Example 4a: Models for Accelerated Longitudinal Designs using Multilevel and Single-Level Structural Equation Modeling (M-SEM and SEM)

(complete syntax, data, and output available for Mplus electronically)

These data come from Hoffman (2015) chapter 10, which examined prediction of a recall outcome from two metrics of time: years since birth (centered at 84 years) and years in study (centered at the first occasion) in a sample of 557 observations from 207 older adults. This example shows how to estimate the time-based models from Example 3 (which used MLM) using multilevel and single-level SEM in Mplus instead.

Model 0. Empty Means, Random Intercept for Recall:

Level-1: $recall_{ti} = \beta_{0i} + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + U_{0i}$

Model 0 STATA Univariate MLM:

```
display "Model 0: Empty Means, Random Intercept Model for Recall Outcome"
mixed recall , || personid: , mle nolog
```

Model 0 R Univariate MLM:

```
print("Model 0: Empty Means, Random Intercept Model for Recall Outcome")
Empty = lmer(data=Example3, REML=FALSE, formula=recall~1+(1|PersonID))
llikAIC(Empty); summary(Empty); icc(Empty); print("Does the random intercept improve model fit?")
ranova(Empty, reduce.term=TRUE) # LRT for removing random intercept
```

```
 $AICtab
      AIC        BIC     logLik   deviance   df.resid
 2863.3550  2876.3227 -1428.6775  2857.3550   554.0000  Mplus provides LL instead

Random effects:
 Groups   Name        Variance Std.Dev.
 PersonID (Intercept) 10.4579  3.2339  Level-2 var(U_0i)
 Residual              5.1646  2.2726  Level-1 var(e_ti)
Number of obs: 557, groups:  PersonID, 207

Fixed effects:
             Estimate Std. Error        df t value  Pr(>|t|)
(Intercept)   9.73491    0.25058 197.01209   38.85 < 2.2e-16

# Intraclass Correlation Coefficient
    Adjusted ICC: 0.669
  Unadjusted ICC: 0.669
```

$$ICC = \frac{10.4579}{10.4579 + 5.1646} = .669$$

So 67% of the variance in recall is initially due to person mean differences.

Model 0 Mplus M-SEM:

```
TITLE:   M-SEM Model 0: Empty Means, Random Intercept
DATA:    FILE = Chapter10.csv;              ! Syntax in same folder as data
         TYPE = INDIVIDUAL; FORMAT = FREE;  ! Defaults

VARIABLE:
! List of variables in long data file IN ORDER
  NAMES = PersonID occasion ageT0 tvage recall time;
! Variables to be analyzed in this model (new defined at end)
  USEVARIABLE = recall;
! Missing data identifier
  MISSING ARE ALL (-999);
! MSEM options
  CLUSTER = PersonID;       ! Level-2 ID
  BETWEEN = ;               ! Observed ONLY level-2 predictors
  WITHIN  = ;               ! Observed ONLY level-1 predictors
```

```
ANALYSIS:   TYPE = TWOLEVEL RANDOM; ESTIMATOR = ML;

MODEL:  ! LEVEL-1 = WITHIN, LEVEL-2 = BETWEEN
%WITHIN%
 recall;       ! L1 R: Residual variance

%BETWEEN%
[recall];      ! Fixed intercept
 recall;       ! L2 G: Random intercept variance only

Number of Free Parameters                        3
Loglikelihood
          H0 Value                       -1428.678

MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Within Level
 Variances
    RECALL             5.170      0.394     13.127      0.000    Level-1 var(e_ti)

Between Level
 Means
    RECALL             9.736      0.251     38.815      0.000    gammma00
 Variances
    RECALL            10.438      1.306      7.991      0.000    Level-2 var(U_0i)
```

Model 0 Mplus SEM:

```
TITLE:  SEM Model 0: Empty Means, Random Intercept
DATA:   FILE = Chapter10.csv;              ! Syntax in same folder as data
        TYPE = INDIVIDUAL; FORMAT = FREE;  ! Defaults

! Unstacking to multivariate format
DATA LONGTOWIDE:
! Names of old stacked former variables (without numbers)
  LONG = recall|time;
! Names of new multivariate variables (that use numbers)
  WIDE = recall0 recall2 recall4 recall6 recall8 |
         time0 time2 time4 time6 time8;
! Variable with level-2 ID info
  IDVARIABLE = PersonID;
! Old level-1 identifier
  REPETITION = occasion (0 2 4 6 8);

VARIABLE:
! List of variables in long data file IN ORDER
  NAMES = PersonID occasion ageT0 tvage recall time;
! Variables to be analyzed in this model (new defined at end)
  USEVARIABLE = recall0 recall2 recall4 recall6 recall8;
! Missing data identifier
  MISSING ARE ALL (-999);

ANALYSIS:   TYPE = GENERAL; ESTIMATOR = ML;
OUTPUT:     STDYX; ! Standardized solution in SEM

MODEL:
[recall0-recall8@0];         ! All variable intercepts fixed to 0
 recall0-recall8 (Resvar);   ! L1 R residual variances held equal

! Recall intercept-only model
  Int BY recall0-recall8@1;

! Level-2 model
  [Int];   ! Fixed intercept
   Int;    ! L2 G: Random intercept variance
```

```
Number of Free Parameters                        3
Loglikelihood
          H0 Value                       -1428.678

MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
 INT      BY
    RECALL0            1.000      0.000    999.000    999.000
    RECALL2            1.000      0.000    999.000    999.000
    RECALL4            1.000      0.000    999.000    999.000
    RECALL6            1.000      0.000    999.000    999.000
    RECALL8            1.000      0.000    999.000    999.000
 Means
    INT                9.735      0.251     38.786      0.000  gamma00
 Intercepts
    RECALL0            0.000      0.000    999.000    999.000
    RECALL2            0.000      0.000    999.000    999.000
    RECALL4            0.000      0.000    999.000    999.000
    RECALL6            0.000      0.000    999.000    999.000
    RECALL8            0.000      0.000    999.000    999.000
 Variances
    INT               10.458      1.310      7.986      0.000  Level-2 var(U_0i)
 Residual Variances
    RECALL0            5.165      0.393     13.141      0.000  Level-1 var(e_ti)
    RECALL2            5.165      0.393     13.141      0.000
    RECALL4            5.165      0.393     13.141      0.000
    RECALL6            5.165      0.393     13.141      0.000
    RECALL8            5.165      0.393     13.141      0.000

STANDARDIZED MODEL RESULTS
STDYX Standardization

R-SQUARE
    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value
    RECALL0            0.669      0.035     19.285      0.000  ICC
    RECALL2            0.669      0.035     19.285      0.000
    RECALL4            0.669      0.035     19.285      0.000
    RECALL6            0.669      0.035     19.285      0.000
    RECALL8            0.669      0.035     19.285      0.000
```

Model 1b. Syntax and Partial Output for Fixed Quadratic Time, Random Intercept for Recall:

Level-1 Time: $recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - ageT0_i) + \beta_{2i}(age_{ti} - ageT0_i)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + U_{0i},\ \beta_{1i} = \gamma_{10},\ \beta_{2i} = \gamma_{20}$

Where $age_{ti} - ageT0_i = time_{ti}$ (as years-in-study rather than years-since-birth)

Model 1b STATA Univariate MLM:

```
display "Model 1b: Fixed Quadratic Time, Random Intercept Model"
mixed recall c.time c.timesq, || personid: , mle nolog
```

Model 1b R Univariate MLM:

```
print("Model 1b: Fixed Quadratic Time, Random Intercept Model")
RITim = lmer(data=Example3, REML=FALSE, formula=recall~1+time+timesq+(1|PersonID))
llikAIC(RITim); summary(RITim)
```

```
$AICtab
       AIC        BIC     logLik   deviance   df.resid
 2856.0088  2877.6216 -1423.0044  2846.0088   552.0000

Random effects:
 Groups   Name        Variance Std.Dev.
 PersonID (Intercept) 10.622   3.2591   Level-2 var(U_0i)
 Residual              4.983   2.2323   Level-1 var(e_ti)

Fixed effects:
             Estimate Std. Error         df t value  Pr(>|t|)
(Intercept)  9.660987   0.274986 282.359753 35.1326 < 2.2e-16  gamma00
time         0.261331   0.119243 377.995056  2.1916  0.029019  gamma10
timesq      -0.046907   0.015826 366.791758 -2.9640  0.003235  gamma20
```

Model 1b Mplus M-SEM:

```
TITLE:   M-SEM Model 1b: Fixed Quadratic Time, Random Intercept
DATA:    FILE = Chapter10.csv;              ! Syntax in same folder as data
         TYPE = INDIVIDUAL; FORMAT = FREE;  ! Defaults

VARIABLE:
! List of variables in long data file IN ORDER
  NAMES = PersonID occasion ageT0 tvage recall time;
! Variables to be analyzed in this model (new defined at end)
  USEVARIABLE = recall time timesq;
! Missing data identifier
  MISSING ARE ALL (-999);
! MSEM options
  CLUSTER = PersonID;       ! Level-2 ID
  BETWEEN = ;               ! Observed ONLY level-2 predictors
  WITHIN  = time timesq;    ! Observed ONLY level-1 predictors

DEFINE:  ! Make squared version of time
  timesq = time*time;

ANALYSIS:   TYPE = TWOLEVEL RANDOM; ESTIMATOR = ML;

MODEL:  ! LEVEL-1 = WITHIN, LEVEL-2 = BETWEEN
%WITHIN%
 recall;               ! L1 R: Residual variance
 recall ON time;       ! No B1i placeholder yet because fixed linear only
 recall ON timesq;     ! No B2i placeholder because fixed quad only

%BETWEEN%
[recall];      ! Fixed intercept
 recall;       ! L2 G: Random intercept variance only
```

```
Number of Free Parameters                        5
Loglikelihood
          H0 Value                       -1423.004

MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Within Level
 RECALL     ON
    TIME               0.261      0.119      2.187      0.029  gamma10
    TIMESQ            -0.047      0.016     -2.963      0.003  gamma20
 Residual Variances
    RECALL             4.983      0.380     13.122      0.000  Level-1 var(e_ti)

Between Level
 Means
    RECALL             9.661      0.275     35.129      0.000  gamma00
 Variances
    RECALL            10.622      1.320      8.044      0.000  Level-2 var(U_0i)
```

Model 1b Mplus SEM:

```
TITLE:   SEM Model 1b: Fixed Quadratic Time, Random Intercept
DATA:    FILE = Chapter10.csv;              ! Syntax in same folder as data
         TYPE = INDIVIDUAL; FORMAT = FREE;  ! Defaults

! Unstacking to multivariate format
DATA LONGTOWIDE:
! Names of old stacked former variables (without numbers)
  LONG = recall|time;
! Names of new multivariate variables (that use numbers)
  WIDE = recall0 recall2 recall4 recall6 recall8 |
         time0 time2 time4 time6 time8;
! Variable with level-2 ID info
  IDVARIABLE = PersonID;
! Old level-1 identifier
  REPETITION = occasion (0 2 4 6 8);

VARIABLE:
! List of variables in long data file IN ORDER
  NAMES = PersonID occasion ageT0 tvage recall time;
! Variables to be analyzed in this model (new defined at end)
  USEVARIABLE = recall0 recall2 recall4 recall6 recall8
                time0 time2 time4 time6 time8;
! Missing data identifier
  MISSING ARE ALL (-999);
! Exact time definition variables
  TSCORES = time0 time2 time4 time6 time8;

ANALYSIS:    TYPE = RANDOM; ESTIMATOR = ML;

MODEL:
[recall0-recall8@0];          ! All variable intercepts fixed to 0
 recall0-recall8 (Resvar);    ! L1 R residual variances held equal

! Recall quadratic growth model using exact time as loadings
  Int Lin Qua | recall0-recall8 AT time0-time8;

! Level-2 model
  [Int Lin Qua];        ! Fixed intercept, linear quad time slopes
   Int Lin@0 Qua@0;     ! L2 G: Random intercept variance (Lin=0 & Quad=0)
```

```
Number of Free Parameters                        5
Loglikelihood
          H0 Value                       -1422.043
MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
 Means
    INT                9.660      0.275     35.168      0.000  gamma00
    LIN                0.261      0.118      2.205      0.027  gamma10
    QUA               -0.047      0.016     -2.983      0.003  gamma20
 Intercepts
    RECALL0            0.000      0.000    999.000    999.000
    RECALL2            0.000      0.000    999.000    999.000
    RECALL4            0.000      0.000    999.000    999.000
    RECALL6            0.000      0.000    999.000    999.000
    RECALL8            0.000      0.000    999.000    999.000
 Variances
    INT               10.687      1.324      8.071      0.000  Level-2 var(U_0i)
    LIN                0.000      0.000    999.000    999.000
    QUA                0.000      0.000    999.000    999.000
 Residual Variances
    RECALL0            4.879      0.377     12.929      0.000  Level-1 var(e_ti)
    RECALL2            4.879      0.377     12.929      0.000
    RECALL4            4.879      0.377     12.929      0.000
    RECALL6            4.879      0.377     12.929      0.000
    RECALL8            4.879      0.377     12.929      0.000
```

Model 2b. Syntax and Partial Output for Fixed Quadratic Time, Random Intercept for Recall, adding Age at Baseline to Introduce Total Cross-Sectional Birth Cohort Effects:

Level-1 Time: $recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - ageT0_i) + \beta_{2i}(age_{ti} - ageT0_i)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(ageT0_i - 84) + \gamma_{02}(ageT0_i - 84)^2 + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(ageT0_i - 84),\ \beta_{2i} = \gamma_{20}$

Model 2b STATA Univariate MLM:

```
display "Model 2b: Fixed Quadratic Time, Random Intercept Model"
display "Controlling for Birth Cohort as Total Effects"
mixed recall time c.timesq c.aget084 c.aget084sq c.time#c.aget084, ///
             || personid: , mle nolog
```

Model 2b R Univariate MLM:

```
print("Model 2b: Fixed Quadratic Time, Random Intercept Model")
print("Controlling for Birth Cohort as Total Effects")
RICohTim = lmer(data=Example3, REML=FALSE, formula=recall~1+time+timesq
                +ageT084+ageT084sq+time:ageT084+(1|PersonID))
llikAIC(RICohTim); summary(RICohTim);

$AICtab
       AIC        BIC     logLik   deviance   df.resid
 2851.3846  2885.9651 -1417.6923  2835.3846   549.0000

Random effects:
 Groups   Name        Variance Std.Dev.
 PersonID (Intercept) 10.2325  3.1988   Level-2 var(U_0i)
 Residual              4.9275  2.2198   Level-1 var(e_ti)

Fixed effects:
               Estimate  Std. Error          df t value  Pr(>|t|)
(Intercept)   9.3880788   0.3414122 263.1359999 27.4978 < 2.2e-16  gamma00
time          0.2876175   0.1192557 380.6947769  2.4118  0.016348  gamma10
timesq       -0.0435466   0.0158152 366.6031559 -2.7535  0.006190  gamma20
ageT084      -0.2879274   0.1000388 250.7639296 -2.8782  0.004345  gamma01
ageT084sq     0.0068459   0.0184967 235.7300477  0.3701  0.711631  gamma02
time:ageT084  0.0389327   0.0177798 400.3882883  2.1897  0.029122  gamma11
```

Model 2b Mplus M-SEM:

```
TITLE:   M-SEM Model 2b: Add Birth Cohort to Fixed Quad Time, Random Intercept
DATA:    FILE = Chapter10.csv;              ! Syntax in same folder as data
         TYPE = INDIVIDUAL; FORMAT = FREE;  ! Defaults

VARIABLE:
! List of variables in long data file IN ORDER
  NAMES = PersonID occasion ageT0 tvage recall time;
! Variables to be analyzed in this model (new defined at end)
  USEVARIABLE = recall time timesq aT084 aT084sq;
! Missing data identifier
  MISSING ARE ALL (-999);
! MSEM options
  CLUSTER = PersonID;        ! Level-2 ID
  BETWEEN = aT084 aT084sq;   ! Observed ONLY level-2 predictors
  WITHIN  = time timesq;     ! Observed ONLY level-1 predictors

DEFINE:  ! Center predictors and make squared versions
  timesq  = time*time;
  aT084   = ageT0-84;
  aT084sq = aT084*aT084;
ANALYSIS:   TYPE = TWOLEVEL RANDOM; ESTIMATOR = ML;
```

```
MODEL:  ! LEVEL-1 = WITHIN, LEVEL-2 = BETWEEN
%WITHIN%
 recall;               ! L1 R: Residual variance
 lin | recall ON time; ! B1i placeholder for linear time slope
 recall ON timesq;     ! No B2i placeholder because fixed quad only

%BETWEEN%
[recall lin];          ! Fixed intercept and linear time slope
 recall lin@0;         ! L2 G: Random intercept variance only
 recall lin ON aT084;  ! Linear Cohort -> recall int and linear time slope
 recall ON aT084sq;    ! Quadratic Cohort -> recall int

Number of Free Parameters                        8
Loglikelihood
          H0 Value                       -1417.679

MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Within Level
 RECALL     ON
    TIMESQ            -0.044      0.016     -2.753      0.006  gamma20
 Residual Variances
    RECALL             4.926      0.375     13.126      0.000  Level-1 var(e_ti)

Between Level
 LIN        ON
    AT084              0.039      0.018      2.189      0.029  gamma11
 RECALL     ON
    AT084             -0.288      0.100     -2.878      0.004  gamma01
    AT084SQ            0.007      0.018      0.370      0.711  gamma02
 Intercepts
    RECALL             9.388      0.341     27.497      0.000  gamma00
    LIN                0.288      0.119      2.408      0.016  gamma10
 Residual Variances
    RECALL            10.233      1.277      8.012      0.000  Level-2 var(U_0i)
    LIN                0.000      0.000    999.000    999.000
```

Model 2b Mplus SEM:

```
TITLE:   SEM Model 2b: Add Birth Cohort to Fixed Quad Time, Random Intercept
DATA:    FILE = Chapter10.csv;              ! Syntax in same folder as data
         TYPE = INDIVIDUAL; FORMAT = FREE;  ! Defaults

! Unstacking to multivariate format
DATA LONGTOWIDE:
! Names of old stacked former variables (without numbers)
  LONG = recall|time;
! Names of new multivariate variables (that use numbers)
  WIDE = recall0 recall2 recall4 recall6 recall8 |
         time0 time2 time4 time6 time8;
! Variable with level-2 ID info
  IDVARIABLE = PersonID;
! Old level-1 identifier
  REPETITION = occasion (0 2 4 6 8);

VARIABLE:
! List of variables in long data file IN ORDER
  NAMES = PersonID occasion ageT0 tvage recall time;
! Variables to be analyzed in this model (new defined at end)
  USEVARIABLE = recall0 recall2 recall4 recall6 recall8
                time0 time2 time4 time6 time8 aT084 aT084sq;
! Missing data identifier
  MISSING ARE ALL (-999);
! Exact time definition variables
  TSCORES = time0 time2 time4 time6 time8;
```

```
DEFINE:  ! Center predictors and make squared versions
  aT084   = ageT0-84;
  aT084sq = aT084*aT084;

ANALYSIS:   TYPE = RANDOM; ESTIMATOR = ML;

MODEL:
[recall0-recall8@0];         ! All variable intercepts fixed to 0
 recall0-recall8 (Resvar);   ! L1 R residual variances held equal

! Recall quadratic growth model using exact time as loadings
  Int Lin Qua | recall0-recall8 AT time0-time8;

! Level-2 model
  [Int Lin Qua];       ! Fixed intercept, linear quad time slopes
   Int Lin@0 Qua@0;    ! L2 G: Random intercept variance (Lin=0 & Quad=0)
   Int Lin ON aT084;   ! Linear Cohort -> Int and linear time slope
   Int ON aT084sq;     ! Quadratic Cohort -> Int
```

```
Number of Free Parameters                        8
Loglikelihood
          H0 Value                       -1416.747

Information Criteria
          Akaike (AIC)                    2849.494
          Bayesian (BIC)                  2876.155
          Sample-Size Adjusted BIC        2850.808
            (n* = (n + 2) / 24)
```

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
INT	ON					
	AT084	-0.288	0.100	-2.875	0.004	**gamma01**
	AT084SQ	0.007	0.018	0.364	0.716	**gamma02**
LIN	ON					
	AT084	0.039	0.018	2.170	0.030	**gamma11**
Means						
	QUA	-0.044	0.016	-2.775	0.006	**gamma20**
Intercepts						
	RECALL0	0.000	0.000	999.000	999.000	
	RECALL2	0.000	0.000	999.000	999.000	
	RECALL4	0.000	0.000	999.000	999.000	
	RECALL6	0.000	0.000	999.000	999.000	
	RECALL8	0.000	0.000	999.000	999.000	
	INT	9.388	0.341	27.520	0.000	**gamma00**
	LIN	0.288	0.119	2.428	0.015	**gamma10**
Variances						
	QUA	0.000	0.000	999.000	999.000	
Residual Variances						
	RECALL0	4.825	0.373	12.935	0.000	**Level-1 var(e_ti)**
	RECALL2	4.825	0.373	12.935	0.000	
	RECALL4	4.825	0.373	12.935	0.000	
	RECALL6	4.825	0.373	12.935	0.000	
	RECALL8	4.825	0.373	12.935	0.000	
	INT	10.295	1.281	8.037	0.000	**Level-2 var(U_0i)**
	LIN	0.000	0.000	999.000	999.000	

The Mplus M-SEM and SEM results are very close, but they are not quite the same as the univariate MLM results. This is (at least partly) because they are using different definitions of the fixed linear time slope (which is the conditional mean of the person mean slopes in the M-SEM framework because it is predicted by cohort, but not in the univariate MLM version in which level-2 units are not distinguished without an accompanying random slope). The Mplus SEM results differ more, though (perhaps because of optimizing on wide data?).

Model 3b. Syntax and Partial Output to add Random Linear Time to Model 2b:

Level-1 Time: $recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - ageT0_i) + \beta_{2i}(age_{ti} - ageT0_i)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(ageT0_i - 84) + \gamma_{01}(ageT0_i - 84)^2 + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(ageT0_i - 84) + U_{1i}$, $\beta_{2i} = \gamma_{20}$

Model 3b STATA Univariate MLM:

```
display "Model 3b Time: Add Random Linear Time to Model 2b"
mixed recall c.time c.timesq c.aget084 c.aget084sq c.tvage84#c.aget084, ///
            || personid: time, mle nolog covariance(unstructured)
```

Model 3b R Univariate MLM:

```
print("Model 3b Time: Add Random Linear Time to Model 2b")
RLCohTim = lmer(data=Example3, REML=FALSE, formula=recall~1+time+timesq
               +ageT084+ageT084sq+time:ageT084+(1+time|PersonID))
llikAIC(RLCohTim); summary(RLCohTim)

$AICtab
       AIC        BIC     logLik   deviance   df.resid
 2838.5453  2881.7709 -1409.2726  2818.5453   547.0000
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 PersonID (Intercept) 12.4835  3.53320
          time         0.1272  0.35665  -0.473  → new Level-2 var(U_1i)
 Residual              3.9405  1.98508

Fixed effects:
               Estimate  Std. Error          df t value  Pr(>|t|)
(Intercept)   9.3402104   0.3515637 230.2831758 26.5676 < 2.2e-16
time          0.3132277   0.1123665 366.1427566  2.7876  0.005588
timesq       -0.0455538   0.0149692 340.1904601 -3.0432  0.002523
ageT084      -0.2972341   0.1050941 205.1743603 -2.8283  0.005144
ageT084sq     0.0091296   0.0183207 229.8606122  0.4983  0.618735
time:ageT084  0.0442743   0.0207981 126.2247447  2.1288  0.035217

print("LRT for random linear TVage slope"); anova(RLCohTim,RICohTim)

         npar     AIC     BIC   logLik deviance   Chisq Df Pr(>Chisq)
RICohTim    8 2851.39 2885.97 -1417.69  2835.39
RLCohTim   10 2838.55 2881.77 -1409.27  2818.55 16.8393  2 0.00022049
```

Model 3b Mplus M-SEM:

```
TITLE:   M-SEM Model 3b: Add Random Linear Time to Model 2b (Fixed Time + Cohort)
DATA:    FILE = Chapter10.csv;              ! Syntax in same folder as data
         TYPE = INDIVIDUAL; FORMAT = FREE;  ! Defaults

VARIABLE:
! List of variables in long data file IN ORDER
  NAMES = PersonID occasion ageT0 tvage recall time;
! Variables to be analyzed in this model (new defined at end)
  USEVARIABLE = recall time timesq aT084 aT084sq;
! Missing data identifier
  MISSING ARE ALL (-999);
! MSEM options
  CLUSTER = PersonID;       ! Level-2 ID
  BETWEEN = aT084 aT084sq;  ! Observed ONLY level-2 predictors
  WITHIN  = time timesq;    ! Observed ONLY level-1 predictors

DEFINE:  ! Center predictors and make squared versions
  timesq = time*time;
  aT084 = ageT0-84;
  aT084sq = aT084*aT084;
```

```
ANALYSIS:   TYPE = TWOLEVEL RANDOM; ESTIMATOR = ML;

MODEL:  ! LEVEL-1 = WITHIN, LEVEL-2 = BETWEEN
%WITHIN%
 recall;               ! L1 R: Residual variance
 lin | recall ON time; ! B1i placeholder for linear time slope
 recall ON timesq;     ! No B2i placeholder because fixed quad only

%BETWEEN%
[recall lin];          ! Fixed intercept and linear time slope
 recall lin;           ! L2 G: Random effects variances
 recall WITH lin;      ! L2 G: Random effects covariance
 recall lin ON aT084;  ! Linear Cohort -> recall int and linear time slope
 recall ON aT084sq;    ! Quadratic Cohort -> recall int

Number of Free Parameters                       10
Loglikelihood
          H0 Value                      -1409.272

MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Within Level
 RECALL     ON
    TIMESQ            -0.046      0.015     -3.026      0.002
 Residual Variances
    RECALL             3.941      0.360     10.957      0.000

Between Level
 LIN        ON
    AT084              0.044      0.021      2.122      0.034
 RECALL     ON
    AT084             -0.297      0.105     -2.826      0.005
    AT084SQ            0.009      0.018      0.496      0.620
 RECALL   WITH
    LIN               -0.596      0.226     -2.644      0.008  new Level-2 covar(U_0i,U_1i)
 Intercepts
    RECALL             9.340      0.352     26.501      0.000
    LIN                0.313      0.113      2.778      0.005
 Residual Variances
    RECALL            12.483      1.596      7.822      0.000
    LIN                0.127      0.043      2.944      0.003  new Level-2 var(U_1i)
```

Model 3b Mplus SEM:

```
TITLE:   SEM Model 3b: Add Random Linear Time to Model 2b (Fixed Time + Cohort)
DATA:    FILE = Chapter10.csv;              ! Syntax in same folder as data
         TYPE = INDIVIDUAL; FORMAT = FREE;  ! Defaults

! Unstacking to multivariate format
DATA LONGTOWIDE:
! Names of old stacked former variables (without numbers)
  LONG = recall|time;
! Names of new multivariate variables (that use numbers)
  WIDE = recall0 recall2 recall4 recall6 recall8 |
         time0 time2 time4 time6 time8;
! Variable with level-2 ID info
  IDVARIABLE = PersonID;
! Old level-1 identifier
  REPETITION = occasion (0 2 4 6 8);

VARIABLE:
! List of variables in long data file IN ORDER
  NAMES = PersonID occasion ageT0 tvage recall time;
! Variables to be analyzed in this model (new defined at end)
  USEVARIABLE = recall0 recall2 recall4 recall6 recall8
                time0 time2 time4 time6 time8 aT084 aT084sq;
```

```
! Missing data identifier
  MISSING ARE ALL (-999);
! Exact time definition variables
  TSCORES = time0 time2 time4 time6 time8;

DEFINE:  ! Center predictors and make squared versions
  aT084 = ageT0-84;
  aT084sq = aT084*aT084;

ANALYSIS:   TYPE = RANDOM; ESTIMATOR = ML;

MODEL:
[recall0-recall8@0];         ! All variable intercepts fixed to 0
 recall0-recall8 (Resvar);   ! L1 R residual variances held equal

! Recall quadratic growth model using exact time as loadings
  Int Lin Qua | recall0-recall8 AT time0-time8;

! Level-2 model
  [Int Lin Qua];       ! Fixed intercept, linear quad time slopes
   Int Lin Qua@0;      ! L2 G: Random effects variances (Quad=0)
   Int WITH Lin;       ! L2 G: Random effects covariance
   Int Lin ON aT084;   ! Linear Cohort -> Int and linear time slope
   Int ON aT084sq;     ! Quadratic Cohort -> Int
```

```
Number of Free Parameters                       10
Loglikelihood
          H0 Value                      -1409.189
```

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
INT ON					
AT084	-0.297	0.105	-2.825	0.005	
AT084SQ	0.009	0.018	0.495	0.620	
LIN ON					
AT084	0.044	0.021	2.122	0.034	
INT WITH					
LIN	-0.589	0.225	-2.613	0.009	**new Level-2 covar(U_0i,U_1i)**
Means					
QUA	-0.046	0.015	-3.011	0.003	
Intercepts					
RECALL0	0.000	0.000	999.000	999.000	
RECALL2	0.000	0.000	999.000	999.000	
RECALL4	0.000	0.000	999.000	999.000	
RECALL6	0.000	0.000	999.000	999.000	
RECALL8	0.000	0.000	999.000	999.000	
INT	9.340	0.352	26.504	0.000	
LIN	0.313	0.113	2.779	0.005	
Variances					
QUA	0.000	0.000	999.000	999.000	
Residual Variances					
RECALL0	3.938	0.360	10.942	0.000	
RECALL2	3.938	0.360	10.942	0.000	
RECALL4	3.938	0.360	10.942	0.000	
RECALL6	3.938	0.360	10.942	0.000	
RECALL8	3.938	0.360	10.942	0.000	
INT	12.480	1.596	7.821	0.000	
LIN	0.121	0.043	2.794	0.005	**new Level-2 var(U_1i)**

Model 4b. Syntax and Partial Output to add Random Linear Baseline Age to Model 3b:

Level-1 Time: $recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - ageT0_i) + \beta_{2i}(age_{ti} - ageT0_i)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(ageT0_i - 84) + \gamma_{02}(ageT0_i - 84)^2 + U_{0i} + ?(ageT0_i - 84)$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(ageT0_i - 84) + U_{1i},\ \beta_{2i} = \gamma_{20}$

Btw, this punctuation mark ‽ is called an interrobang

Model 4b STATA Univariate MLM (converged but NPD):

```
display "Model 4b: Add Random Linear AgeCoh to Model 3b -- extra iterations"
mixed recall c.time c.timesq c.aget084 c.aget084sq c.tvage84#c.aget084, ///
            || personid: time aget084, mle nolog emiterate(100) covariance(unstructured)
------------------------------------------------------------------------------
             recall |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------------------+---------------------------------------------------------
               time |   .3135874   .1123693     2.79   0.005     .0933477    .5338271
             timesq |  -.0455589   .0149707    -3.04   0.002     -.074901   -.0162169
           ageT084 |  -.2974387   .1053805    -2.82   0.005    -.5039806   -.0908968
         ageT084sq |  -.0351895   .0268436    -1.31   0.190    -.0878019    .0174229
c.tvage84#c.ageT084 |   .0447116    .020784     2.15   0.031     .0039757    .0854474
              _cons |   9.336693   .3540654    26.37   0.000     8.642737    10.03065
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.      [95% Conf. Interval]
-----------------------------+------------------------------------------------
PersonID: Unstructured       |
                   var(time) |   .1269905   .0431637      .0652304    .2472249
                 var(ageT084) |   .0005878     .00043      .0001401    .0024653 → new term!
                   var(_cons) |   12.53215   1.599177      9.759045    16.09326
             cov(time,ageT084) |  -.0085325          .             .           .
               cov(time,_cons) |  -.6000924   .2264234     -1.043874   -.1563108
           cov(ageT084,_cons) |   .0520106          .             .           .
-----------------------------+------------------------------------------------
                var(Residual) |   3.941342   .3611464      3.293429    4.716719
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 238.31                Prob > chi2 = 0.0000
```

Model 4b R Univariate MLM (won't even run):

```
print("Model 4b: Add Random Linear Time to Model 3b -- won't run at all")
RL2CohTim = lmer(data=Example3, REML=FALSE, formula=recall~1+time+timesq
                 +ageT084+ageT084sq+time:ageT084+(1+time+ageT084|PersonID))
llikAIC(RL2CohTim); summary(RL2CohTim)
```

Model 4b Mplus M-SEM (sort of converged; SEM version would not run at all):

```
TITLE:   M-SEM Model 3b: Add Random Linear Cohort to Model 3b (Random Time + Cohort)
DATA:    FILE = Chapter10.csv;              ! Syntax in same folder as data
         TYPE = INDIVIDUAL; FORMAT = FREE;  ! Defaults

VARIABLE:
! List of variables in long data file IN ORDER
  NAMES = PersonID occasion ageT0 tvage recall time;
! Variables to be analyzed in this model (new defined at end)
  USEVARIABLE = recall time timesq aT084 aT084sq;
! Missing data identifier
  MISSING ARE ALL (-999);
! MSEM options
  CLUSTER = PersonID;       ! Level-2 ID
  BETWEEN = aT084 aT084sq;  ! Observed ONLY level-2 predictors
  WITHIN  = time timesq;    ! Observed ONLY level-1 predictors

DEFINE:  ! Center predictors and make squared versions
  timesq = time*time;
  aT084  = ageT0-84; aT084sq = aT084*aT084;
```

```
ANALYSIS:   TYPE = TWOLEVEL RANDOM; ESTIMATOR = ML;
            ALGORITHM = INTEGRATION;  ! Required for interobang

MODEL:  ! LEVEL-1 = WITHIN, LEVEL-2 = BETWEEN
%WITHIN%
 recall;               ! L1 R: Residual variance
 lin | recall ON time; ! B1i placeholder for linear time slope
 recall ON timesq;     ! No B2i placeholder because fixed quad only

%BETWEEN%
[recall lin];          ! Fixed intercept and linear time slope
 recall lin;           ! L2 G: Random effects variances
 recall WITH lin;      ! L2 G: Random effects covariance
 recall lin ON aT084;  ! Linear Cohort -> recall int and linear time slope
 recall ON aT084sq;    ! Quadratic Cohort -> recall int

Intero | recall ON aT084;  ! Define random slope for level-2 predictor
recall ON Intero;          ! Add L2 predictor random slope to predict recall
```

```
     WARNING: THE MODEL ESTIMATION HAS REACHED A SADDLE POINT OR A POINT WHERE THE
     OBSERVED AND THE EXPECTED INFORMATION MATRICES DO NOT MATCH.
     AN ADJUSTMENT TO THE ESTIMATION OF THE INFORMATION MATRIX HAS BEEN MADE.
     THE CONDITION NUMBER IS      -0.138D-05.
     THE PROBLEM MAY ALSO BE RESOLVED BY DECREASING THE VALUE OF THE
     MCONVERGENCE OR LOGCRITERION OPTIONS OR BY CHANGING THE STARTING VALUES
     OR BY INCREASING THE NUMBER OF INTEGRATION POINTS OR BY USING THE MLF ESTIMATOR.

     THE STANDARD ERRORS OF THE MODEL PARAMETER ESTIMATES MAY NOT BE
     TRUSTWORTHY FOR SOME PARAMETERS DUE TO A NON-POSITIVE DEFINITE
     FIRST-ORDER DERIVATIVE PRODUCT MATRIX.  THIS MAY BE DUE TO THE STARTING
     VALUES BUT MAY ALSO BE AN INDICATION OF MODEL NONIDENTIFICATION.  THE
     CONDITION NUMBER IS       0.590D-18.  PROBLEM INVOLVING THE FOLLOWING PARAMETER:
     Parameter 8, %BETWEEN%: RECALL ON AT084

THE MODEL ESTIMATION TERMINATED NORMALLY
Number of Free Parameters                       13
Loglikelihood
          H0 Value                       -1409.293

MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Within Level
 RECALL     ON
    TIMESQ            -0.043      0.015     -2.842      0.004
 Residual Variances
    RECALL             3.945      0.360     10.952      0.000

Between Level
 LIN        ON
    AT084              0.044      0.022      2.022      0.043
 RECALL     ON
    INTERO             1.052      4.148      0.254      0.800 → new variance
 RECALL     ON
    AT084             -0.199      3.989     -0.050      0.960
    AT084SQ            0.009      0.019      0.490      0.624
 RECALL   WITH
    LIN               -0.592      0.227     -2.604      0.009
 Means
    INTERO            -0.099      3.963     -0.025      0.980
 Intercepts
    RECALL             9.458      4.625      2.045      0.041
    LIN                0.293      0.114      2.580      0.010
 Variances
    INTERO             0.000      0.364      0.001      0.999
 Residual Variances
    RECALL            12.478      2.935      4.252      0.000
    LIN                0.126      0.043      2.929      0.003
```