Example 3: Models for Change over Time in Latent Factors using Single-Level Structural Equation Modeling (SEM) (complete syntax and output available for Mplus electronically)

These real data (N = 653) come from the Octogenarian Twin Study of Aging in Sweden. I am analyzing three measures of cognition—block design, digit–symbol substitution, and prose recall—whose pattern of correlation is consistent with a single latent factor at each occasion. For the sake of this example, I am only using four occasions (collected at two-year intervals) and pretending these occasions are completely balanced (given that these models are more difficult to estimate for unbalanced occasions). Likewise, I am ignoring the nesting of individuals in twin pairs to use as many observations as possible. This analysis will involve three main steps: (1) verifying the factor structure across occasions as a *configural invariance* model (model 1), (2) testing *longitudinal measurement invariance* to ensure comparable meaning of the latent factor over time (models 2a–4b), and (3) examining whether higher-order factors for an intercept and latent basis change can adequately describe the pattern of means, variances, and covariances over time in the latent factor (models 5a–5b).

Model 1. Mplus Syntax for Configural Invariance—all measurement model parameters estimated separately over time, with all factor means=0 and factor variances=1 fixed for identification:



Model 1. Mplus Output for Configural Invariance:

Number of Free Parameters 60 → 12 load, 12 int, 12 resvar, 18 res cov, Loglikelihood and 6 factor cov -13135.677 -> Our configural invariance model LL HO Value H0 Scaling Correction Factor 1.0873 > Deviation from multiv normality=1 for MLR -13121.771 > Saturated=best model LL H1 Value H1 Scaling Correction Factor 1.0595 → Deviation from multiv normality=1 for MLR Information Criteria **>** Smaller is better (because they start with -2LL) Akaike (AIC) 26391.355 Bayesian (BIC) 26660.250 Sample-Size Adjusted BIC 26469.750 $(n^* = (n + 2) / 24)$ Chi-Square Test of Model Fit Value 27.704* \rightarrow LRT for configural against saturated=best Degrees of Freedom 30 P-Value 0.5861 MLR estimation requires a modified LRT formula Scaling Correction Factor 1.0039 using the scaling correlation factors given above for MLR The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option. RMSEA (Root Mean Square Error Of Approximation) → How much worse than saturated model=0 Estimate 0.000 90 Percent C.I. 0.000 0.027 Probability RMSEA <= .05 1.000 CFI/TLI CFI 1.000 \rightarrow How much better than null model=0 TLI 1.000 Chi-Square Test of Model Fit for the Baseline Model 3516.779 → LRT for null vs saturated (don't need) Value Degrees of Freedom 66 P-Value 0.0000 SRMR (Standardized Root Mean Square Residual) \rightarrow How much worse than saturated model=0 Value 0.010 Two-Tailed S.E. Est./S.E. Estimate P-Value Factor loadings \rightarrow slope of factor predicting each outcome т1 BY BLOCK1 6.046 0.239 25.275 0.000 DIGIT1 10.648 0.434 24.522 0.000 PROSE1 3.272 0.147 22.209 0.000 т2 BY 0.220 BLOCK2 6.449 29.371 0.000 10.975 0.416 26.400 0.000 DIGIT2 PROSE2 3.558 0.152 23.400 0.000 ͲЗ BY BLOCK3 6.610 0.253 26.118 0.000 DIGIT3 11.624 0.453 25.672 0.000 PROSE3 3.866 0.177 21.809 0.000 т4 BY 24.373 6.976 0.286 0.000 BLOCK4

21.464

24.172

0.000

0.000

0.596

0.194

12.787

4.690

DIGIT4 PROSE4

					Two-Tailed	
		Estimate	S.E. 1	Est./S.E.	P-Value	
FACTOR	COVARIANCES	(= CORRELATIO	NS BECAUS	E FACTOR VA	ARIANCES=1)	
Τ1	WITH	•			.	
Т2		0.952	0.014	66.221	0.000	
Т3		0.871	0.030	28.985	0.000	
Т4		0.825	0.032	25.386	0.000	
Т2	WITH					
Т3		0.942	0.022	41.877	0.000	
Т4		0.911	0.022	40.934	0.000	
тЗ	WITH					
т4		0.954	0.014	69.532	0.000	
RESIDUA	L COVARIANCE	ES FOR SAME OU	TCOME OVE	R TIME		
BLOCK1	WITH					
BLO	CK2	7.565	1.274	5.940	0.000	
BLO	CK3	7 778	1 261	6 1 6 9	0 000	
BLO	CKN	5 987	1 111	1 155	0.000	
BLOCK2	<u>штт</u> н	5.507	T • 111	1.100	0.000	
BIOCK2	CK3	6 900	1 256	5 192	0 000	
BLO	CKJ	0.900	1 207	2 200	0.000	
DIOCKS	WT MU	4.110	1.20/	3.200	0.001	
BLUCKS	WIIH	E 400	1 472	2 (07	0 000	
BLO	CK4	5.432	1.4/3	3.68/	0.000	
DIGITI	WITH WITH	0 0 7 0	2 4 2 6	0 654	0 0 0 0	
DIG	1T2 	9.279	3.496	2.654	0.008	
DIG	IT3	7.746	3.521	2.200	0.028	
DIG	IT4	8.503	3.979	2.137	0.033	
DIGIT2	WITH					
DIG	IT3	8.249	3.404	2.423	0.015	
DIG	IT4	8.766	3.571	2.455	0.014	
DIGIT3	WITH					
DIG	IT4	4.525	3.863	1.171	0.241	
PROSE1	WITH					
PRO	SE2	5.181	0.647	8.011	0.000	
PRO	SE3	4.403	0.708	6.218	0.000	
PRO	SE4	3.932	0.767	5.127	0.000	
PROSE2	WITH					
PRO	SE3	5.568	0.736	7.566	0.000	
PRO	SE4	4.697	0.857	5.480	0.000	
PROSE3	WITH					
PRO	SE4	5 233	0 779	6 720	0 0 0 0	
FACTOR	MEANS (IS "N	TEAN" FOR ANY	VARTABLE	IN THE LIKI	TITHOOD NOT PR	EDICTED)
Means						
пеано т1		0 000	0 000	999 000	999 000	
тт то		0.000	0.000	999.000	999.000	
12 T2		0.000	0.000	999.000	999.000	
1J 		0.000	0.000	999.000	999.000	
	тышерсерше					
Tatorgo	TNIERCEP15	(EXPECIED OUT	COME WHEN	FACTOR PRI	DICIOR-0)	
TILETCE	CV1	10 172	0 202	22 617	0 000	
BLO	CK1	10.173	0.302	33.047	0.000	
BLO	CKZ CK2	9.004	0.311	27 205	0.000	
BLO	CKS	0.752	0.321	27.303	0.000	
BLO	CK4	7.519	0.364	20.653	0.000	
DIG	1T1 	21.039	0.511	41.135	0.000	
DIG	IT2	19.923	0.526	37.908	0.000	
DIG	IT3	18.714	0.573	32.682	0.000	
DIG	IT4	15.602	0.710	21.974	0.000	
PRO	SE1	8.503	0.187	45.513	0.000	
PRO	SE2	8.097	0.211	38.412	0.000	
PRO	SE3	7.274	0.239	30.412	0.000	
PRO	SE4	6.521	0.289	22.582	0.000	
FACTOR	VARIANCES ()	IS "VARIANCE"	FOR ANY V	ARIABLE IN	THE LIKELIHOO	D NOT PREDICTED)
Varian	ces					
Т1		1.000	0.000	999.000	999.000	
Т2		1.000	0.000	999.000	999.000	
Т3		1.000	0.000	999.000	999.000	
Т4		1.000	0.000	999.000	999.000	

OUTCOME LEFTOVER	VARIANCES (IS	"RESIDUAL	VARIANCE"	FOR ANY PREDICT	ED OUTCOME)
Residual Variance	es				
BLOCK1	19.334	1.707	11.329	0.000	
BLOCK2	14.178	1.456	9.736	0.000	
BLOCK3	12.465	1.739	7.168	0.000	
BLOCK4	12.533	1.807	6.935	0.000	
DIGIT1	32.716	4.583	7.138	0.000	
DIGIT2	24.595	3.834	6.414	0.000	
DIGIT3	24.554	4.088	6.006	0.000	
DIGIT4	24.878	4.918	5.058	0.000	
PROSE1	9.981	0.680	14.686	0.000	
PROSE2	10.664	0.774	13.778	0.000	
PROSE3	9.803	1.017	9.643	0.000	
PROSE4	7.431	0.960	7.739	0.000	

Given the excellent fit of this model, it appears that the outcome means, variances, and covariances are well recreated by the four correlated factors (one for each occasion), along with residual covariances for the same outcome over time. Next, we examine **longitudinal measurement invariance** for each parameter separately: **loadings** (called **metric** or weak), **intercepts** (called **scalar** or strong), and **residual variances** (called **residual** or strict). To compare each layer of constraints as nested models, we will use **rescaled likelihood ratio tests**, which is the $-2\Delta LL$ accounting for the scaling correction factors. At each layer, we will hope that global model fit is **not significantly worse** from enforcing the invariance constraints, and we will also examine modification indices to see if any specific parameters want to be noninvariant (different) over time (as local fit). For more explanation and examples of testing invariance, please see Lecture 7 and Examples 7a–7d from <u>my SEM class</u>.

Model 2a. Mplus Syntax for Full Metric Invariance—Model 1 except the factor loadings for the same outcome are now constrained equal over time, and the factor variance =1 at T1 for identification but is free at T2–T4:



T1 T2 T3 T4 WITH T1* T2* T3* T4*;

Model 2a. Mplus Output for Full Metric Invariance:

Number of Free Parameters 54 \rightarrow Saved DF=6 (212cad vs. 31cad + 3FactVar) Leglikelihod HO Value 13141.701 \rightarrow Our metric invariance model LL HO Scaling Correction Factor 1.1194 For MIR -13121.771 \rightarrow Saturated=best model LL 1.0595 Information Criteria -13121.771 \rightarrow Saturated=best model LL Information Criteria 1.0595 $(n^+ = (n + 2) / 24)$ Chi-Square Test of Model Fit Value 26391.403 Pacevent index of the configural model (12) Makike (ATC) 26391.403 Newse, modification indices (below) suggest (n^+ = (n + 2) / 24) 26461.956 In examining why the constrained model fits wores, modification indices (below) suggest Scaling Correction Factor 0.505 1.000 Inter invariance model nonve forward.* Statimate 0.015 90 Percent C.I. 0.000 0.015 90 Percent C.I. 0.0000 0.015 7.272 0.506 7.11 M Statimate 0.028 1.272 0.506 FTLI 0.997 1.10 Nove Failed 1.282 0.526 SMDEL RESULTS (RELEVANT FARAMETERS ONLY) Two-Tailed<	MODEL FIT	INFORMATION			
HIG Value -13141.701 → Our metric invariance model LL HG Scaling Correction Factor 1.1194 HG Scaling Correction Factor 1.0595 for MER -1312.771 → Saturated=best model LL 1.0595 for MER 1.0595 (a* = (n + 2) / 24) 26431.403 Bayesian (RC) 26431.403 Bayesian (RC) 26461.958 (a* = (n + 2) / 24) 0.0256 Statimate Square Error 0f Approximation) Estimate G.15 Estimate Square Error 0f Approximation) Est. MER Estimate Subar (Richardired Root Mean Square Residual) Mole Freed the factor loading at T4, the rescaled -2ALL should improve by 7.285, and the T4 loading should be greater by 0.348 MODEL RESULTS (RELEVANT PARAMETERS ONLY) Two-Tailed FY ROSE4 5.917 0.215 MODEL RESULTS (RELEVANT PARAMETERS ONLY) Two-Tailed FY ROSE4 5.917 0.215 Di GY 5.917	Number of	Free Parameters	54	\rightarrow s	aved DF=6 (12load vs. 3load + 3FactVar)
H0 Scaling Correction Factor for MLR H1 Value Akaise (ATC) Akaise (DOGITICAT	H0 Value	-13141.701	→ c	Our metric invariance model LL
HI Value -13121.771 > Saturated=best model LL HI Value -13121.771 > Saturated=best model LL Information Criteria 1.0595 Akalke (ArC) 26391.403 Bayesian (BrC) 26633.408 (n* = (n + 2) / 24) 266461.958 Chi-Square Test of Model Fit 0.0000 Value 41.112+ Degrees of Freedon 36 P-Value 0.2586 Scaling Correction Factor 0.9666 for MLR 0.000 RMSEA (Root Mean Square Error Of Approximation) Estimate Estimate 0.0155 90 Percent C.I. 0.000 90 Percent C.I. 0.000 CFI/TLI 0.9997 TI 0.997 SRME (Standardized Root Mean Square Residual) Mole Hould improve by 7.285, and the T4 loading should be greater by 0.348. MODEL MODIFICATION INDICES Formation T4 (the rescaled -2ALL should improve by 7.285, and the T4 loading should be greater by 0.348. MODEL RESULTS (RELEVANT FARAMETERS ONLY) Two-Tailed FACTOR LADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME Fvalue FACTOR LADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME F		HO Scaling Correction Factor	1.1194		
H1 Scaling Correction Factor 1.0595 For MLR Information Criteria Akaike (ATC) 2631.403 Sample-Size Adjusted BIC 26461.998 ($n^+ = (n + 2) / 24$) Chi-Square Test of Model Fit Value 41.112* Degrees of Freedom 36 P-Value 0.2566 Scaling Correction Factor 0.8696 For MLR EMSRA (Root Mean Square Error Of Approximation) Estimate 0.015 90 Percent C.I. 0.000 0.033 Probability RMSEA <= .05 1.000 CFI/TLI CFI 0.9997 TLI 0.9997 SERME (Standardized Root Mean Square Residual) Value 0.028 MODEL MODIFICATION INDICES (truncated) BY SLAMEMENTS 0.159 TV Probability RMSEA <= .05 1.000 CFI/TLI CFI 0.9997 TLI 0.9997 SERME (Standardized Root Mean Square Residual) Value 0.028 MODEL RESULTS (RELEVANT FARAMETERS ONLY) TWO-Tailed Factors 10.484 0.388 27.047 0.000 = BL DiGK1 5.917 0.215 27.569 DIGT1 0.484 0.388 27.047 0.000 = BL DIGK3 5.917 0.215 27.569 DIGT1 0.484 0.388 27.047 0.000 = BL DIGT3 0.484 5.917 0.215 27.569 DIGT1 0.484 0.388 27.047 0.000 = BL DIGT3 0.484 5.917 0.215 27.569 DIGT3 0.484 0.388 27.047 0.000 = BL DIGT3 0.484 0.388 27.047 0.000 = BL DIGT3 0.484 0.388 27.047 0.000 = BL DIGTA 5.917 0.215 27.569 DIGT3 0.484 0.388 27.047 0.000 = BL DIGTA 5.917 0.215 27.569 DIGT3 0.484 0.388 27.047 0.000 = BL DIGTA 5.917 0.215 27.569 DIGT3 0.484 0.388 27.047 0.000 = BL DIGTA 5.917 0.215 27.569 DIGTA 5.917 0.215 27.569		H1 Value	-13121.771	→ s	aturated=best model LL
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Information Criteria Akaike (AIC) 26391.403 Bayesian (BC) 26633.408 Sample-Size Adjusted BIC 26631.498 ($n^{+} = (n + 2) / 24$) Chi-Square Test of Model Fit Value 41.112+ Degrees of Freedom 36 P^{-Value} 0.2566 Scaling Correction Factor 0.9696 for MLR RMSEA (Root Mean Square Error Of Approximation) Estimate 0.000 0.033 Probability RMSEA <= .05 1.000 CFI/TLI CEI 0.999 TLI 0.999 TLI 0.999 TLI 0.999 TLI 0.028 MODEL RESULTS (RELEVANT FARAMETERS ONLY) Value 0.028 MODEL RESULTS (RELEVANT FARAMETERS ONLY) Value 0.028 MODEL RESULTS (RELEVANT FARAMETERS ONLY) Two-Tailed P-Cable Model Fit 0.999 DIGITI 10.484 0.388 27.047 0.000 = BL PCOSE 1.3.455 0.121 28.641 0.000 = FL FACTOR LOADINGS NOW EQULFOR SAME OVTCOME OVER TIME TL BY BLOCK2 5.917 0.215 27.569 0.000 = BL DIGITI 10.484 0.388 27.047 0.000 = DL PROSEL 3.455 0.121 28.641 0.000 = PL PROSEL 3.455 0.121 28.641 0.000 = PL PROSEL 3.455 0.121 28.641 0.000 = DL PROSEL 3.455 0.121 28.641 0.000 = PL PROSEL 3.455 0.121 28.641 0.000		for MLR			Does the full metric invariance model (2a)
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Bayestan (BIC) 26633.408 Sample-Size Adjusted BIC 266461.958 (n* = (n + 2) / 24) In examining why the constrained model fits worse, modification indices (below) suggest the loading of proses wants to be greater at T4, so we can fee that loading to create a partial metric invariance model to move forward.* Value 0.2566 Scaling Correction Factor 0.9696 for MLR 0.015 Systements 0.015 90 Percent C.I. 0.000 0.033 Probability RMSEA <= .05		Akaike (AIC)	26391.403		Yes, $-2\Delta LL(df=6) = 15.09, p = .0196$
Chi-Square Test of Model Fit worse, modification indices (below) suggest Value 41.112* Degrees of Freedom 36 P-Value 0.2566 Scaling Correction Factor 0.366 for MLR 0.2566 Secaling Correction Factor 0.361 go Percent C.I. 0.000 0.033 Probability RMSEA <= .05		Bayesian (BIC) Sample-Size Adjusted BIC	26633.408		
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P-Value 0.2566 Scaling Correction Factor 0.9696 for MLR RMSEA (Root Mean Square Error Of Approximation) Estimate 0.000 0.005 90 Percent C.I. 0.000 0.001 0.000 0.015 73 BY PROSE4 7.872 7.285 0.506 7.372 0.506 7.285 CFI/TLI CFI CFI 0.999 TLI 0.999 0.997 0.999 0.997 0.002 0.484 MODEL RESULTS (RELEVANT PARAMETERS ONLY) Value Two-Tailed Estimate F-Value F-Value MODEL RESULTS (RELEVANT PARAMETERS ONLY) Value Two-Tailed Estimate S.E. Est./S.E. P-Value TI D.917 0.215 27.569 0.000 = BL DIGITI 0.000 = DL PROSE1 TI D.917 0.215 27.569 0.000 = BL DIGITI 0.000 = DL PROSE1 BLOCK1 5.917 0.215 27.569 0.000 = BL DIGITI 0.000 = DL PROSE2 TI D.044 0.388 27.047 0.000 = DL PROSE3 5.917 0.215 TI D.044 0.388 27.047 0.000 = DL PROSE3 5.917 0.215 27.569 0.000 = BL PROSE3 0.121 28.641 0.000 = PL TI D.044 0.388 27.047 0.000 = DL PROSE3<		Value Degrees of Freedom	41.112	~	so we can free that foading to create a partial
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RMSEA (Root Mean Square Error Of Approximation) Estimate 0.015 90 Percent C.I. 0.000 0.015 12 BY PROSE4 7.879 7.372 0.510 73 BY PROSE4 CFI/TLI CFI 0.999 TLI 0.999 0.997 1.000 1.000 1.000 SRMR (Standardized Root Mean Square Residual) Value 0.028 If we freed the factor loading at T4, the rescaled -2ALL should improve by 7.285, and the T4 loading should be greater by 0.348. MODEL RESULTS (RELEVANT PARAMETERS ONLY) Two-Tailed Estimate S.E. Est./S.E. P-Value FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME Two-Tailed PROSE1 3.455 0.121 20.641 DIGTT1 10.484 0.388 27.047 0.000 = BL PROSE1 3.455 0.121 28.641 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = PL T3 BY ELOCKA 5.917 0.215 27.569 0.000 = DL		tor MLR			MODEL MODIFICATION INDICES (truncated)
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90 Percent C.I. 0.000 0.033 T3 BY PROSE4 7.879 0.506 CFI/TLI CFI 0.999 0.997 TLI 0.999 CSRMR (Standardized Root Mean Square Residual) Value 0.028 If we freed the factor loading at T4, the rescaled -2ALL should improve by 7.285, and the T4 loading should be greater by 0.348. MODEL RESULTS (RELEVANT PARAMETERS ONLY) Two-Tailed FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME P-Value T1 BY BLOCK1 5.917 0.215 27.569 DIGIT1 10.484 0.388 27.047 0.000 = BL PROSEZ 5.917 0.215 27.569 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSEZ 5.917 0.215 27.569 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSEZ 5.917 0.215 27.569 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = DL PROSE3 5.917		Estimate	0.015		T2 BY PROSE4 7.372 0.510
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		90 Percent C.I.	0.000 0.033		T3 BY PROSE4 7.879 0.506
CFI/TLI CFI 0.939 If we freed the factor loading at T4, the rescaled $-2\Delta LL$ should improve by 7.285, and the T4 loading should be greater by 0.348. SRMR (Standardized Root Mean Square Residual) 0.028 If we freed the factor loading at T4, the rescaled $-2\Delta LL$ should improve by 7.285, and the T4 loading should be greater by 0.348. MODEL RESULTS (RELEVANT PARAMETERS ONLY) Two-Tailed FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME P-Value T1 BY BLOCK1 5.917 0.215 27.569 0.000 = BL DIGIT1 10.484 0.388 27.047 0.000 = DL PROSE1 3.455 0.121 28.641 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = BL PROSE2 3.455 0.121 28.641 0.000 = BL PROSE3 5.917 0.215 27.569 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSE3 5.917 0.215 27.569 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 <t< td=""><td></td><td>TIODADITICY MISEA (= .05</td><td>1.000</td><td></td><td>T4 BI PROSE4 7.285 0.348</td></t<>		TIODADITICY MISEA (= .05	1.000		T4 BI PROSE4 7.285 0.348
CFI TLI 0.999 0.997 rescaled -2∆LL should improve by 7.285, and the T4 loading should be greater by 0.348. SRMR (Standardized Root Mean Square Residual) Value 0.028 MODEL RESULTS (RELEVANT PARAMETERS ONLY) Two-Tailed Estimate S.E. Est./S.E. P-Value FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME Two-Tailed PI BLOCK1 5.917 0.215 27.569 0.000 = BL PROSE1 3.455 0.121 28.641 0.000 = DL PROSE1 3.455 0.121 28.641 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = DL PROSE3 5.917 0.215 27.569 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 5.917 0.215 27.569 0.000 = DL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = DL	CFI/TLI				If we freed the factor loading at T4, the
III 0.997 SRMR (Standardized Root Mean Square Residual) Value 0.028 MODEL RESULTS (RELEVANT PARAMETERS ONLY) Two-Tailed Estimate SEXER (Standardized Root Mean Square Residual) Value Two-Tailed PACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME P-Value FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME P-Value T1 BY BLOCK1 5.917 0.215 27.569 0.000 = BL DIGIT1 10.484 0.388 27.047 0.000 = DL PROSE1 3.455 0.121 28.641 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = BL PROSE2 3.455 0.121 28.641 0.000 = BL PROSE3 3.455 0.121 28.641 0.000 = BL PROSE3 3.455 0.121 28.641 0.000 = PL T4 BY BUCK4 5.917 0.215 27.569 0.000 = BL DIGTA 10.484 0.388 27.047 0.000 = DL PROSE4		CFI	0.999		rescaled $-2\Delta LL$ should improve by 7.285, and
SRMR (Standardized Root Mean Square Residual) Value 0.028 MODEL RESULTS (RELEVANT PARAMETERS ONLY) Estimate S.E. Est./S.E. P-Value FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME T1 EY ELOCK1 5.917 0.215 27.569 0.000 = BL DIGIT1 10.484 0.388 27.047 0.000 = DL PROSE1 3.455 0.121 28.641 0.000 = PL T3 ELOCK3 5.917 0.215 27.569 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = PL T3 ELOCK3 5.917 0.215 27.569 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = PL T4 BY ELOCK4 5.917 0.215 27.569 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = PL FACTOR VARIANCES FREE AFTER T1 > INCREASING VARIABILITY OVER TIME Variances T1 1.000 0.000 999.000 999.000 999.000 999.000			0.997		the T4 loading should be greater by 0.348.
Value 0.028 MODEL RESULTS (RELEVANT PARAMETERS ONLY) Two-Tailed Estimate Estimate S.E. Est./S.E. FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME T1 BY BLOCK1 5.917 0.215 27.569 0.000 BL PROSE1 3.455 0.121 28.641 0.000 PL T2 BY BLOCK1 5.917 0.215 27.569 0.000 PL T3 BY BLOCK3 5.917 0.215 27.569 0.000 PL T3 BY BLOCK3 5.917 0.215 27.569 0.000 PL T3 BY BLOCK3 5.917 0.215 27.569 0.000 PL FROSE3 3.455 0.121 28.641 0.000 PL FROSE3 3.455 0.121 28.641 0.000 PL FROSE4 5.917 0.215 27.569 0.000 PL FROSE3 3.455 0.121 <	SRMR (Sta	ndardized Root Mean Square Re	esidual)		
MODEL RESULTS (RELEVANT PARAMETERS ONLY) Two-Tailed P-Value FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME T1 BY BLOCK1 5.917 0.215 27.569 0.000 = BL DIGIT1 10.484 0.388 27.047 0.000 = DL PROSE1 3.455 0.121 28.641 0.000 = PL T2 BY BLOCK2 5.917 0.215 27.569 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = PL T3 BY BLOCK3 5.917 0.215 27.569 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = PL T3 BY BLOCK3 5.917 0.215 27.569 0.000 = PL PROSE3 3.455 0.121 28.641 0.000 = PL T4 BY BLOCK4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 <t< td=""><td></td><td>Value</td><td>0.028</td><td></td><td></td></t<>		Value	0.028		
MODEL RESULTS (RELEVANT PARAMETERS ONLY) Two-Tailed Estimate S.E. Est./S.E. P-Value FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME T1 BY T1 BY BLOCK1 5.917 0.215 27.569 0.000 = BL DIGIT1 10.484 0.388 27.047 0.000 = DL PROSE1 3.455 0.121 28.641 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSE2 5.917 0.215 27.569 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = BL PROSE4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE4 3.455 0.121 28.641					
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FACTOR LOADINGS NOW EQUAL FOR SAME OUTCOME OVER TIME T1 BY BLOCK1 5.917 0.215 27.569 0.000 = BL DIGIT1 10.484 0.388 27.047 0.000 = PL T2 BY BLOCK2 5.917 0.215 27.569 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSE1 3.455 0.121 28.641 0.000 = DL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = DL PROSE3 5.917 0.215 27.569 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = DL PROSE4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL <td></td> <td>Estimate S.B</td> <td>. Est./S.E.</td> <td>1 00</td> <td>P-Value</td>		Estimate S.B	. Est./S.E.	1 00	P-Value
T1 BY BLOCK1 5.917 0.215 27.569 0.000 = BL DIGIT1 10.484 0.388 27.047 0.000 = DL PROSE1 3.455 0.121 28.641 0.000 = PL T2 BY BLOCK2 5.917 0.215 27.569 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = PL T3 BY BLOCK3 5.917 0.215 27.569 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = PL T4 BY BLOCK4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = PL T4 BY BLOCK4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = PL FACTOR VARIANCES FREE AFTER T1 \rightarrow INCREASING VARIABILITY OVER TIME Variances T1 1.000 0.000 999.000 999.000 T2 1.124 0.055 20.307 0.000	FACTOR LO	ADINGS NOW EQUAL FOR SAME OUT	COME OVER TIN	ME	
BLOCKI 3.917 0.215 27.369 0.000 = HL DIGIT1 10.484 0.388 27.047 0.000 = DL PROSE1 3.455 0.121 28.641 0.000 = PL T2 BY BLOCK2 5.917 0.215 27.569 0.000 = DL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = PL T3 BY BL BY BL 0.000 = DL PROSE3 5.917 0.215 27.569 0.000 = DL PROSE3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = PL T4 BY BLOCK4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = PL PL PROSE4 3.455 0.121 28.641 0.000 = PL <td>T1 PIOCK</td> <td>BY 1 5 917 0 21</td> <td>5 27 569</td> <td></td> <td>0 000 - BI</td>	T1 PIOCK	BY 1 5 917 0 21	5 27 569		0 000 - BI
PROSE1 3.455 0.121 28.641 $0.000 = PL$ T2BYBLOCK2 5.917 0.215 27.569 $0.000 = BL$ DIGIT2 10.484 0.388 27.047 $0.000 = DL$ PROSE2 3.455 0.121 28.641 $0.000 = PL$ T3BYBIOCK3 5.917 0.215 27.569 $0.000 = BL$ DIGIT3 10.484 0.388 27.047 $0.000 = DL$ PROSE3 3.455 0.121 28.641 $0.000 = PL$ T4BY BY $BIOCK4$ 5.917 0.215 27.569 $0.000 = PL$ T4BY BY $BIOCK4$ 5.917 0.215 27.569 $0.000 = PL$ T4BY BY $BIOCK4$ 5.917 0.215 27.569 $0.000 = PL$ T4BY BY $BIOCK4$ 5.917 0.215 27.569 $0.000 = PL$ T4BY BY $BIOCK4$ 5.917 0.215 27.569 $0.000 = PL$ FACTOR VARIANCES FREEAFTER T1 \rightarrow INCREASING VARIABILITY OVER TIMEVariances $T1$ 1.000 0.000 999.000 T2 1.124 0.055 20.307 0.000	DIGIT	1 10.484 0.38	27.047		0.000 = DL
T2 BY BLOCK2 5.917 0.215 27.569 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = PL T3 BY BLOCK3 5.917 0.215 27.569 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 5.917 0.215 27.569 0.000 = BL PROSE3 3.455 0.121 28.641 0.000 = PL T4 BY BLOCK4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = PL FACTOR VARIANCES FREE AFTER T1 \rightarrow INCREASING VARIABILITY OVER TIME Variances T1 1.000 0.000 999.000 999.000 T2 1.124 0.055 20.307 0.000 999.000	PROSE	1 3.455 0.12	28.641		0.000 = PL
BLOCK2 5.917 0.215 27.569 0.000 = BL DIGIT2 10.484 0.388 27.047 0.000 = DL PROSE2 3.455 0.121 28.641 0.000 = PL T3 BY BLOCK3 5.917 0.215 27.569 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = PL T4 BY BLOCK4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = PL FACTOR VARIANCES FREE AFTER T1 \rightarrow INCREASING VARIABILITY OVER TIME Variances T1 1.000 0.000 999.000 999.000 T2 1.124 0.055 20.307 0.000	Т2	BY			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	BLOCK	2 5.917 0.21 2 10.484 0.38	27.569		0.000 = BL
T3 BY BLOCK3 5.917 0.215 27.569 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = PL T4 BY BLOCK4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = PL	PROSE	2 3.455 0.12	27.047		0.000 = PL
BLOCK3 5.917 0.215 27.569 0.000 = BL DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = PL T4 BY BLOCK4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = PL FACTOR VARIANCES FREE AFTER T1 \rightarrow INCREASING VARIABILITY OVER TIME Variances T1 1.000 0.000 999.000 999.000 T2 1.124 0.055 20.307 0.000	ΤЗ	ВҮ			
DIGIT3 10.484 0.388 27.047 0.000 = DL PROSE3 3.455 0.121 28.641 0.000 = PL T4 BY BLOCK4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = PL FACTOR VARIANCES FREE AFTER T1 \rightarrow INCREASING VARIABILITY OVER TIME Variances T1 1.000 0.000 999.000 999.000 T2 1.124 0.055 20.307 0.000	BLOCK	3 5.917 0.21	5 27.569		0.000 = BL
PROSES 3.433 0.121 28.641 $0.000 = PL$ T4 BY BLOCK4 5.917 0.215 27.569 $0.000 = BL$ DIGIT4 10.484 0.388 27.047 $0.000 = DL$ PROSE4 3.455 0.121 28.641 $0.000 = PL$ FACTOR VARIANCES FREE AFTER T1 > INCREASING VARIABILITY OVER TIME Variances T1 1.000 0.000 999.000 T2 1.124 0.055 20.307 0.000	DIGIT		38 27.047		0.000 = DL
BLOCK4 5.917 0.215 27.569 0.000 = BL DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = PL FACTOR VARIANCES FREE AFTER T1 > INCREASING VARIABILITY OVER TIME Variances T1 1.000 0.000 999.000 999.000 T2 1.124 0.055 20.307 0.000	T4	BY 5.400 0.12	20.041		0.000 – FL
DIGIT4 10.484 0.388 27.047 0.000 = DL PROSE4 3.455 0.121 28.641 0.000 = PL FACTOR VARIANCES FREE AFTER T1 → INCREASING VARIABILITY OVER TIME Variances T1 1.000 0.000 999.000 T2 1.124 0.055 20.307 0.000	BLOCK	4 5.917 0.21	.5 27.569		0.000 = BL
PROSE4 3.455 0.121 28.641 0.000 = PL FACTOR VARIANCES FREE AFTER T1 → INCREASING VARIABILITY OVER TIME Variances T1 1.000 0.000 999.000 999.000 T2 1.124 0.055 20.307 0.000	DIGIT	4 10.484 0.38	27.047		0.000 = DL
FACTOR VARIANCES FREE AFTER T1 → INCREASING VARIABILITY OVER TIME Variances T1 1.000 0.000 999.000 T2 1.124 0.055 20.307 0.000	PROSE	4 3.455 0.12	28.641		0.000 = PL
Variances T1 1.000 0.000 999.000 999.000 T2 1.124 0.055 20.307 0.000	FACTOR VA	RIANCES FREE AFTER T1 \rightarrow INCR	EASING VARIAE	SILII	Y OVER TIME
TI I.000 0.000 999.000 999.000 T2 1.124 0.055 20.307 0.000	Variance	s			222.222
	тт Т2	1.124 0.05	5 20.307		0.000

* Note: Although one could argue that the metric model is "good enough" based on its absolute fit, I wanted to show an example of how to trouble-shoot sources of noninvariance and create partial invariance models.

0.000

0.000

17.149

0.108 14.053

0.072

1.233

1.522

ΤЗ

т4

Model 2b. Mplus Syntax for Partial Metric Invariance—Model 2a except the factor loading for prose at T4 is now allowed to differ from its factor loadings at T1–T3:



! Latent factor covariances (all possible pairs)
T1 T2 T3 T4 WITH T1* T2* T3* T4*;

Model 2b. Mplus Output for Partial Metric Invariance:

Number of	Free Parameters	55
Loglikelik	nood	
	HO Value -131	.37.301
	HO Scaling Correction Factor for MLR	1.1146
	H1 Value -131	21.771
	H1 Scaling Correction Factor for MLR	1.0595
Informatio	on Criteria	
	Akaike (AIC) 263	384.603
	Bayesian (BIC) 266	531.089
	Sample-Size Adjusted BIC $(n^* = (n + 2) / 24)$	56.465
Chi-Square	e Test of Model Fit	
-	Value	31.925*
	Degrees of Freedom	35
	P-Value	0.6173
	Scaling Correction Factor for MLR	0.9729
RMSEA (Roc	ot Mean Square Error Of Approximati	on)
	Estimate	0.000
	90 Percent C.I. 0.000	0.025
	Probability RMSEA <= .05	1.000
CFI/TLI	1	
	CFI	1.000
	TLI	1.000
SRMR (Star	ndardized Root Mean Square Residual)
	Value	0.017

Does the partial metric invariance model (2b) still fit *worse* than the **configural model (1)**? No, $-2\Delta LL(df=5) = 4.127$, p = .5313

This means that differences in the factor variances over time were sufficiently responsible for the prior differences in the factor loadings over time. In other words, outcomes are related to the latent factor equivalently across time.

Now we can move forward to test equality of the outcome intercepts (scalar).

MODEL RESULTS (RELEVANT PARAMETERS ONLY)

						ŗ	Two-Tai	led			
			Estimate	s.	E. E	st./S.E.	P-Val	ue			
FACI	OR LOADINGS	NOW	EQUAL FOR	SAME OU	TCOME	OVER TIME	EXCEPT	PROSE4			
Τ1	BY										
	BLOCK1		5.987	0.2	14	28.027	0.0	00			
	DIGIT1		10.553	0.3	87	27.288	0.0	00			
	PROSE1		3.361	0.1	26	26.618	0.0	00			
т2	BY										
	BLOCK2		5.987	0.2	14	28.027	0.0	00			
	DIGIT2		10.553	0.3	87	27.288	0.0	00			
	PROSE2		3.361	0.1	26	26.618	0.0	00			
т3	BY										
	BLOCK3		5.987	0.2	14	28.027	0.0	00			
	DIGIT3		10.553	0.3	87	27.288	0.0	00			
	PROSE3		3.361	0.1	26	26.618	0.0	00			
Τ4	BY										
	BLOCK4		5.987	0.2	14	28.027	0.0	00			
	DIGIT4		10.553	0.3	87	27.288	0.0	00			
	PROSE4		<mark>3.915</mark>	0.1	94	20.158	0.0	00 = PL4	at T4	is >	т1,т2,т3
Var	riances										
	Т1		1.000	0.0	00	999.000	999.0	00			
	Т2		1.119	0.0	55	20.486	0.0	00			
	Т3		1.231	0.0	71	17.345	0.0	00			
	т4		1.410	0.1	07	13.228	0.0	00			

Model 3a. Mplus Syntax for Full Scalar Invariance—Model 2b except the intercepts for the same outcome are constrained equal (including prose4, given how few outcomes there are per factor), and the factor mean = 0 at T1 for identification but is free at T2–T4:



[!] Latent factor covariances (all possible pairs)
T1 T2 T3 T4 WITH T1* T2* T3* T4*;

Model 3a. Mplus Output for Full Scalar Invariance:

Number of	Free	Parame	eters		49	\rightarrow Saved DF=6 (1
Loglikeli	hood	_				
	HO Va	alue			-13140.311	Does the full
	HO So	caling	Correction	Factor	1.1311	than the ner
	İ0:	r MLR				
	H1 Va	alue			-13121.771	No, $-2\Delta LL(c$
	H1 So	caling	Correction	Factor	1.0595	
	to:	r MLR				This means the
Informati	on Cr	iteria	~ `		0.0000.001	means over ti
	Akaı.	ke (AI	C)		26378.621	responsible f
	Bayes	sian (i	SIC)		26598.219	responsible it
	Samp.	le-Size	e Adjusted	BIC	26442.644	outcome mea
a1 ' a	(n)	* = (n	+ 2) / 24)			
Chi-Squar	e Tesi	C OI MO	Dael Fit		20 075	Now we can
	Value	=			30.073	of the outcom
	Degre	ees or	Freedom		41 0 6014	
	r-va.	ing Co.	mostion Po	atan	0.0014	
	SCAL.	ing co. • Mid	LIECTION FA	CLOI	0.9739	
DMCEA (Do	ot Mo:	L MLK	no Error (of Approx	imation	
NHSLA (NO	Fatir	an Syua moto	ale biloi c	Appiox		
	OU D	arcont	СТ	0	0.000	
	Droh		DMCEA /-	05	1 000	
CET/TIT	FLOD		Y NHOLA (-	.05	1.000	
011/111	CET				1 000	
	TT.T				1.000	
SRMR (Sta	ndard	ized Ro	oot Mean So	uare Res	idual)	
	Value	9			0.020	
MODEL RES	ULTS	(RELEVA	ANT PARAMEI	ERS ONLY)	
						Two-Tailed
		I	Estimate	S.E.	Est./S.E.	P-Value
FACTOR ME	ANS SI	HOW DE	CLINE OVER	TIME		
Means						
Τ1			0.000	0.000	999.000	999.000
Т2			-0.110	0.027	-4.030	0.000
т3			-0.255	0.037	-6.936	0.000
Т4			-0.479	0.049	-9.741	0.000
OUTCOME I	NTERCI	SPTS NO	DW EQUAL FO	OR SAME O	UTCOME OVER	TIME
Incercep	1		10 222	0 205	25 0/0	0 000 - PT
BLOCK	2		10.232	0.205	25 0/0	0.000 – BI
BLOCK	2		10.232	0.205	25 040	0.000
BLOCK	.S 1		10.232	0.200	25 049	0.000
BLUCK	1 1		10.232 21 067	0.200	JJ.949 NJ 010	0.000 - r
DIGIT	1 2		21.00/	0.480	43.919	0.000 = DI
DIGIT	2		21.00/	0.480	43.919 12 010	0.000
DIGIT	1		21.007	0.400	4J.919 ND 010	0.000
DIGIT	ኳ 1		21.00/	0.400	43.919	$0.000 - \mathbf{p}$
PRUSE	1 2		0.4ZZ 8 /22	0.176	41.033 N7 035	0.000 = PI
PDOGE	2		0.422 8 /00	0.176	47.033 47.033	0.000
11/001	J		0.722	0.1/0	-1.000	0.000

8.422

PROSE4

0.176

47.835

0.000

→ Saved DF=6 (12int vs. 3int + 3FactMean)

Does the full scalar model (3a) fit *worse* **than the partial metric model (2a)?** No, $-2\Delta LL(df=6) = 6.144$, p = .4073

This means that differences in the factor means over time were sufficiently responsible for the differences in the outcome means (now intercepts) over time.

Now we can move forward to test equality of the outcome residual variances.

Model 4a. Mplus Syntax for Full Residual Variance Invariance—Model 3a except the residual variances for the same outcome are constrained equal over time (including prose4 to start with):



[!] Latent factor covariances (all possible pairs)
T1 T2 T3 T4 WITH T1* T2* T3* T4*;

Model 4a. Mplus Output for Full Residual Variance Invariance:

Number of Loglikelik	Free Parameters		40
- 5 -	HO Value	-13157	. 694
	HO Scaling Correction Factor	1.1	1780
	H1 Value ·	-13121.	.771
	H1 Scaling Correction Factor for MLR	1.()595
Informatio	on Criteria		
	Akaike (AIC)	26395	.388
	Bayesian (BIC)	26574.	.651
	Sample-Size Adjusted BIC	26447	.651
	$(n^* = (n + 2) / 24)$		
Chi-Square	e Test of Model Fit		
	Value	74.	.477*
	Degrees of Freedom		50
	P-Value	0.0	0140
	Scaling Correction Factor for MLR	0.9	9647
RMSEA (Roc	ot Mean Square Error Of Approxim	mation)
	Estimate	0.	027
	90 Percent C.T. 0.(013 0	040
	Probability RMSEA <= 05	0	999
CFT/TLT	riobability raibbili (.00	0	• • • • •
011/121	CFI	0	993
		0	991
SRMR (Star	dardized Root Mean Square Pesi	dual)	•
SINIX (Stai	Value	0	.032

➔ Saved DF=9 (12resvar vs. 3resvar)

Does the full residual variance model
(4a) fit <i>worse</i> than the full scalar
model (3a)?
Yes, $-2\Delta LL(df=9) = 37.680, p < .0001$

MODEL	MODIFICATION	INDICES (truncated)	
	M.I.	E.P.C.	
Varia	nces/Residual	Variances	
BLOCK	21.897	4.079	
DIGIT	10.763	7.267	

If we freed the block residual variance at T1, the rescaled $-2\Delta LL$ would improve by 21.897, and the residual variance should be greater by 4.079. To save a step, I will free both of these residual variances at once.

MODEL RESULTS (RELEVANT PARAMETERS ONLY)

				Two-Tailed	
	Estimate	S.E.	Est./S.E.	P-Value	
RESIDUAL VARIANCES	= AMOUNT OF	"NOT THE	FACTOR" VAF	RIANCE EQUAL OVER	TIME
BLOCK1	15.848	1.193	13.282	0.000 = BR	
BLOCK2	15.848	1.193	13.282	0.000	
BLOCK3	15.848	1.193	13.282	0.000	
BLOCK4	15.848	1.193	13.282	0.000	
DIGIT1	26.480	3.211	8.246	0.000 = DR	
DIGIT2	26.480	3.211	8.246	0.000	
DIGIT3	26.480	3.211	8.246	0.000	
DIGIT4	26.480	3.211	8.246	0.000	
PROSE1	10.032	0.538	18.661	0.000 = PR	
PROSE2	10.032	0.538	18.661	0.000	
PROSE3	10.032	0.538	18.661	0.000	
PROSE4	10.032	0.538	18.661	0.000	

Model 4b. Mplus Syntax for Partial Residual Variance Invariance—Model 4a except the residual variances for block and digit at T1 can differ from those at T2–T4:



! Latent factor covariances (all possible pairs)
T1 T2 T3 T4 WITH T1* T2* T3* T4*;

Model 4b. Mplus Output for Partial Residual Variance Invariance:

Number of Free Parameters	42	→ sa	aved I	DF=7	(12resvar v	/s.	3+2resvar)
Loglikelihood							
HO Value	-13144.753						
HO Scaling Correction Factor	1.1650						
for MLR							
H1 Value	-13121.771						
H1 Scaling Correction Factor	1.0595						
for MLR							
Information Criteria							
Akaike (AIC)	26373.506						
Bayesian (BIC)	26561.732						
Sample-Size Adjusted BIC	26428.382						

	(n* =	= (n + 2) / 24)			
Chi-Squar	e Test d	of Model Fit			
	Value			47.525*	Does the partial residual variance
	Degrees	s of Freedom		48	model (4b) still fit <i>worse</i> than the full
	P-Value	5		0.4922	scalar model (3a)?
	Scaling	g Correction Fact	lor	0.9672	Scalar model (5a): N. $2ALL(16.7) = 0.57(-1.2120)$
	for N	1LR			No, $-2\Delta LL(df=7) = 9.5/6, p = .2139$
RMSEA (Ro	ot Mean	Square Error Of	Approxim	ation)	
	Estimat	te		0.000	This will be our new baseline moving
	90 Perc	rent C T	0 0	00 0 025	forward with respect to the structure
	Probabi	lity RMSEA <= (15	1 000	forward with respect to the structural
CET/UTT	TTODADI	LILUY INISER (C	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	1.000	model, which is saturated here (all
CF1/1L1	0.77			1 000	possible means, variances, and
	CF.T			1.000	covariances are estimated excent where
	TLI			1.000	covariances are estimated except where
Chi-Squar	e Test d	of Model Fit for	the Base	line Model	constrained for identification).
	Value			3516.779	
	Degrees	s of Freedom		66	But we will need to change the method
	P-Value	2		0.0000	But we will need to enalige the method C^{+}_{1}
SRMR (Star	ndardize	ed Root Mean Squa	are Resid	ual)	of identification for our change model
(Value			0 025	so that all the lower-order factor
	Varue			0.025	variances can be estimated instead
MODET הבס	III.TPC				
NODET KE2	CITO			m	o-modilod
				'l'w	U-IALLEQ
		Estimate	S.E.	ESt./S.E.	P-value
FACTOR LO	ADINGS E	EQUAL FOR SAME OU	JTCOME OV	ER TIME EXCE	PT PROSE4
T1 1	BY				_
BLOCK	1	5.972	0.215	27.823	0.000 = BL \rightarrow to be used next
DIGIT	1	10.579	0.385	27.475	0.000 = DL
PROSE	1	3.371	0.125	26.973	0.000 = PL
Ͳ2	BY				
BLOCK	2	5 972	0 215	27 823	0 000 = BI .
DICTT	2	10 579	0.215	27.025	0.000 - DI
DIGII	2	2 271	0.305	27.475	0.000 = DI
PROSE.	2	3.3/1	0.125	20.9/3	0.000 = PL
'1'3	BY				
BLOCK	3	5.972	0.215	27.823	0.000 = BL
DIGIT	3	10.579	0.385	27.475	0.000 = DL
PROSE	3	3.371	0.125	26.973	0.000 = PL
Т4	BY				
BLOCK	4	5.972	0.215	27.823	0.000 = BL
DIGIT	4	10.579	0.385	27.475	0.000 = DL
PROSE	4	3.911	0.195	20.103	0.000 = PL4
FACTOR CO	- VARTANCE	ידת חיד האטגנאג S ALLOWE	GANO BAA		ORRELATIONS HERE)
T1 1	WTTH			111111 (1101 0	
 Ͳ2		1 009	0 028	36 545	0 000
<u>⊥ د</u> س ک		T.009	0.020	22.0240	0.000
L J		0000	0.042	23.020	0.000
1.4		0.983	0.052	19.024	0.000
TZ = c	M T .T,H		0 0	10 505	0.000
Т3		1.109	0.059	18.727	0.000
Т4		1.150	0.067	17.099	0.000
тЗ	WITH				
Т4		1.263	0.077	16.482	0.000
RESIDUAL	COVARIAN	ICES FOR SAME OUT	COME OVE	R TIME (FREE	LY ESTIMATED)
BLOCK1	WITH				
BLOCK	2	7.453	1.193	6.247	0.000
BLOCK	3	8,263	1.248	6.620	0.000
BI.OCK	-	6 584	1 448	4 548	0.000
BLOCKS	- ₩TΨ₽	0.001	1.110	1.010	
	3 MTTII	7 1 5 0	1 100	5 070	0 000
BLUCK.	J 1	1.109	1 210	2.2/0	0.000
BLOCK	4	4.482	1.319	3.398	0.001
BLOCK3	WITH				
BLOCK	4	6.331	1.359	4.658	0.000
DIGIT1	WITH				
DIGIT	2	8.909	3.339	2.668	0.008
DIGIT	3	7.459	3.531	2.113	0.035
DIGIT	4	7.823	3.728	2.099	0.036
DIGIT2	WITH				
	3	7 300	3 183	2 1 2 4	0 034
DIGII	1	טעניי מרר ר	3 116	2.124 0.057	0.024
DIGIL	ч мттт	1.119	3.440	2.201	0.024
DTGT.I.2	M T T H				

DIGIT4	2.729	3.671	0.743	0.457		
PROSE1 WITH						
PROSE2	4.916	0.619	7.944	0.000		
PROSE3	4.368	0.681	6.418	0.000		
PROSE4	4.717	0.848	5.560	0.000		
PROSE2 WITH						
PROSE3	5.261	0.622	8.461	0.000		
PROSE4	5.325	0.853	6.240	0.000		
PROSE3 WITH						
PROSE4	6.301	0.680	9.261	0.000		
FACTOR MEANS SHOW	INCREASING DE	CLINE OVER	TIME			
Means						
Т1	0.000	0.000	999.000	999.000		
т2	-0.110	0.027	-4.032	0.000		_
тЗ	-0.256	0.037	-6.944	0.000 -	\land T2 =1	<mark>46</mark>
т4	-0.484	0.049	-9.791	0.000 🚽	A T3 = −.2:	<mark>28</mark>
INTERCEPTS FOR SA	ME OUTCOME HELD	D EQUAL OV	ER TIME (SO	CHANGE IS	DUE TO FACTO	RS ONLY!)
Intercepts				_		
BLOCK1	10.238	0.284	35.996	0.000 =	BI	
BLOCK2	10.238	0.284	35.996	0.000		
BLOCK3	10.238	0.284	35.996	0.000		
BLOCK4	10.238	0.284	35.996	0.000		
DIGIT1	21.086	0.481	43.876	0.000 =	DI	
DIGIT2	21.086	0.481	43.876	0.000		
DIGIT3	21.086	0.481	43.876	0.000		
DIGIT4	21.086	0.481	43.8/6	0.000		
PROSEI	8.423	0.176	47.934	0.000 =	PI	
PROSEZ	8.423	0.176	47.934	0.000		
PROSE3	8.423	0.176	47.934	0.000		
PROSE4	8.423	0.176	47.934	0.000		
FACTOR VARIANCES	SHOW INCREASING	J VARIABIL	ITY OVER TI	ME		
variances	1 000	0 000	000 000	000 000		
11 m2	1.000	0.000	999.000	999.000		
12 m2	1.120	0.034	20.007	0.000		
13 T4	1.231	0.070	13 534	0.000		
PESTDUAL VARIANCE	S = MOINT OF	NOT THE E	ACTOR VART	ANCE FOULT	FYCEDT BLOCK	1 AND DIGIT
BLOCK1	19 552	1 624	12 0/1		BD1	I AND DIGIII
BLOCK2	13 573	1 220	11 127	0.000 =	BR	
BLOCK3	13 573	1 220	11 127	0.000 =	BR	
BLOCK4	13 573	1 220	11 127	0.000 =	BR	
DIGITI	32 968	4 390	7 510	0.000 =		
DIGIT2	23 577	3,147	7,492	0,000 =	DR	
DIGIT3	23 577	3,147	7,492	0.000 =	DR	
DIGIT4	23.577	3.147	7.492	0.000 =	DR	
PROSE1	9,918	0.542	18.283	0.000 =	PR	
PROSE2	9.918	0.542	18,283	0,000 =	PR	
PROSE3	9.918	0.542	18.283	0.000 =	PR	
PROSE4	9.918	0.542	18.283	0.000 =	PR	

Model 5a. Mplus Syntax for Latent Basis Change Model—Keeping non-invariant parameters from prior measurement models, but using a "marker item" identification method for the factor variances:

```
MODEL: ! DATA, VARIABLE, ANALYSIS, OUTPUT are same
!!!!!! 5a. Latent Basis Change Model !!!!!!
! Define latent factors (Factor = indicator loadings)
! Factor loadings held equal over time except prose4
  T1 BY block105.972; T1 BY digit1* prose1* (DL PL);
  T2 BY block205.972; T2 BY digit2* prose2* (DL PL);
  T3 BY block305.972; T3 BY digit3* prose3* (DL PL);
  T4 BY block405.972; T4 BY digit4* prose4* (DL PL4);
! Indicator intercepts all held equal over time
  [block1-block4*] (BI);
  [digit1-digit4*] (DI);
  [prose1-prose4*] (PI);
```

Because our time-specific factor variances need to be free to become leftover (= "disturbances"), we need to change our model identification to use a "marker item" whose factor loading is fixed (and still equal over time). Rather than fixing that loading to 1, we are fixing it to the value corresponding to the previous T1 factor (with mean=0 and variance=1), that way the total SD ~= 1 for the T1 factor.



FIGURE 14.3. Path diagram of a second-order growth model.

Model 5a. Mplus Output for Latent Basis Change Model:

Number of Loglikeli	Free Parameters hood		36	→	Saved DF=6
	H0 Value H0 Scaling Corre	ction Factor	-13151.623 1.1915		Saved DF=6 how? 3 factor means \rightarrow 1 fixed change slope
	H1 Value		-13121.771		3 factor variances and 6 covariances \rightarrow
	H1 Scaling Corre for MLR	ction Factor	1.0595		2 loadings, 1 intercept factor variance, 1
Informati	on Criteria				slope factor variance, and i covariance
	Akaike (AIC)		26375.247		
	Bayesian (BIC)		26536.583		Does the latent basis change model
	Sample-Size Adju	sted BIC	26422.284		(5a) fit <i>worse</i> than the partial residual
	$(n^* = (n + 2))$	/ 24)			variance model (4b)?
Cni-Squar	Test of Model F	10	C1 4E0	+	Yes, $-2\Delta LL(df=6) = 13.658, p = .0337$
	Value Dogroop of Erood	0m	61.438 54	^	
	P-Value	IOIII	0 2265		MODEL MODIFICATION INDICES (truncated)
	Scaling Correcti	on Factor	0.2205		M.I. E.P.C.
	for MLR	OII FACCOL	0.9713		Means/Intercepts/Thresholds
RMSEA (RC	ot Mean Square Er	ror Of Approx	imation)		[T4] 10.295 -0.194
1010111 (100	Estimate	TOT OF HPPION	0.015		
	90 Percent C.I.	0	.000 0.030		If we freed the factor intercept at T4, the
	Probability RMSF	A <= .05	1.000		rescaled $-2\Delta LL$ would improve by 10.295,
CFI/TLI	11020011107 1002		2.000		and the factor intercent should be lower by
	CFI		0.998		0.104 (And no moving the fixed leading of
	TLI		0.997		0.194 . (And no, moving the fixed loading of $1.6 \pm 1.6 \pm $
SRMR (Sta	ndardized Root Me	an Square Res	idual)		1 for the change factor to 12 instead of 14
	Value	-	0.028		doesn't solve the problem)
MODEL RES	SULTS - NEW PARAME	TERS ONLY:		Т	wo-Tailed
	Estima	te S.E.	Est./S.E.		P-Value
NEW HIGHE	R-ORDER FACTOR LC	ADINGS			
INT	BY				
T1	1.0	0.000	999.000		999.000
T2	1.0	0.000	999.000		999.000
T3	1.0	0.000	999.000		999.000
'1'4 GID	1.0	0.000	999.000		999.000
SLP m1	BI		000 000		000,000
11	0.0	0.000	999.000		999.000
12	0.2	0.045	6.057		$0.000 \rightarrow 27.0\%$ of change by T2
·T-3	0.6	0.074	8.439		0.000 7 62.6% of change by 13
14	L.U		999.000		999.000
TNT	MITTU	ANCE = RANDOM	EFECT COVA	RIA	NCE (IN G MATRIX)
CT D	MTTII U U	25 0.054	0 111		0 659
UTCUED-OF	U.U	- FIVED INTER	0.441 CEDT-0 EOD	тпр	NUTETON FILED SLOPE
Means	DEN FACTOR MEANS	- EIVED INTER	CHFI-0 FOR	TOR	MITTICATION, FIAD SLOPE
тит	0 0		999 000		999 000
ST.P	-0.4	66 0.047	-9 890		$0.000 \rightarrow$ Total mean decline over time
FACTOR VA	$\mathbf{RIANCES} = \mathbf{RANDOM}$	EFFECTS VARIA	NCES (TN C	ጠልም	RIX)
Variance	S S S S S S S S S S S S S S S S S S S	TITOID AUNTA			,
TNT	n c	0.069	14.304		0.000
ST.P	0.3	72 0 083	4 494		0.000
Residual	Variances = REST	UAL VARIANCE	OF LOWER-OR	DER	FACTORS (IN R MATRIX DIAGONAL)
т1	0.0	44 0.011	4.110		0.000
т2 т2	0.0	44 0.011	4.110		0.000
т3 Т	0.0	44 0.011	4.110		0.000
т4	0.0	44 0.011	4.110		0.000

Model 5b. Mplus Syntax for Revised Latent Basis Change Model—Model 5a, except freeing the factor intercept at T4:

```
MODEL: ! DATA, VARIABLE, ANALYSIS, OUTPUT are same
!!!!!! 5b. Revised Latent Basis Change Model !!!!!!
! Define latent factors (Factor = indicator loadings)
! Factor loadings held equal over time except prose4
  T1 BY block1@5.972; T1 BY digit1* prose1* (DL PL);
  T2 BY block2@5.972; T2 BY digit2* prose2* (DL PL);
 T3 BY block3@5.972; T3 BY digit3* prose3* (DL PL);
  T4 BY block4@5.972; T4 BY digit4* prose4* (DL PL4);
! Indicator intercepts all held equal over time
  [block1-block4*] (BI);
  [digit1-digit4*]
                   (DI)
  [prose1-prose4*] (PI);
! Indicator residual variances held equal over time
! except block1 and digit1
 block1* (BR1); block2-block4* (BR);
digit1* (DR1); digit2-digit4* (DR);
 prose1-prose4* (PR);
! Same-outcome residual covariances over time
 block1-block4 WITH block1-block4*;
  digit1-digit4 WITH digit1-digit4*;
 prose1-prose4 WITH prose1-prose4*;
! Latent factor mean=0 at all occasions so that all mean change
! is captured by the intercept and slope factors' fixed effects
  [T100 T200 T300 T4*]; ! T4 int now free
 Latent factor variance held equal over time (like diagonal R matrix)
! so all heterogeneity of variance is captured by slope factor variance
   T1* T2* T3* T4* (ResVar);
! Latent factor covariances (all possible pairs) SHUT OFF @0 so that
! all covariance over time is captured by intercept and slope factor variances
   T1 T2 T3 T4 WITH T1@0 T2@0 T3@0 T4@0;
! Define new higher-order intercept and latent basis change factors
  Int BY T1@1 T2@1 T3@1 T4@1;
 Slp BY T1@0 T2* T3* T4@1;
! Higher-order factor means = fixed effects
  [Int@0 Slp*]; ! Fixed int = 0 for identification
! Higher-order factor variances = random effect variances
```

Model 5b. Mplus Output for Revised Latent Basis Change Model:

! Higher-order factor covariance = random effects covariance

Number of Free Parameters Loglikelihood	37 → Saved DF=5 no
H0 Value H0 Scaling Correction H for MLR H1 Value H1 Scaling Correction H for MLR	-13146.993 Pactor 1.1808 Saved DF 3 factor n -13121.771 3 factor v 2 loading
Information Criteria	factor var
Akaike (AIC)	26367.987
Bayesian (BIC) Sample-Size Adjusted BI $(n^* = (n + 2) / 24)$	26533.805 26416.330 Does the (5b) fit w
Chi-Square Test of Model Fit Value Degrees of Freedom P-Value	51.749* 53 0.5230

Int* Slp*;

Int WITH Slp*;

Saved DF=5... how? 3 factor means \rightarrow 1 fixed change slope +1 int 3 factor variances and 6 covariances \rightarrow 2 loadings, 1 intercept factor variance, 1 slope factor variance, and 1 covariance

Does the revised latent basis change model (5b) fit *worse* than the partial residual variance model (4b)? No, $-2\Delta LL(df=5) = 4.274$, p = .5106

Scaling Correction Factor	0.9748	
for MLR		
RMSEA (Root Mean Square Error Of Approxim	mation)	
Estimate	0.000	
90 Percent C.I.	0.000	0.024
Probability RMSEA <= .05	1.000	
CFI/TLI		
CFI	1.000	
TLI	1.000	
SRMR (Standardized Root Mean Square Resid	dual)	
Value	0.027	

FULL MODEL RESULTS

		Estimate	S.E.	Est./S.E.	P-Value	
FACT	OR LOADINGS	EQUAL FOR SAME	OUTCOME O	VER TIME EXC	CEPT PROSE4	
Т1	BY					
	BLOCK1	5.972	0.000	999.000	999.000	
	DIGITI DDOGE1	10.574	0.34/	30.459	0.000	
щΟ	PROSEI	3.362	0.128	26.327	0.000	
12	BI OCK3	5 072	0 000	999 000	999 000	
	DIGIT2	10 574	0.000	30 459	0 000	
	PROSE2	3 362	0.128	26 327	0.000	
тì	RV	3.302	0.120	20.527	0.000	
10	BTOCK3	5 972	0 000	999 000	999 000	
		10 574	0.000	30 459	0 000	
	PROSE3	3 362	0 128	26 327	0.000	
Ͳ4	BY	3.302	0.120	20.027	0.000	
	BLOCK4	5,972	0.000	999.000	999.000	
	DIGIT4	10.574	0.347	30.459	0.000	
	PROSE4	3,921	0.177	22.130	0.000 =	PL4
NEW	HIGHER-ORDER	R FACTOR LOADING	S			
INT	BY					
	Т1	1.000	0.000	999.000	999.000	
	Т2	1.000	0.000	999.000	999.000	
	тЗ	1.000	0.000	999.000	999.000	
	Τ4	1.000	0.000	999.000	999.000	
SLP	BY					
	Т1	0.000	0.000	999.000	999.000	
	Т2	0.329	0.057	5.792	0.000 →	▶ 32.9% of change by T2
	тЗ	0.752	0.084	8.977	0.000 →	▶ 75.2% of change by T3
	Τ4	1.000	0.000	999.000	999.000	
DIST	URBANCES COV	VARIANCES FOR FA	CTORS SHU	T OFF (LIKE	NO RESIDUAL	COVARIANCE IN R)
т1	WITH					
	Т2	0.000	0.000	999.000	999.000	
	Т3	0.000	0.000	999.000	999.000	
- 0	14	0.000	0.000	999.000	999.000	
Τ2	WITH	0 000	0 000			
	T3	0.000	0.000	999.000	999.000	
	14	0.000	0.000	999.000	999.000	
13	WITH	0 000	0 000	000 000	000 000	
итон	14 ED ODDED EXC			999.000 FFFFCUS COV	999.000	C MAMDIY)
птеп. тмт	MTTH	TOR COVARIANCE	- KANDOM	EFFECIS COVF	MIANCE (IN	G MAIRIX)
TINT	STP	0.009	0.052	0.182	0.856	
RESI	DUAL COVARIA	NCES FOR SAME O		ER TIME (FRE	ELY ESTIMAT	ED)
BLO	CK1 WITH			•		
	BLOCK2	7.535	1.199	6.282	0.000	
	BLOCK3	8.122	1.251	6.490	0.000	
	BLOCK4	6.569	1.455	4.516	0.000	
BLO	CK2 WITH					
	BLOCK3	7.210	1.176	6.130	0.000	
	BLOCK4	4.510	1.304	3.458	0.001	
BLO	CK3 WITH					
	BLOCK4	6.207	1.367	4.539	0.000	
DIG	IT1 WITH					
	DIGIT2	9.229	3.285	2.809	0.005	
	DIGIT3	6.952	3.520	1.975	0.048	
	DIGIT4	7.552	3.622	2.085	0.037	
DIG	IT2 WITH					
	DIGIT3	7.658	3.452	2.218	0.027	
	DIGIT4	7.915	3.410	2.321	0.020	

Two-Tailed

DIGIT3 WI	TH					
DIGIT4	2.184	3.642	0.600	0.549		
PROSE1 WI	ΨН					
PROSE2	4.942	0.618	8.001	0.000		
PROSE3	4 335	0 677	6 403	0 000		
PROSE4	4 732	0.846	5 596	0.000		
DDOGE2 WT	·mu	0.010	5.550	0.000		
DDOGE3	5 273	0 619	0 537	0 000		
PROSES	J.2/3	0.010	0.337	0.000		
PROSE4	5.327	0.849	6.275	0.000		
PROSE3 WI	.TH	0 650	0.045			
PROSE4	6.2/4	0.6/3	9.31/	0.000		
HIGHER-ORDER	FACTOR MEANS =	FIXED INTERC	EPT=0 FOR II	DENTIFICATION	I, FIXED SLOPE	
Means						
INT	0.000	0.000	999.000	999.000		
SLP	-0.340	0.050	-6.752	0.000 →	• Total mean decl	ine over time
INTERCEPTS F	OR SAME OUTCOME	HELD EQUAL O	VER TIME (SO	CHANGE IS I	JUE TO FACTORS ON	1LX ;)
Intercepts						
BLOCK1	10.245	0.282	36.364	0.000		
BLOCK2	10.245	0.282	36.364	0.000		
BLOCK3	10.245	0.282	36.364	0.000		
BLOCK4	10.245	0.282	36.364	0.000		
DIGIT1	21.095	0.479	44.045	0.000		
DIGIT2	21.095	0.479	44.045	0.000		
DIGIT3	21.095	0.479	44.045	0.000		
DIGIT4	21.095	0.479	44.045	0.000		
PROSE1	8.423	0.175	48.083	0.000		
PROSE2	8 423	0.175	48 083	0.000		
PROSE3	8 423	0.175	48 083	0.000		
PROSES	8 123	0.175	40.003	0.000		
m1	0.423	0.175	0.005	0.000		
11	0.000	0.000	999.000	999.000		
12	0.000	0.000	999.000	999.000		
13	0.000	0.000	999.000	999.000		
14	-0.145	0.040	-3.638	0.000 7	NEW MEAN DEVIAT	ION FOR T4
FACTOR VARIA	NCES = RANDOM EF	FECT VARIANC	ES (IN G MAT	TRIX)		
Variances						
INT	0.994	0.070	14.106	0.000		
SLP	0.366	0.076	4.837	0.000		
OUTCOME "NOT	' THE FACTOR" LEF	TOVER VARIAN	CES AND RES	DUAL VARIANC	E (IN R MATRIX I	DIAGONAL)
Residual Va	riances					
BLOCK1	19.393	1.615	12.005	0.000 =	BR1	
BLOCK2	13.651	1.211	11.271	0.000		
BLOCK3	13.651	1.211	11.271	0.000		
BLOCK4	13.651	1.211	11.271	0.000		
DIGIT1	32.163	4.317	7.450	0.000 =	DR1	
DIGIT2	23.748	3.110	7.637	0.000		
DIGIT3	23.748	3.110	7.637	0.000		
DIGIT4	23.748	3.110	7.637	0.000		
PROSE1	9.920	0.541	18.334	0.000		
PROSE2	9.920	0.541	18.334	0.000		
PROSE3	9.920	0.541	18.334	0.000		
PROSE4	9,920	0.541	18.334	0.000		
т1	0 040	0 010	3 915	0 000	Comparing mod	del-predicted factor
т т2	0.040	0.010	3 915	0.000	means and vari	ances as given by
т2 т3	0.040	0.010	3 915	0.000		
1J 17	0.040	0.010	3 015	0.000	TECH4 output	(at the very end):
14	0.040	0.010	3.913	0.000		
	——4c Residual Inv	ariance Model			-4c Residual Inv	variance Model
		Model			5a Latent Basis	Model
	-5h Latent Basis	Model + T4int				
0.1 —	SS Eatent Dasis			47	-56 Latent Basis	s Model + 14int
				1./		
S 0 1				% 1.5 ──		
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<u>щ</u>				6.0 없		
-0.5			4	<u>ш</u>		
				0.7		
-0.7				0.5		
	Т1 Т2	та	Т4	0.0	רד דו	то ти
	14 14	15	17		11 12	15 14

Sample results section for these analyses:

The extent of individual differences in change over time (four occasions collected at two-year intervals) in a latent factor of cognitive functioning (with three observed outcomes: block design, digit–symbol substitution, and prose recall) was examined using *Mplus* v. 8.8 (Muthén & Muthén, 1998–2017). Robust maximum likelihood (MLR) estimation was used for all analyses; accordingly, nested model comparisons were conducted using the rescaled difference in the model –2LL values with degrees of freedom equal to the difference in the number of model parameters. Prior to examining change in the latent factor over time, partial longitudinal measurement invariance was established by a series of nested models, as described next.

[Table 1 would have the fit of each model, as shown in the excel workbook for this example. Depending on the journal, you may need to add text defining each fit index and what is considered "good fit" for each. You could also make a Table 2 for all the LRTs instead of giving them in the text as I did below.]

Table 1 Model Fit										
Model	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	Chi-Square DF	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower Cl	RMSEA Higher Cl	RMSEA p-value
1. Configural Model	60	27.704	1.0039	30	0.5861	1.000	0.000	0.000	0.027	1.000
2a. Full Metric Invariance	54	41.112	0.9696	36	0.2566	0.999	0.015	0.000	0.033	1.000
2b. Partial Metric (- PL4)	55	31.925	0.9729	35	0.6173	1.000	0.000	0.000	0.025	1.000
3a. Full Scalar Invariance	49	38.075	0.9739	41	0.6014	1.000	0.000	0.000	0.024	1.000
4a. Full Residual Variance	40	74.477	0.9647	50	0.0140	0.993	0.027	0.013	0.040	0.999
4b. Partial Residual Variance (- BR1, -DR1)	42	47.525	0.9672	48	0.4922	1.000	0.000	0.000	0.025	1.000
5a. Latent Basis	36	61.458	0.9715	54	0.2265	0.998	0.015	0.000	0.030	1.000
5b. Revised Latent Basis	37	51.749	0.9748	53	0.5230	1.000	0.000	0.000	0.024	1.000

First, a configural invariance model was specified in which four correlated factors (i.e., one factor for each occasion) were estimated simultaneously; all factor means were fixed to 0 and all factor variances were fixed to 1 for identification. Residual covariances for the same outcome across the four occasions were also estimated. As shown in Table 1, the configural invariance model had excellent fit by every index, indicating that the 12 outcome means, variances, and covariances were well recreated by the model.

Equality of the unstandardized factor loadings across occasions was then examined in a metric invariance model. The factor variance was fixed to 1 at occasion 1 for identification but was freely estimated at occasions 2, 3, and 4. The factor means were all fixed to 0 for identification. All factor loadings were constrained equal across occasions, but all outcome intercepts and residual variances varied over time. Factor covariances and outcome residual covariances were estimated as described previously. Although the metric invariance model had excellent global fit, it fit significantly worse than the configural invariance model $-2\Delta LL(6) = 15.09$, p = .020. Modification indices suggested that the loading of prose recall at occasion 4 was a significant source of local misfit and should be freed. After doing so, the partial metric invariance model, $-2\Delta LL(5) = 4.13$, p = .531. The fact that partial metric invariance (i.e., "weak invariance") held indicates that the same latent factor was being measured at each occasion, or that the outcomes were related to their latent factor equivalently over time (except for prose recall, which was slightly more related to its factor at occasion 4 than at occasions 1, 2, or 3).

Equality of the unstandardized outcome intercepts across occasions was then examined in a scalar invariance model. The factor mean and variance at occasion 1 were fixed to 0 and 1, respectively, for identification, but the factor mean and variance were then estimated at occasions 2, 3, and 4. All factor loadings (except for prose recall at occasion 4) and all outcome intercepts were constrained equal across occasions; all outcome residual variances still differed over time. Factor covariances and residual covariances were estimated as described previously. The scalar invariance model had excellent fit (as shown in Table 1) that was not significantly worse than the partial metric invariance model, $-2\Delta LL(6) = 6.14$, p = .407. The fact that full scalar invariance (i.e.,

"strong invariance") held indicates that all occasions have the same expected response for each outcome at the same absolute level of the latent factor, or that the observed difference in the outcome means across occasions 1-4 was due to factor mean differences only.

Equality of the unstandardized outcome residual variances across occasions was then examined in a residual variance invariance model. As in the scalar invariance model, the factor mean and variance were fixed to 0 and 1, respectively, at occasion 1 for identification, but the factor mean and variance were estimated at occasions 2, 3, and 4. All factor loadings (except for prose recall at occasion 4), all outcome intercepts, and all outcome residual variances were constrained to be equal over time. Factor covariances and outcome residual covariances were estimated as described previously. Although the residual variance invariance model had excellent global fit, it fit significantly worse than the scalar invariance model, $-2\Delta LL(9) = 37.68$, p < .001. Modification indices suggested that the residual variances of block design and digit–symbol substitution at occasion 1 were the largest sources of misfit and should be freed. After doing so, the partial residual variance invariance model, $-2\Delta LL(7) = 9.58$, p = .214. The fact that partial residual variance invariance (i.e., "strict invariance") held indicates that the amount of outcome variance not accounted for by the latent factor was the same across time (except for block design and digit–symbol substitution at occasion 1).

In the final invariance model, the factor means showed increasing decline over time, while the factor variances showed increasing individual differences over time. The factors were highly correlated across occasions ($r \approx .8$ to .9). The extent to which two higher-order factors-for an intercept and latent basis change-could recreate the lower-order factor means, variances, and covariances was then examined. To create a meaningful scale by which to identify the model, the factor loading for block design was fixed to 5.972, its value from the last invariance model in which the occasion 1 factor variance was fixed to 1. Consequently, the total SD will be ≈ 1 for occasion 1, setting the scale of the latent outcome to be predicted. All lower-order factor variances were estimated but constrained equal over time so that any heterogeneity of variance over time in the lower-order factors would be captured by the higher-order factor for latent basis change. Likewise, all lower-order factor covariances were fixed to 0 so that all factor correlation over time would be captured by the estimated variance of the higher-order factors for intercept and latent basis change (and their estimated covariance). All lower-order factor intercepts and the mean of the higher-order intercept factor were fixed to 0 for identification given the estimation of the outcome intercepts. All residual covariances for the same outcome over time were estimated as in previous models. Finally, the latent basis factor loadings were fixed to 0 and 1 at occasions 1 and 4, respectively, with estimated factor loadings at occasions 2 and 3. Consequently, the higher-order intercept factor will capture the expected latent factor at occasion 1, and the mean of the higher-order latent basis change factor will capture the amount of overall change in the latent factor across the four occasions.

Although the latent basis change model had excellent fit, it fit significantly worse than the last invariance model, $-2\Delta LL(6) = 13.66$, p = .034. Modification indices suggested that the occasion 4 factor intercept was the largest source of misfit and should be freed. After doing so, the latent basis change invariance model had excellent fit (as shown in Table 1) that was not significantly worse than the last invariance model, $-2\Delta LL(5) = 4.27$, p = .511. Figure 1 displays the predicted lower-order factor means and variances for each occasion. There was a significant average decline of 0.340 (as given by the mean of the higher-order factor for latent basis change), 32.9% and 75.2% of which happened by occasions 2 and 3, respectively. The occasion 4 intercept (capturing its deviation from the predicted trajectory) was significantly negative (-0.145). Wald tests* indicated significant individual differences in the predicted latent outcome at occasion 1 and in its subsequent decline, as captured by the variances of the higher-order intercept and latent basis change factors, respectively.

* Yes, I know that Wald tests should not be used for testing the significance of variances, but this is very commonly done in the SEM world. In this case, the likelihood ratio tests would have agreed, and so I didn't report those additional model comparisons.