Example 3 using MLM: Alternative Metrics of Time in Accelerated Longitudinal Designs *(complete syntax, data, and output available for STATA, R, and SAS electronically)*

Time = current age – baseline age, which was created in order to represent the purely longitudinal variance of time-varying age.

Also, I am building squared variables to simplify the model syntax below (but these versions won't work with the predicted outcomes code used in the chapter 10 syntax at the book's website).

These data come from Hoffman (2015) chapter 10, which examined prediction of a recall outcome from two metrics of time: years since birth (centered at 84 years) and years in study (centered at the first occasion) in a sample of 557 observations from 207 older adults. This example first estimates empty models for each time-varying variable (age, time, recall), and then saturated means for recall by age and time. This is followed by a series of unconditional and conditional models for change as a function of age to evaluate age convergence (i.e., age cohort effects), and then using change by time in study instead (in which age at baseline is one of many potential time-related moderators that could be included). I used maximum likelihood estimation to facilitate all model comparisons (and Example 4's translation into SEM and M-SEM).

STATA Syntax for Importing and Preparing Data for Analysis:

```
// Define working directory for file location to be replaced in code below
// \\Client\ precedes path in Virtual Desktop outside H drive
cd "C:\Dropbox\25_PSQF7375_AdvLong\PSQF7375_AdvLong_Example3_MLM"

// Import Example 3 long data
clear // clear memory in case a dataset is already open
import excel "Excel_Chapter10.xlsx", sheet("Chapter10") firstrow case(preserve) clear

// Create time in study (and squared version)
gen time = tvage-ageT0
gen timesq = time*time

// Fix 1 case rounded to 9
replace occasion=8 if (occasion==9)

// Create rounded age (years since birth)
gen roundage = round(tvage,1)
// Fix 2 cases above 95
replace roundage=95 if (roundage==97)
replace roundage=95 if (roundage==100)

// Center TV age and age at time 0 at 84 (and squared versions)
gen tvage84 = tvage-84
gen tvage84sq = tvage84*tvage84
gen ageT084 = ageT0-84
gen ageT084sq = ageT084*ageT084

// Label variables
label variable time       "time: Years since Time 0"
label variable timesq     "timesq: Squared Years since Time 0"
label variable roundage   "roundage: Age Rounded to Nearest Year"
label variable tvage84    "tvage84: Time-Varying Age (0=84 years)"
label variable tvage84sq  "tvage84sq: Squared Time-Varying Age (0=84 years)"
label variable ageT084    "ageT084: Age at Time 0 (0=84 years)"
label variable ageT084sq  "ageT084: Squared Age at Time 0 (0=84 years)"

// Subset sample to complete cases for all predictors
egen nummiss = rowmiss(tvage ageT0 recall)
drop if nummiss>0
```

R Syntax for Importing and Preparing Data for Analysis (after loading packages *readxl*, *psych*, *TeachingDemos*, *emmeans*, *lmerTest*, and *performance*, as well as two custom functions for effect sizes as shown online):

```
# Set working directory (to import and export files to)
setwd("C:/Dropbox/25_PSQF7375_AdvLong/PSQF7375_AdvLong_Example3_MLM")

# Import Example 3 long-format data from excel -- path = file name
Example3 = read_excel(path="Excel_Chapter10.xlsx", sheet="Chapter10")
# Convert to data frame to use for analysis
Example3 = as.data.frame(Example3)

# Create time in study (and squared version)
Example3$time=Example3$tvage-Example3$ageT0
Example3$timesq=Example3$time*Example3$time

# Fix 1 case that rounded to 9
Example3$occasion[which(Example3$occasion==9)]=8

# Create rounded age (years since birth)
Example3$roundage=round(Example3$tvage,digits=0)
# Fix 2 cases above 95
Example3$roundage[which(Example3$roundage>95)]=95

# Center TV age and age at time 0 at 84 years (and squared version)
Example3$tvage84=Example3$tvage-84
Example3$tvage84sq=Example3$tvage84*Example3$tvage84
Example3$ageT084=Example3$ageT0-84
Example3$ageT084sq=Example3$ageT084*Example3$ageT084

# Subset sample to complete cases for all predictors
Example3 = Example3[complete.cases(Example3[ , c("tvage","ageT0","recall")]),]
```

Syntax and Output for Data Description and Correlations:

```
display "STATA Descriptive Statistics and Correlations"
summarize ageT0 tvage time recall
pwcorr ageT0 tvage time recall, sig

print("R Descriptive Statistics and Correlations")
describe(x=Example2[ , c("ageT0","tvage","time","recall")])
corr.test(x=Example2[ , c("ageT0","tvage","time","recall")])
```

```
    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
       ageT0 |        557    82.97113    2.688563   79.41918   97.77808
       tvage |        557    85.64534     3.55728   79.41918   99.89863
        time |        557    2.674216    2.603101          0   8.502732
      recall |        557     10.1939    3.826512          0         16

             |    ageT0    tvage     time   recall
-------------+------------------------------------
       ageT0 |   1.0000 
             |
       tvage |   0.6852   1.0000 
             |   0.0000
             |
        time |  -0.0965   0.6589   1.0000 
             |   0.0228   0.0000
             |
      recall |  -0.1230  -0.0630   0.0409   1.0000 
             |   0.0037   0.1375   0.3352
```

Because baseline age and time do have some (negative) correlation, they do not provide a complete separation of the between-person and within-person variance in age (i.e., as would have been obtained by centering using person mean age instead of baseline age to create time, analogous to chapter 8). Instead, we will think of **baseline age** as representing **cross-sectional** age variance and **time** as representing **longitudinal** age variance.

Syntax and Partial Output for Empty Means, Random Intercept Model for Age:

```
display "STATA Empty Means, Random Intercept Model for Age"
mixed tvage , || personid: , mle nolog

print("R Empty Means, Random Intercept Model for Age")
Age = lmer(data=Example2, REML=FALSE, formula=tvage~1+(1|PersonID))
summary(Age); icc(Age); print("Does the random intercept improve model fit?")
ranova(Age, reduce.term=TRUE) # LRT for removing random intercept
```

```
Random effects:
 Groups   Name        Variance Std.Dev.
 PersonID (Intercept) 5.0911   2.2564
 Residual             7.7728   2.7880

# Intraclass Correlation Coefficient
     Adjusted ICC: 0.396
   Unadjusted ICC: 0.396
```

$$\text{ICC} = \frac{5.0911}{5.0911 + 7.7728} = .396$$

So 40% of the variance in age is actually cross-sectional—due to age mean differences! This means that age can have both cross-sectional (~BP) and longitudinal (~WP) effects simultaneously.

```
               npar  logLik     AIC     LRT Df Pr(>Chisq)
<none>            3 -1460.21 2926.43
(1 | PersonID)    2 -1496.68 2997.36 72.9318  1 < 2.22e-16
```

Syntax and Partial Output for Empty Means, Random Intercept Model for Time:

```
display "STATA Empty Means, Random Intercept Model for Time"
mixed time , || personid: , mle nolog
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
personid: Identity           |
                  var(_cons) |   3.74e-20   1.03e-19      1.70e-22    8.23e-18
-----------------------------+------------------------------------------------
               var(Residual) |   6.763969   .4053182      6.014436    7.606909
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 0.00          Prob >= chibar2 = 1.0000

estat icc  // Intraclass correlation
------------------------------------------------------------------------------
                       Level |        ICC   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
                    personid |   5.53e-21           0      5.53e-21    5.53e-21
------------------------------------------------------------------------------
```

$$\text{ICC} = \frac{0}{0 + 6.764} = 0$$

All variance in *time* is within persons—this means time can only have a longitudinal (~WP) effect.

```
print("R Empty Means, Random Intercept Model for Time")
Time = lmer(data=Example2, REML=FALSE, formula=time~1+(1|PersonID))
summary(Time); icc(Time); print("Does the random intercept improve model fit?")
ranova(Time, reduce.term=TRUE) # LRT for removing random intercept
```

Syntax and Partial Output for Empty Means, Random Intercept Model for <u>Recall Outcome</u>:

Level-1: $recall_{ti} = \beta_{0i} + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + U_{0i}$

```
display "STATA Model 0: Empty Means, Random Intercept Model for Recall Outcome"
mixed recall , || personid: , mle nolog
matrix Empty = r(table)     // Save results for computations below

print("R Model 0: Empty Means, Random Intercept Model for Recall Outcome")
Empty = lmer(data=Example2, REML=FALSE, formula=recall~1+(1|PersonID))
llikAIC(Empty); summary(Empty); icc(Empty)
```

```
$AICtab
      AIC        BIC      logLik   deviance   df.resid
 2863.3550  2876.3227 -1428.6775  2857.3550   554.0000  deviance = −2LL (for homework)

Random effects:
 Groups   Name        Variance Std.Dev.
 PersonID (Intercept) 10.4579  3.2339  Level-2 var(U_0i)
 Residual              5.1646  2.2726  Level-1 var(e_ti)
Number of obs: 557, groups:  PersonID, 207

Fixed effects:
             Estimate Std. Error        df t value  Pr(>|t|)
(Intercept)   9.73491    0.25058 197.01209   38.85 < 2.2e-16

# Intraclass Correlation Coefficient
    Adjusted ICC: 0.669
  Unadjusted ICC: 0.669
```

$$ICC = \frac{10.4579}{10.4579 + 5.1646} = .669$$

So 67% of the variance in recall is initially due to person mean differences.

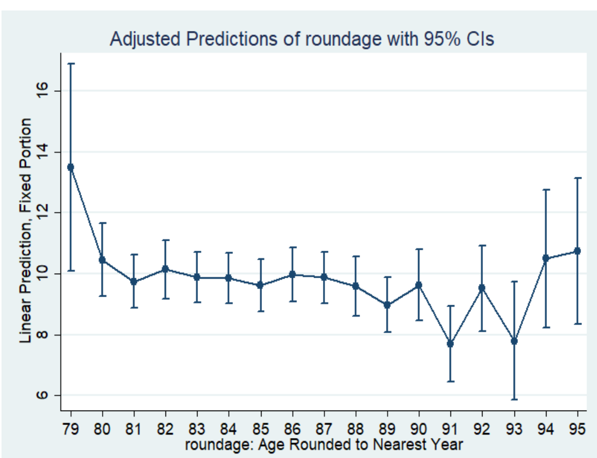
```
print("Does the random intercept improve model fit?")
ranova(Empty, reduce.term=TRUE) # LRT for removing random intercept
              npar   logLik     AIC     LRT Df Pr(>Chisq)
<none>           3 -1428.68 2863.36
(1 | PersonID)   2 -1537.32 3078.63 217.278  1 < 2.22e-16
```

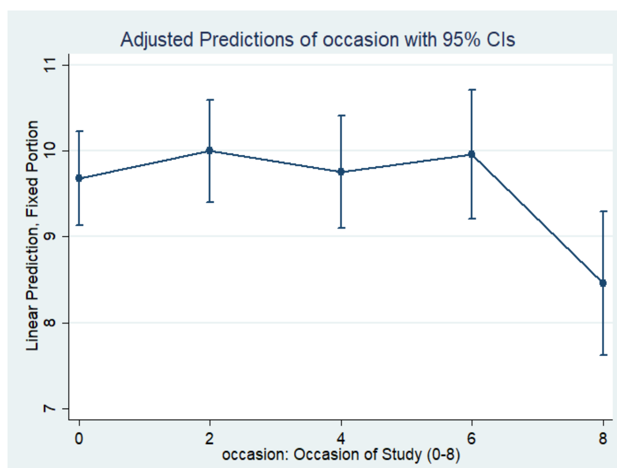
Next, we will see what the mean trajectory for recall looks like over age and time…

STATA Syntax and Plots for Saturated Means for Recall by Age and Time

```
display "Saturated Means by Rounded Age, Random Intercept Model"
mixed recall i.roundage, || personid: , mle nolog
margins  i.roundage  // get saturated means per age and plot them
marginsplot, xdimension(roundage) name(by_age, replace)
graph export "STATA plots\STATA Recall by Age.png", replace

display "Saturated Means by Rounded Time, Random Intercept Model"
mixed recall i.occasion, || personid: , mle nolog
margins  i.occasion  // get saturated means per occasion and plot them
marginsplot, xdimension(occasion) name(by_time, replace)
graph export "STATA plots\STATA Recall by Time.png", replace
```





R Syntax for Saturated Means for Recall by Age and Time (see syntax online for plots)

```
print("Saturated Means by Rounded Age, Random Intercept Model")
SatAge = lmer(data=Example2, REML=FALSE, formula=recall~0+as.factor(roundage)+(1|PersonID))
summary(SatAge)

print("Saturated Means by Rounded Occasion, Random Intercept Model")
SatTim = lmer(data=Example2, REML=FALSE, formula=recall~0+as.factor(occasion)+(1|PersonID))
summary(SatTim)
```

Age-as-Time Models

Model 1a. Syntax and Partial Output for Fixed Quadratic Age, Random Intercept for Recall:

Level-1 Age: $recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - 84) + \beta_{2i}(age_{ti} - 84)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + U_{0i}, \ \beta_{1i} = \gamma_{10}, \ \beta_{2i} = \gamma_{20}$

```
display "STATA Model 1a: Fixed Quadratic Age, Random Intercept Model"
mixed recall c.tvage84 c.tvage84sq, || PersonID: , mle nolog
matrix RIAge = r(table)      // Save results for computations below
display "-2LL = " e(ll)*-2   // Print -2LL for model
estimates store FitRIAge     // Save for LRT

print("R Model 1a: Fixed Quadratic Age, Random Intercept Model")
RIAge = lmer(data=Example2, REML=FALSE, formula=recall~1+tvage84+tvage84sq+(1|PersonID))
llikAIC(RIAge); summary(RIAge)

$AICtab
       AIC        BIC     logLik   deviance    df.resid
 2860.6264  2882.2393 -1425.3132  2850.6264    552.0000   deviance = -2LL

Random effects:
 Groups   Name        Variance Std.Dev.
 PersonID (Intercept) 10.4804  3.2373   Level-2 var(U_0i)
 Residual              5.0716  2.2520   Level-1 var(e_ti)

Fixed effects:
               Estimate  Std. Error          df t value Pr(>|t|)
(Intercept)   9.8196505   0.2634293 230.9528321 37.2762  < 2e-16  gamma00
tvage84      -0.1189889   0.0516498 465.3551305 -2.3038  0.02168  gamma10
tvage84sq     0.0047917   0.0075791 474.9664062  0.6322  0.52755  gamma20
```

Interpret these fixed effects:

Intercept γ_{00} =

Slope for age_{ti} γ_{10} =

Slope for age_{ti}^2 γ_{20} =

```
// Build and save total-R2
predict PredAge, xb             // Save fixed-effect predicted outcome
quietly corr recall PredAge     // Get total r to make R2
global R2Age = r(rho)^2         // Save total-R2 for comparison
display "Total-R2 = " $R2Age    // Print total-R2 relative to empty model

# Total R2 using custom function
TotalR2(data=Example3, dvName="recall", model1=RIAge, name1="TVAge")
[1] 0.0046802504

// Build pseudo-R2
matrix list RIAge  // Show saved results (variances are saved as log of SD)
display "Pseudo-R2 for Intercept = " 1-(exp(RIAge[1,4])^2/exp(Empty[1,2])^2)
display "Pseudo-R2 for Residual = "  1-(exp(RIAge[1,5])^2/exp(Empty[1,3])^2)

# Pseudo-R2 relative to empty model using custom function
PseudoR2(data=Example2, baseModel=Empty, model1=RIAge, name1="TVAge")

Pseudo-R2 for TVAge
         term       base     model1 pseudoR2.model1
1 (Intercept) 10.4579158 10.4804309         -0.0022
2    Residual  5.1645871  5.0716102          0.0180
```

Below, I am adding terms in a different order than I usually recommend so that I can make some pedagogical points. First, I add effects of **baseline age** ($ageT0_i - 84$) to the fixed quadratic age, random intercept mode—we will see later how its fixed slopes differ in interpretation when using *age* versus *time* in the level-1 model. Then I add **random linear slopes** for each level-1 predictor to see how their level-2 random effect variances differ between models. Finally, I show syntax and partial output for novel models (relative to what was covered in chapter 10) that try to repair the mis-specification introduced into the random slope age-as-time model. Additionally, I am inter-weaving the code from STATA and R that do the same things for easier comparison.

Model 2a. Syntax and Partial Output for Fixed Quadratic Age, Random Intercept for Recall, adding Age at Baseline to Add Contextual Birth Cohort Effects (that Test Age Convergence):

Level-1 Age: $recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - 84) + \beta_{2i}(age_{ti} - 84)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(ageT0_i - 84) + \gamma_{02}(ageT0_i - 84)^2 + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(ageT0_i - 84)$, $\beta_{2i} = \gamma_{20}$

```
display "STATA Model 2a: Fixed Quadratic Age, Random Intercept Model"
display "Controlling for Birth Cohort as Contextual Effects"
mixed recall c.tvage84 c.tvage84sq c.ageT084 c.ageT084sq c.tvage84#c.ageT084, ///
           || PersonID: , mle nolog
matrix RICohAge = r(table)  // Save results for computations below
display "-2LL = " e(ll)*-2  // Print -2LL for model

print("R Model 2a: Fixed Quadratic Age, Random Intercept Model")
print("Controlling for Birth Cohort as Contextual Effects")
RICohAge = lmer(data=Example2, REML=FALSE, formula=recall~1+tvage84+tvage84sq
                +ageT084+ageT084sq+tvage84:ageT084+(1|PersonID))
llikAIC(RICohAge); summary(RICohAge)
```

```
$AICtab
       AIC         BIC      logLik   deviance   df.resid
 2851.3846   2885.9651 -1417.6923  2835.3846   549.0000   deviance = -2LL

Random effects:
 Groups   Name        Variance Std.Dev.
 PersonID (Intercept) 10.2325  3.1988   Level-2 var(U_0i)
 Residual              4.9275  2.2198   Level-1 var(e_ti)

Fixed effects:
                  Estimate Std. Error         df t value  Pr(>|t|)
(Intercept)       9.388079   0.341412 263.136000 27.4978 < 2.2e-16   gamma00
tvage84           0.287618   0.119256 380.694774  2.4118 0.0163477   gamma10
tvage84sq        -0.043547   0.015815 366.603154 -2.7535 0.0061904   gamma20
ageT084          -0.575545   0.157213 556.003996 -3.6609 0.0002753   gamma01
ageT084sq        -0.075633   0.028853 549.018161 -2.6213 0.0090012   gamma02
tvage84:ageT084   0.126026   0.034676 377.633390  3.6344 0.0003172   gamma11
```

Interpret these fixed effects:

Intercept γ_{00} =

Slope for age_{ti} γ_{10} =

Slope for age_{ti}^2 γ_{20} =

Slope for $ageT0_i$ γ_{01} =

Slope for $ageT0_i^2$ γ_{02} =

Slope for $age_{ti} * ageT0_i^2$ γ_{11} =

```
estimates store FitRICohAge      // Save for LRT
lrtest FitRICohAge FitRIAge      // LRT for birth cohort contextual fixed slopes

print("LRT for birth cohort contextual fixed slopes"); anova(RICohAge,RIAge)
         npar     AIC     BIC   logLik deviance   Chisq Df Pr(>Chisq)
RIAge       5 2860.63 2882.24 -1425.31  2850.63
RICohAge    8 2851.38 2885.97 -1417.69  2835.38 15.2423  3  0.0016209
```

Above, the slopes of baseline age represent **contextual** birth cohort effects (or age non-convergence). The **total** birth cohort effects of baseline age are then linear combinations as shown below (see chapter 10 for the math).

```
// Linear Combinations to get Total Effects (g = gamma below)
// Total Linear Birth Cohort on Intercept = g10+g01
   lincom c.ageT084*1 + c.tvage84*1
// Total Quadratic Birth Cohort on Intercept = g20+g02+g11
   lincom c.ageT084sq*1 + c.tvage84#c.ageT084*1 + c.tvage84sq*1
// Total Linear Birth Cohort on Linear Slope = 2*g20+g11
   lincom c.tvage84#c.ageT084*1 + c.tvage84sq*2

print("Linear Combinations to get Total Effects") # g = gamma below
print("Total Linear    Birth Cohort on Intercept"); contest1D(RICohAge, L=c(0,1,0,1,0,0)) # g10+g01
print("Total Quadratic Birth Cohort on Intercept"); contest1D(RICohAge, L=c(0,0,1,0,1,1)) # g20+g02+g11
print("Total Linear Birth Cohort on Linear Slope"); contest1D(RICohAge, L=c(0,0,2,0,0,1)) # 2*g20+g11
```

Estimates (from SAS for better organization)					
Label	Estimate	SE	DF	t Value	Pr > |t|
Total Linear Birth Cohort on Intercept	-0.2879	0.1000	251	-2.88	0.0043
Total Quadratic Birth Cohort on Intercept	0.006846	0.01850	236	0.37	0.7116
Total Linear Birth Cohort on Linear Slope	0.03893	0.01778	400	2.19	0.0291

```
// Build and save total-R2
predict PredCohAge, xb           // Save fixed-effect predicted outcome
quietly corr recall PredCohAge   // Get total r to make R2
global R2CohAge = r(rho)^2       // Save total-R2 for comparison
display "Total-R2 = " $R2CohAge // Print total-R2 relative to empty model
display "Change in Total-R2 = " $R2CohAge - $R2Age

# Total R2 and change using custom function
TotalR2(data=Example3, dvName="recall", model1=RIAge, name1="TVage",
                                        model2=RICohAge, name2="TVage+Cohort")

Total-R2 and Change in Total-R2 for TVage vs TVage+Cohort
     totalR2.1    totalR2.2     changeR2
1 0.0046802504 0.025146123 0.020465873

// Build pseudo-R2
matrix list RICohAge   // Show saved results (variances are saved as log of SD)
display "Pseudo-R2 for Intercept = " 1-(exp(RICohAge[1,7])^2/exp(Empty[1,2])^2)
display "Pseudo-R2 for Residual = "  1-(exp(RICohAge[1,8])^2/exp(Empty[1,3])^2)

display "Change in Pseudo-R2 for Intercept = " ///
(1-(exp(RICohAge[1,7])^2/exp(Empty[1,2])^2)) - (1-(exp(RIAge[1,4])^2/exp(Empty[1,2])^2))

display "Change in Pseudo-R2 for Residual = " ///
(1-(exp(RICohAge[1,8])^2/exp(Empty[1,3])^2)) - (1-(exp(RIAge[1,5])^2/exp(Empty[1,3])^2))

# Pseudo-R2 and change relative to empty model using custom function
PseudoR2(data=Example2, baseModel=Empty, model1=RIAge, name1="TVage",
                                         model2=RICohAge, name2="TVage+Cohort")

Pseudo-R2 and Change in Pseudo-R2 for TVage vs TVage+Cohort
         term       base     model1     model2 pseudoR2.model1 pseudoR2.model2 pseudoR2.change
1 (Intercept) 10.4579158 10.4804309 10.2324666         -0.0022          0.0216          0.0237
2    Residual  5.1645871  5.0716102  4.9274692          0.0180          0.0459          0.0279
```

Model 3a. Syntax and Partial Output to add Random Linear Age to Model 2a:

Level-1 Age: $Recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - 84) + \beta_{2i}(age_{ti} - 84)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(ageT0_i - 84) + \gamma_{01}(ageT0_i - 84)^2 + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(ageT0_i - 84) + U_{1i}, \;\; \beta_{2i} = \gamma_{20}$

```
display "STATA Model 3a: Add Random Linear TVage to Model 2a"
mixed recall c.tvage84 c.tvage84sq c.ageT084 c.ageT084sq c.tvage84#c.ageT084, ///
           || PersonID: tvage84, mle nolog covariance(unstructured)
estat recovariance, relevel(PersonID) correlation  // GCORR matrix
display "-2LL = " e(ll)*-2       // Print -2LL for model

print("R Model 3a: Add Random Linear TVage to Model 2a")
RLCohAge = lmer(data=Example2, REML=FALSE, formula=recall~1+tvage84+tvage84sq
                +ageT084+ageT084sq+tvage84:ageT084+(1+tvage84|PersonID))
llikAIC(RLCohAge); summary(RLCohAge)

$AICtab
      AIC       BIC    logLik  deviance  df.resid
 2843.8455  2887.0711 -1411.9227  2823.8455  547.0000

Random effects:
 Groups   Name        Variance  Std.Dev. Corr
 PersonID (Intercept) 11.154725 3.33987
          tvage84      0.090726 0.30121  -0.340  → new random linear age slope variance
 Residual              4.110764 2.02750

Fixed effects:
                  Estimate Std. Error         df t value  Pr(>|t|)
(Intercept)       9.414130   0.350985 256.483500 26.8220 < 2.2e-16
tvage84           0.295904   0.113458 353.980589  2.6081 0.0094915
tvage84sq        -0.045388   0.015083 333.443196 -3.0092 0.0028184
ageT084          -0.579338   0.154367 522.386360 -3.7530 0.0001943
ageT084sq        -0.077490   0.030650 162.364800 -2.5282 0.0124203
tvage84:ageT084   0.125601   0.034497 356.744786  3.6409 0.0003119

estimates store FitRLCohAge      // Save for LRT
lrtest FitRLCohAge FitRICohAge   // LRT for random linear TVage slope

print("LRT for random linear TVage slope"); anova(RLCohAge,RICohAge)
         npar    AIC     BIC   logLik deviance   Chisq Df Pr(>Chisq)
RICohAge    8 2851.39 2885.97 -1417.69  2835.39
RLCohAge   10 2843.84 2887.07 -1411.92  2823.84 11.5391  2  0.0031211
```

Foreshadowing variance explained by each model (within subtype of age vs. time at level 1):

Model	Random Intercept Variance	Residual Variance	Random Intercept Pseudo-R2	Residual Variance Pseudo-R2	Change in Intercept Pseduo-R2	Change in Residual Pseudo-R2
Model 0: Empty Means, Random Intercept	10.4578	5.1646				
Model 1a Age: Fixed Quadratic, Random Intercept Model	10.4803	5.0716	-0.002	0.018		
Model 2a Age: Fixed Quadratic, Random Intercept Model + Cohort	10.2323	4.9275	0.022	0.046	0.024	0.028
Model 0: Empty Means, Random Intercept	10.4578	5.1646				
Model 1b Time: Fixed Quadratic, Random Intercept Model	10.6213	4.9831	-0.016	0.035		
Model 2b Time: Fixed Quadratic, Random Intercept Model + Cohort	10.2323	4.9275	0.022	0.046	0.037	0.011

Time-in-Study Models: (time = as years-in-study rather than years-since-birth)

Model 1b. Syntax and Partial Output for Fixed Quadratic Time, Random Intercept for Recall:

Level-1 Time: $recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - ageT0_i) + \beta_{2i}(age_{ti} - ageT0_i)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + U_{0i},\ \ \beta_{1i} = \gamma_{10},\ \ \beta_{2i} = \gamma_{20}$

```
display "STATA Model 1b: Fixed Quadratic Time, Random Intercept Model"
mixed recall c.time c.timesq, || PersonID: , mle nolog
matrix RITim = r(table)     // Save results for computations below
display "-2LL = " e(ll)*-2  // Print -2LL for model
estimates store FitRITim    // Save for LRT

print("R Model 1b: Fixed Quadratic Time, Random Intercept Model")
RITim = lmer(data=Example2, REML=FALSE, formula=recall~1+time+timesq+(1|PersonID))
llikAIC(RITim); summary(RITim)

$AICtab
       AIC        BIC     logLik   deviance   df.resid
 2856.0088  2877.6216 -1423.0044  2846.0088   552.0000   deviance = -2LL

Random effects:
 Groups   Name        Variance Std.Dev.
 PersonID (Intercept) 10.622   3.2591   Level-2 var(U_0i)
 Residual              4.983   2.2323   Level-1 var(e_ti)

Fixed effects:
              Estimate Std. Error         df t value  Pr(>|t|)
(Intercept)   9.660987   0.274986 282.359753 35.1326 < 2.2e-16  gamma00
time          0.261331   0.119243 377.995056  2.1916  0.029019  gamma10
timesq       -0.046907   0.015826 366.791758 -2.9640  0.003235  gamma20
```

Interpret these fixed effects:

Intercept γ_{00} =

Slope for $time_{ti}$ γ_{10} =

Slope for $time_{ti}^2$ γ_{20} =

```
// Build and save total-R2
predict PredTim, xb              // Save fixed-effect predicted outcome
quietly corr recall PredTim      // Get total r to make R2
global R2Tim = r(rho)^2          // Save total-R2 for comparison
display "Total-R2 = " $R2Tim     // Print total-R2 relative to empty model

# Total R2 using custom function
TotalR2(data=Example3, dvName="recall", model1=RITim, name1="Time")

Total R2 for Time
[1] 0.0027298944

// Build pseudo-R2
matrix list RITim  // Show saved results (variances are saved as log of SD)
display "Pseudo-R2 for Intercept = "  1-(exp(RITim[1,4])^2/exp(Empty[1,2])^2)
display "Pseudo-R2 for Residual = "   1-(exp(RITim[1,5])^2/exp(Empty[1,3])^2)

# Pseudo-R2 relative to empty model using custom function

PseudoR2(data=Example2, baseModel=Empty, model1=RITim, name1="Time")
         term       base     model1 pseudoR2.model1
1 (Intercept) 10.4579158 10.6216155         -0.0157
2    Residual  5.1645871  4.9830202          0.0352
```

Level-1 time explained almost twice as much variance as level-1 age!

Model 2b. Syntax and Partial Output for Fixed Quadratic Time, Random Intercept for Recall, adding Age at Baseline to Introduce Total Cross-Sectional Birth Cohort Effects:

Level-1 Time: $recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - ageT0_i) + \beta_{2i}(age_{ti} - ageT0_i)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(ageT0_i - 84) + \gamma_{02}(ageT0_i - 84)^2 + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(ageT0_i - 84),\ \ \beta_{2i} = \gamma_{20}$

In which $age_{ti} - ageT0_i = time_{ti}$ (as years-in-study rather than years-since-birth)

Although it may not appear so, this is an equivalent model to the previous Model 2a using age as the level-2 predictor (centered at age 84) instead…. Even though the difference is in the level-1 predictor, it's going to be the level-2 slopes and cross-level interactions slopes for baseline age that change their values and interpretation!

```
display "STATA Model 2b: Fixed Quadratic Time, Random Intercept Model"
display "Controlling for Birth Cohort as Total Effects"
mixed recall time c.timesq c.ageT084 c.ageT084sq c.time#c.ageT084, ///
             || PersonID: , mle nolog
matrix RICohTim = r(table)  // Save results for computations below
display "-2LL = " e(ll)*-2  // Print -2LL for model

print("R Model 2b: Fixed Quadratic Time, Random Intercept Model")
print("Controlling for Birth Cohort as Total Effects")
RICohTim = lmer(data=Example2, REML=FALSE, formula=recall~1+time+timesq
                +ageT084+ageT084sq+time:ageT084+(1|PersonID))
llikAIC(RICohTim); summary(RICohTim);
```

```
$AICtab
       AIC        BIC     logLik   deviance   df.resid
 2851.3846  2885.9651 -1417.6923  2835.3846   549.0000  deviance = -2LL

Random effects:
 Groups   Name        Variance Std.Dev.
 PersonID (Intercept) 10.2325  3.1988   Level-2 var(U_0i)
 Residual              4.9275  2.2198   Level-1 var(e_ti)

Fixed effects:
                Estimate  Std. Error          df t value  Pr(>|t|)
(Intercept)    9.3880788   0.3414122 263.1359999 27.4978 < 2.2e-16  gamma00
time           0.2876175   0.1192557 380.6947769  2.4118  0.016348  gamma10
timesq        -0.0435466   0.0158152 366.6031559 -2.7535  0.006190  gamma20
ageT084       -0.2879274   0.1000388 250.7639296 -2.8782  0.004345  gamma01
ageT084sq      0.0068459   0.0184967 235.7300477  0.3701  0.711631  gamma02
time:ageT084   0.0389327   0.0177798 400.3882883  2.1897  0.029122  gamma11
```

Interpret these fixed effects:

Intercept γ_{00} =

Slope for $time_{ti}$ γ_{10} =

Slope for $time_{ti}^2$ γ_{20} =

Slope for $ageT0_i$ γ_{01} =

Slope for $ageT0_i^2$ γ_{02} =

Slope for $time_{ti} * ageT0_i^2$ γ_{11} =

```
estimates store FitRICohTim      // Save for LRT
lrtest FitRICohTim FitRITim      // LRT for birth cohort total fixed slopes
```

```
print("LRT for birth cohort total fixed slopes"); anova(RICohTim,RITim)
         npar     AIC     BIC   logLik deviance   Chisq Df Pr(>Chisq)
RITim       5 2856.01 2877.62 -1423.00  2846.01
RICohTim    8 2851.39 2885.97 -1417.69  2835.39 10.6242  3   0.013942
```

Above, the slopes of baseline age represent **total** birth cohort age effects. The **contextual** birth cohort effects (that test age convergence) are then linear combinations as shown below (see chapter 10 for the math).

```
// Linear Combinations to get Contextual Effects (g = gamma)
// Contextual Linear Birth Cohort on Intercept = g01 - g10
   lincom c.ageT084*1 + time*-1
// Contextual Quadratic Birth Cohort on Intercept = g20 + g01 – g11
   lincom c.ageT084sq*1 + c.time#c.ageT084*-1 + c.timesq*1
// Contextual Linear Birth Cohort on Linear Slope = g11 – 2*g20
   lincom c.time#c.ageT084*1 + c.timesq*-2
```

```
print("Linear Combinations to get Contextual Effects")
print("Contextual Linear    Birth Cohort on Intercept"); contest1D(RICohTim, L=c(0,-1, 0,1,0, 0))
print("Contextual Quadratic Birth Cohort on Intercept"); contest1D(RICohTim, L=c(0, 0, 1,0,1,-1))
print("Contextual Linear Birth Cohort on Linear Slope"); contest1D(RICohTim, L=c(0, 0,-2,0,0, 1))
```

Estimates					
Label	Estimate	SE	DF	t Value	Pr > |t|
Contextual Linear Birth Cohort on Intercept	-0.5755	0.1572	556	-3.66	0.0003
Contextual Quadratic Birth Cohort on Intercept	-0.07563	0.02885	549	-2.62	0.0090
Contextual Linear Birth Cohort on Linear Slope	0.1260	0.03468	378	3.63	0.0003

```
Fixed effects from current Model 2b repeated:
                Estimate  Std. Error          df  t value  Pr(>|t|)
(Intercept)    9.3880788   0.3414122 263.1359999 27.4978  < 2.2e-16   gamma00
time           0.2876175   0.1192557 380.6947769  2.4118   0.016348   gamma10
timesq        -0.0435466   0.0158152 366.6031559 -2.7535   0.006190   gamma20
ageT084       -0.2879274   0.1000388 250.7639296 -2.8782   0.004345   gamma01
ageT084sq      0.0068459   0.0184967 235.7300477  0.3701   0.711631   gamma02
time:ageT084   0.0389327   0.0177798 400.3882883  2.1897   0.029122   gamma11

Fixed effects from equivalent Model 2a repeated:
```

Estimates (from SAS for better organization)					
Label	Estimate	SE	DF	t Value	Pr > |t|
Total Linear Birth Cohort on Intercept	-0.2879	0.1000	251	-2.88	0.0043
Total Quadratic Birth Cohort on Intercept	0.006846	0.01850	236	0.37	0.7116
Total Linear Birth Cohort on Linear Slope	0.03893	0.01778	400	2.19	0.0291

```
                 Estimate Std. Error          df  t value  Pr(>|t|)
(Intercept)      9.388079   0.341412 263.136000 27.4978  < 2.2e-16   gamma00
tvage84          0.287618   0.119256 380.694774  2.4118  0.0163477   gamma10
tvage84sq       -0.043547   0.015815 366.603154 -2.7535  0.0061904   gamma20
ageT084         -0.575545   0.157213 556.003996 -3.6609  0.0002753   gamma01
ageT084sq       -0.075633   0.028853 549.018161 -2.6213  0.0090012   gamma02
tvage84:ageT084  0.126026   0.034676 377.633390  3.6344  0.0003172   gamma11
```

In comparing the two solutions directly, the age-as-time Model 2a provides *contextual* effects of age cohort, whereas time-in-study Model 2b provides *total* effects of age cohort. However, the −2LL and variance components are all the same (i.e., they are equivalent models) because there are no random slopes (yet).

```
// Build and save total-R2
predict PredCohTim, xb          // Save fixed-effect predicted outcome
quietly corr recall PredCohTim  // Get total r to make R2
global R2CohTim = r(rho)^2      // Save total-R2 for comparison
display "Total-R2 = " $R2CohTim // Print total-R2 relative to empty model
display "Change in Total-R2 = " $R2CohTim - $R2Tim
```

```
# Total R2 and change using custom function
TotalR2(data=Example3, dvName="recall", model1=RITim, name1="Time",
                                        model2=RICohTim, name2="Time+Cohort")
```

```
Total-R2 and Change in Total-R2 for Time vs Time+Cohort
     totalR2.1   totalR2.2    changeR2
1 0.0027298944 0.025146123 0.022416229

// Build pseudo-R2
matrix list RICohAge  // Show saved results (variances are saved as log of SD)
display "Pseudo-R2 for Intercept = " 1-(exp(RICohAge[1,7])^2/exp(Empty[1,2])^2)
display "Pseudo-R2 for Residual = "  1-(exp(RICohAge[1,8])^2/exp(Empty[1,3])^2)

display "Change in Pseudo-R2 for Intercept = " ///
(1-(exp(RICohTim[1,7])^2/exp(Empty[1,2])^2)) - (1-(exp(RITim[1,4])^2/exp(Empty[1,2])^2))

display "Change in Pseudo-R2 for Residual = " ///
(1-(exp(RICohTim[1,8])^2/exp(Empty[1,3])^2)) - (1-(exp(RITim[1,5])^2/exp(Empty[1,3])^2))

# Pseudo-R2 and change relative to empty model using custom function
PseudoR2(data=Example2, baseModel=Empty, model1=RITim, name1="Time",
                                          model2=RICohTim, name2="Time+Cohort")

Pseudo-R2 and Change in Pseudo-R2 for Time vs Time+Cohort
         term       base     model1     model2 pseudoR2.model1 pseudoR2.model2 pseudoR2.change
1 (Intercept) 10.4579158 10.6216155 10.2324666         -0.0157          0.0216          0.0372
2    Residual  5.1645871  4.9830202  4.9274692          0.0352          0.0459          0.0108
```

Model 3b. Syntax and Partial Output to add Random Linear Time to Model 2b:

Level-1 Time: $recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - ageT0_i) + \beta_{2i}(age_{ti} - ageT0_i)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(ageT0_i - 84) + \gamma_{01}(ageT0_i - 84)^2 + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(ageT0_i - 84) + U_{1i}, \;\; \beta_{2i} = \gamma_{20}$

```
display "STATA Model 3b: Add Random Linear Time to Model 2b"
mixed recall c.time c.timesq c.ageT084 c.ageT084sq c.tvage84#c.ageT084, ///
             || PersonID: time, mle nolog covariance(unstructured)
estat recovariance, relevel(PersonID) correlation  // GCORR matrix
display "-2LL = " e(ll)*-2      // Print -2LL for model

print("R Model 3b: Add Random Linear Time to Model 2b")
RLCohTim = lmer(data=Example2, REML=FALSE, formula=recall~1+time+timesq
                +ageT084+ageT084sq+time:ageT084+(1+time|PersonID))
llikAIC(RLCohTim); summary(RLCohTim)

$AICtab
      AIC       BIC     logLik   deviance   df.resid
 2838.5453  2881.7709 -1409.2726  2818.5453   547.0000
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 PersonID (Intercept) 12.4835  3.53320
          time         0.1272  0.35665  -0.473  → new random time slope variance
 Residual              3.9405  1.98508

Fixed effects:
                Estimate  Std. Error          df t value  Pr(>|t|)
(Intercept)    9.3402104   0.3515637 230.2831758 26.5676 < 2.2e-16
time           0.3132277   0.1123665 366.1427566  2.7876  0.005588
timesq        -0.0455538   0.0149692 340.1904601 -3.0432  0.002523
ageT084       -0.2972341   0.1050941 205.1743603 -2.8283  0.005144
ageT084sq      0.0091296   0.0183207 229.8606122  0.4983  0.618735
time:ageT084   0.0442743   0.0207981 126.2247447  2.1288  0.035217

estimates store FitRLCohTim     // Save for LRT
lrtest FitRLCohTim FitRICohTim  // LRT for random linear time slope

print("LRT for random linear TVage slope"); anova(RLCohTim,RICohTim)
```

```
         npar     AIC     BIC   logLik deviance   Chisq Df Pr(>Chisq)
RICohTim    8 2851.39 2885.97 -1417.69  2835.39
RLCohTim   10 2838.55 2881.77 -1409.27  2818.55 16.8393  2 0.00022049
```

Comparing variance components across random level-1 slope models:

Model	Random Intercept Variance	Random L1 Slope Variance	Residual Variance		Ratio of Intercept Variance	Ratio L1 Slope Variance	Ratio of Residual Variance
Model 3a Age: Add Random Linear TVage to Model 2a	11.1546	0.0907	4.1108				
Model 3b Time: Add Random Linear Time to Model 2b	12.4837	0.1272	3.9406				
Ratio of Variance Components					0.89	0.71	1.04

The models with random level-1 linear age/time slopes are no longer equivalent because the age-as-time model assumes the same pattern of variance heterogeneity occurs across longitudinal age (as time) and cross-sectional age (as baseline age). This is a testable assumption in theory, but no software I tried (SAS, STATA, or R) would cleanly estimate the model needed to do so below! That's because it requires a random level-2 "slope" of baseline age (a level-2 predictor)!

Model 4a. Syntax and Partial Output to add Random Linear Baseline Age to Model 3a (with invented notation because the two-level schema was not set up to anticipate this):

Level-1 Age: $Recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - 84) + \beta_{2i}(age_{ti} - 84)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(ageT0_i - 84) + \gamma_{01}(ageT0_i - 84)^2 + U_{0i} + \text{‽}(ageT0_i - 84)$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(ageT0_i - 84) + U_{1i},\ \ \beta_{2i} = \gamma_{20}$

Btw, this punctuation mark ‽ is called an interrobang

```
print("Model 4a Age: Add Random Linear AgeCoh to Model 3a -- won't run at all")
RL2CohAge = lmer(data=Example3, REML=FALSE, formula=recall~1+tvage84+tvage84sq
                 +ageT084+ageT084sq+tvage84:ageT084+(1+tvage84+ageT084|PersonID))
```

```
display "Model 4a Age: Add Random Linear AgeCoh to Model 3a"
display "Converged onto NPD solution even after extra iterations"
mixed recall c.tvage84 c.tvage84sq c.ageT084 c.ageT084sq c.tvage84#c.ageT084, ///
             || PersonID: tvage84 ageT084, mle nolog covariance(unstructured) emiterate(100)
------------------------------------------------------------------------------------
             recall |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------------------+---------------------------------------------------------------
            tvage84 |   .3135812   .1123666     2.79   0.005     .0933467    .5338156
          tvage84sq |   -.045559   .0149703    -3.04   0.002    -.0749003   -.0162176
            ageT084 |  -.6110154   .1582065    -3.86   0.000    -.9210945   -.3009363
          ageT084sq |  -.0807516   .0299604    -2.70   0.007    -.1394729   -.0220303
c.tvage84#c.ageT084 |   .1358312   .0348349     3.90   0.000      .067556    .2041063
              _cons |   9.336708   .3540698    26.37   0.000     8.642744    10.03067
------------------------------------------------------------------------------------
-----------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+-----------------------------------------------
personid: Unstructured       |  all estimates now closer to time level-1 model!
                var(tvage84) |   .1269792          .             .            .
                var(ageT084) |   .1446499          .             .            .  → new term!
                  var(_cons) |   12.53305          .             .            .
        cov(tvage84,ageT084) |  -.1355204          .             .            .
          cov(tvage84,_cons) |  -.6000266          .             .            .
          cov(ageT084,_cons) |    .6519452         .             .            .
-----------------------------+-----------------------------------------------
               var(Residual) |   3.941131          .
-----------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 238.31                 Prob > chi2 = 0.0000
```

```
display "-2LL = " e(ll)*-2          // Print -2LL for model
-2LL = 2818.5058

estat recovariance, relevel(personid) correlation  // GCORR matrix
Random-effects correlation matrix for level personid

             |   tvage84    aget084      _cons
-------------+---------------------------------
     tvage84 |         1
     aget084 | -.9999523          1
       _cons | -.4756344   .4842071          1
```

Model 4b. Syntax and Partial Output to add Random Linear Baseline Age to Model 3b:

Level-1 Time: $recall_{ti} = \beta_{0i} + \beta_{1i}(age_{ti} - ageT0_i) + \beta_{2i}(age_{ti} - ageT0_i)^2 + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(ageT0_i - 84) + \gamma_{02}(ageT0_i - 84)^2 + U_{0i} + \text{‽}(ageT0_i - 84)$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(ageT0_i - 84) + U_{1i},\ \beta_{2i} = \gamma_{20}$

Btw, this punctuation mark ‽ is called an interrobang

```
print("R Model 4b: Add Random Linear AgeCoh to Model 3b -- won't run")
RL2CohTim = lmer(data=Example2, REML=FALSE, formula=recall~1+time+timesq
                 +ageT084+ageT084sq+time:ageT084+(1+time+ageT084|PersonID))

display "Model 4b: Add Random Linear AgeCoh to Model 3b"
display "Converged onto NPD solution even after extra iterations"
mixed recall c.time c.timesq c.ageT084 c.ageT084sq c.tvage84#c.ageT084,    ///
             || PersonID: time ageT084, mle nolog covariance(unstructured) emiterate(100)

------------------------------------------------------------------------------------
              recall |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------------------+--------------------------------------------------------------
                time |   .3135874   .1123693     2.79   0.005     .0933477    .5338271
              timesq |  -.0455589   .0149707    -3.04   0.002     -.074901   -.0162169
             ageT084 |  -.2974387   .1053805    -2.82   0.005    -.5039806   -.0908968
           ageT084sq |  -.0351895   .0268436    -1.31   0.190    -.0878019    .0174229
c.tvage84#c.ageT084 |   .0447116     .020784     2.15   0.031     .0039757    .0854474
               _cons |   9.336693   .3540654    26.37   0.000     8.642737    10.03065
------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
PersonID: Unstructured       |
                   var(time) |   .1269905   .0431637      .0652304    .2472249
                var(ageT084) |   .0005878     .00043      .0001401    .0024653  → new term!
                  var(_cons) |   12.53215   1.599177      9.759045    16.09326
           cov(time,ageT084) |  -.0085325          .             .           .
             cov(time,_cons) |  -.6000924   .2264234     -1.043874   -.1563108
          cov(ageT084,_cons) |   .0520106          .             .           .
-----------------------------+------------------------------------------------
               var(Residual) |   3.941342   .3611464      3.293429    4.716719
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 238.31                Prob > chi2 = 0.0000

display "-2LL = " e(ll)*-2          // Print -2LL for model
-2LL = 2818.5058

estat recovariance, relevel(personid) correlation  // GCORR matrix
Random-effects correlation matrix for level personid
             |      time    ageT084      _cons
-------------+---------------------------------
        time |         1
     ageT084 |  -.987605          1
       _cons | -.4756852   .6060007          1
```

```
estimates store FitRL2CohTim      // Save for LRT
lrtest FitRL2CohTim FitRLCohTim   // LRT for random linear baseline age slope

Likelihood-ratio test                                 LR chi2(3)  =      0.04
(Assumption: FitRLCohTim nested in FitRL2CohTim)      Prob > chi2 =    0.8430
```

So it appears there is basically 0 variance for the level-2 random "slope" for baseline age, which is why the level-2 random slope variance for level-1 age was too small—it was being downwardly biased by the incorrect assumption of equivalent quadratic heterogeneity of variance across level-2 baseline years-since-birth (as cross-sectional age) and level-1 years-in-study (time as longitudinal age). **Take-home point: Use within-person time as your level-1 predictor instead of time-varying age (or any accelerated time metric that has both cross-sectional and longitudinal variance) to avoid smushed random slopes!!!!!**

Chapter 10 contains an example results section using these models, as well as predicted trajectories by age and time, as shown below. **The vertical lines show where the intercept is for each model; the dotted continuous part of the lines convey impossible extrapolations predicted by the age-as-time model!**