE	DZ	MZ	HCE
Level-3 Pair Variance	54.992	106.060		0.000	0.633	
Level-2 Twin Variance	70.517	18.713		1.207	1.325	
ICC = L3 / (L3 + L2)	0.438	0.850		0.000	0.323	
H2 = 2*(ICC MZ - ICC DZ)			0.824			0.647
C2 = ICC MZ - H2			0.026			-0.323
E2 = 1 - (H2 + C2)			0.150			0.677

Sample Results Section:

The extent of individual change in cognition (as measured by the information test, an indicator of crystallized intelligence) and the extent of heritability therein was examined in a sample of 340 same-sex twin pairs measured every two years for up to four occasions. Multilevel models were estimated using residual maximum likelihood. Accordingly, the significance of fixed effects was evaluated with Wald tests using Satterthwaite denominator degrees of freedom, whereas the significance of random effects was evaluated via likelihood ratio tests (i.e., −2ΔLL with degrees of freedom equal to the number of new random effects variances and covariances). Pseudo-R^2 effect sizes for the fixed effects were calculated as the proportion reduction in each variance component.

A two-level empty means, random intercept model of occasions at level 1 nested in persons at level 2 was initially estimated; its intraclass correlation (ICC) indicated that 85.1% of the outcome variance was between persons. The addition of a level-3 random intercept for twin pair resulted in significantly better model fit, −2ΔLL(1) = 111.37, p < .001, and revealed that, of the 85.1% of the outcome variance that was between persons, 63.6% was actually due to twin pair (i.e., shared variance between twins). Stated more directly, of the total variance, 14.9% was at level 1 (within persons over time), 31.0% was at level 2 (between twins from the same pair), and 54.2% was at level 3 (between twin pairs). Next, a three-level empty means, random intercept model to partition the variance in time-varying age revealed that 63.6% was between pairs (given that the twins varied in age from 79 to 100 at baseline), whereas the remaining 36.4% was within persons over time; there was no detectable level-2 age variance in these twins (as expected given the strategic sampling design in which twins began the study as close in time as possible). Thus, the level-3 between-pair (cross-sectional) and level-1 within-person (longitudinal) effects of age were modeled separately using baseline age (centered so 0 = 85) and time in study (with 0 = baseline), respectively.

Based on the pattern of model-estimated (saturated) means, fixed linear and quadratic effects of time were first added, which accounted for 8.3% of the level-1 residual variance. Although adding a variance for the level-2 (twin) random linear time slope (and its covariance with the level-2 twin intercept) significantly improved model fit, −2ΔLL(2) = 136.52, p < .001, the subsequent addition of a variance for the level-3 (pair) random linear time slope (and its covariance with the level-3 pair intercept) did not significantly improve model fit, −2ΔLL(2) = 0.29, p = .86. Results indicated that 64.1% of the between-person random intercept variance was due to twin pair, whereas only 6.8% of the between-person random linear time slope variance was due to twin pair (the latter of which was not distinguishable from 0). Given our interest in examining heritability, though, both levels of random linear time slope variances were retained. Random quadratic time slopes were not significant at either level 2 or level 3, and these were not retained.

Linear effects of baseline age on the intercept, linear time slope, and quadratic time slope were then added, which resulted in a significant improvement to model prediction, F(3, 428.3) = 8.83, p < .001. These slopes explained 7.9% and 39.0% of the level-3 intercept and linear time slope variance, respectively, as well as 0% of the level-1 residual variance. We then added zygosity (0=MZ, 1=DZ) as a moderator of each fixed. Although these six new fixed effects also resulted in a significant improvement in model prediction, F(6, 413.1) = 2.81, p = .011, only the effect of zygosity on the intercept was significant (which together with the interaction with pair mean age at wave 1 reduced the level-3 pair intercept variance by 2.2%). Finally (using SAS MIXED), we added zygosity differences in all variance model parameters—three at level 3, three at level 2, and in residual variance at level 1, which resulted in significant model improvement, −2ΔLL(7) = 48.9, p < .001.

Results for the final model are given in Table X. Given the centering of the model predictors, the reference for the intercept and linear time slope is an MZ twin pair who were 85 years at baseline (when time = 0). Older age at baseline was related to a significantly lower intercept at wave 1 (time = 0), equivalently so in both MZ and DZ twins.
(see text above for interpretation of heritability results). There was a significantly negative instantaneous linear time slope at wave 1 in MZ twin pairs, the extent of which did not differ in DZ twin pairs. There was a nonsignificant acceleration of decline in MZ twin pairs that was significantly stronger in DZ twin pairs. There was no significant moderation of the effects pair mean wave 1 age on the intercept or change over time. Genetic decomposition of variance indicated heritability of 82.4% in the intercept and 64.7% in linear change. However, while the influence of common environment was computed 2.6% for the intercept, it was a nonsensical −32.3% for linear change.