Example 4b: Generalized Linear Models and Quantile Regression for Positive Skewed Outcomes
(complete syntax, data, and output available for STATA, R, and SAS electronically)

The data for this example come from chapter 4 of Agresti (2015) available here: http://users.stat.ufl.edu/~aa/glm/data/ We will be predicting the sale price of 100 homes from four characteristics: whether they are brand new (0=no, 1=yes), square footage in 100s (centered at 1500), number of bedrooms (2, 3, or 4+), and number of bathrooms (1, 2, or 3+). Because this sample's distribution of home sale prices is bounded by 0 and is positively skewed, we will compare three types of generalized linear models (all with the same linear predictor) estimated using maximum likelihood: identity link with a normal distribution (typical regression), a log-transformed outcome in a typical regression (which is equivalent to an identity link with a lognormal distribution), and a log link with a gamma distribution. In addition, because this outcome also had several univariate outliers, we will use quantile regression to predict the median home price instead of the mean and to examine predictor slope differences across other percentiles.

For the generalized linear models: In SAS, I am still using GLIMMIX (even though these are not mixed-effects models). Because the corresponding STATA options (using GLM to get conditional distribution fit, also using LGAMMA) do not have denominator degrees of freedom, they were set to "none" in SAS GLIMMIX so that the SAS Wald test results (still labeled as t or F) will match those of STATA (using z or χ^2). In R, I am using the base function GLM (also using z or χ^2). For quantile regression: In SAS, I am using QUANTREG. In STATA, I am using SQREG and IQREG, and in R I am using QUANTREG (although I have not yet figured out all the options for obtaining standard errors).

<u>STATA</u> Syntax for Importing and Preparing Data for Analysis:

```
// Defining global variable for file location to be replaced in code below
// \\Client\ precedes path in Virtual Desktop outside H drive;
global filesave "C:\Dropbox\23_PSQF6270\PSQF6270_Example4b"

// Import Houses XLSX data
import excel "$filesave\Houses.xlsx", firstrow case(preserve) clear

// Categories for number of bedrooms
gen bed3v2=.
gen bed3v4=.
replace bed3v2=1 if beds==2
replace bed3v4=0 if beds==2
replace bed3v2=0 if beds==3
replace bed3v4=0 if beds==3
replace bed3v2=0 if beds==4
replace bed3v4=1 if beds==4
replace bed3v2=0 if beds==5
replace bed3v4=1 if beds==5
// Categories for number of bathrooms
gen bath2v1=.
gen bath2v3=.
replace bath2v1=1 if baths==1
replace bath2v3=0 if baths==1
replace bath2v1=0 if baths==2
replace bath2v3=0 if baths==2
replace bath2v1=0 if baths==3
replace bath2v3=1 if baths==3
replace bath2v1=0 if baths==4
replace bath2v3=1 if baths==4
// Center and rescale size into per 100 square feet (0=1500)
gen sqft150=(size-1500)/100
// Generate quadratic sqft150 for use in some routines
gen sqft150sq=sqft150*sqft150
// Log-transform price for lognormal model
gen logprice=log(price)
// Label existing and new variables
label variable price    "price: Sale Price in 100,000 units"
label variable new      "new: House is new construction (0=no, 1=yes)"
label variable bed3v2   "bed3v2: 2 bedrooms instead of 3 (0=no, 1=yes)"
label variable bed3v4   "bed3v4: 4 bedrooms instead of 3 (0=no, 1=yes)"
label variable bath2v1  "bath2v1: 1 bathroom instead of 2 (0=no, 1=yes)"
label variable bath2v3  "bath2v3: 3 bathrooms instead of 2 (0=no, 1=yes)"
```

```
label variable sqft150  "sqft150: Square Footage per 100 feet (0=150)"
label variable logprice "logprice: Natural log of sale price in 100,000 units"

// Install user-written packages for gamma
search lgamma // install from window
```

<u>R</u> Syntax for Importing and Preparing Data for Analysis (after loading packages *readxl*, *TeachingDemos*, *psych*, *multcomp*, and *quantreg*, as shown online):

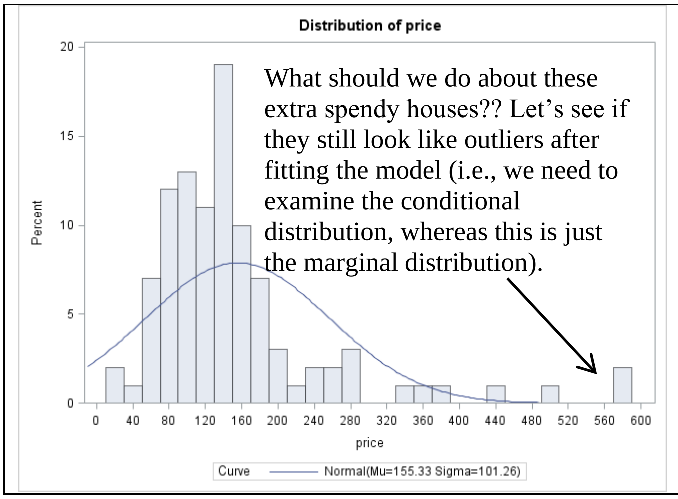
```
# Define variables for working directory and data name
filesave = "C:\\Dropbox/23_PSQF6270/PSQF6270_Example4b/"
filename = "Houses.xlsx"
setwd(dir=filesave)

# Import Houses XLSX data
Example4b = read_excel(paste0(filesave,filename))
# Convert to data frame without labels to use for analysis
Example4b = as.data.frame(Example4b)

# Categories for number of bedrooms
Example4b$bed3v2=NA; Example4b$bed3v4=NA
Example4b$bed3v2[which(Example4b$beds==2)]=1
Example4b$bed3v4[which(Example4b$beds==2)]=0
Example4b$bed3v2[which(Example4b$beds==3)]=0
Example4b$bed3v4[which(Example4b$beds==3)]=0
Example4b$bed3v2[which(Example4b$beds==4)]=0
Example4b$bed3v4[which(Example4b$beds==4)]=1
Example4b$bed3v2[which(Example4b$beds==5)]=0
Example4b$bed3v4[which(Example4b$beds==5)]=1
# Categories for number of bathrooms
Example4b$bath2v1=NA; Example4b$bath2v3=NA
Example4b$bath2v1[which(Example4b$baths==1)]=1
Example4b$bath2v3[which(Example4b$baths==1)]=0
Example4b$bath2v1[which(Example4b$baths==2)]=0
Example4b$bath2v3[which(Example4b$baths==2)]=0
Example4b$bath2v1[which(Example4b$baths==3)]=0
Example4b$bath2v3[which(Example4b$baths==3)]=1
Example4b$bath2v1[which(Example4b$baths==4)]=0
Example4b$bath2v3[which(Example4b$baths==4)]=1
# Center and rescale size into per 100 square feet (0=1500)
Example4b$sqft150=(Example4b$size-1500)/100
# Make squared version for use
Example4b$sqftsq=Example4b$sqft150^2
# Log-transform price for lognormal model
Example4b$logprice=log(Example4b$price)
```

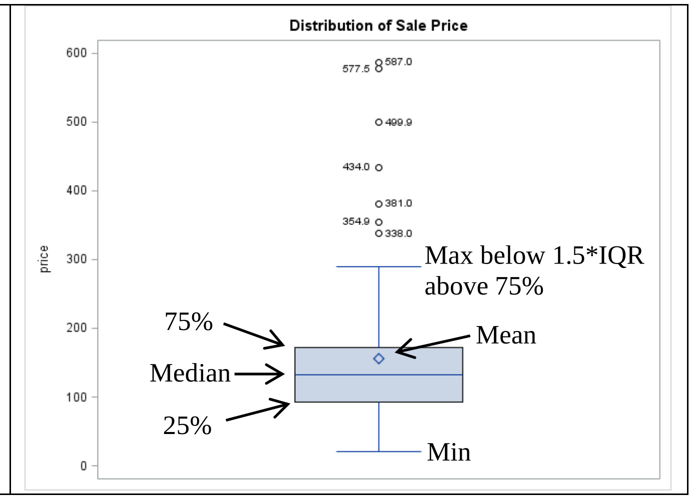
Syntax and SAS Output for Data Description:

```
display "STATA Distribution of Sale Price Outcome"
summarize price
hist price, percent start(0) width(20)
graph export "$filesave\STATA Price Histogram.png", replace
graph box price
graph export "$filesave\STATA Price Box Plot.png", replace

display "STATA Descriptive Stats for Example Variables"
summarize price size
tabulate beds
tabulate baths
tabulate new

# to save a plot: open a file, create the plot, then close the file
png(file = "R Price Histogram.png")  # open file
hist(x=Example4b$price, freq=FALSE,
     ylab="Density",xlab="Sale Price in 100,000 units") # axis labels
dev.off()  # close file
png(file = "R Price Boxplot.png")  # open file
boxplot(x=Example4b$price)
dev.off()  # close file
```

```
print("R Descriptive Stats for Example Variables")
describe(x=Example4b$price); describe(x=Example4b$size)
table(x=Example4b$beds,useNA="ifany")
table(x=Example4b$baths,useNA="ifany")
table(x=Example4b$new,useNA="ifany")
```

Plots from SAS GLIMMIX:





Every model we fit in this example will have the same linear predictor so that the reference house is old (i.e., not new construction) and has 3 bedrooms, 2 bedrooms, and 1500 square feet:

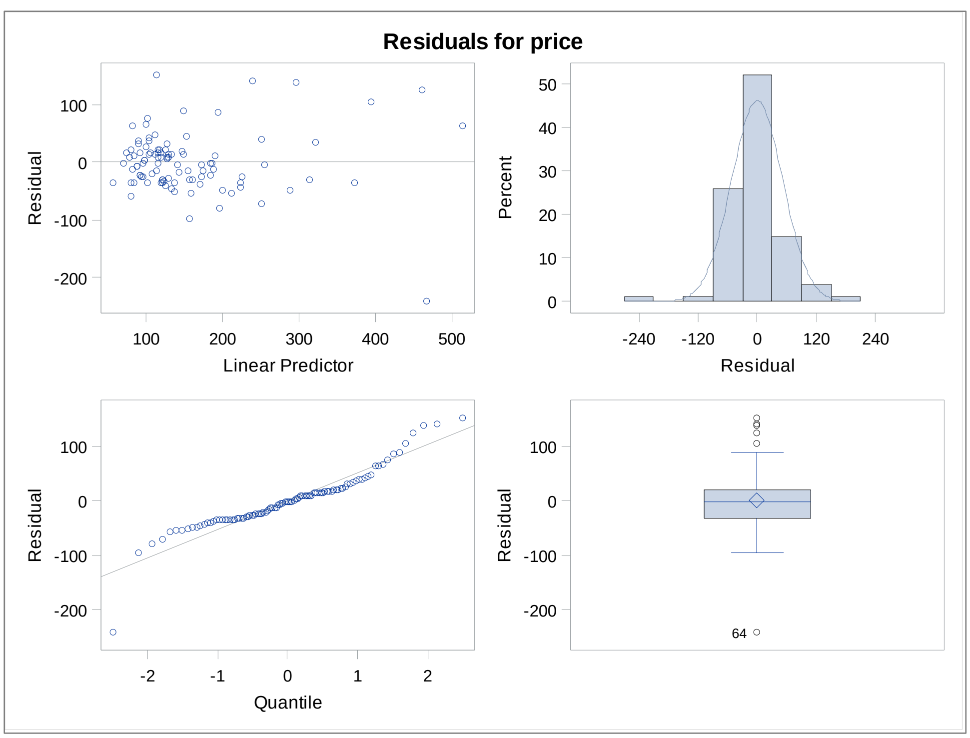
$$\hat{y}_i = \beta_0 + \beta_1(New_i) + \beta_2(Bed3v2_i) + \beta_3(Bed3v4_i) + \beta_4(Bath2v1_i) + \beta_5(Bath2v3_i)$$
$$+\beta_6(SqFt_i - 150) + \beta_7(SqFt_i - 150)^2$$

1) Predict Original Price with Identity Link and Normal Conditional Distribution:

$Price_i \sim Normal(\hat{y}_i, \sigma_e^2)$ → Regular general linear model, but using ML estimation for comparability

```
display "STATA Predict Price using Identity Link, Normal Distribution"
glm price c.new c.bed3v2 c.bed3v4 c.bath2v1 c.bath2v3 c.sqft150 ///
        c.sqft150#c.sqft150, ml link(identity) family(gaussian) nolog

Generalized linear models                     No. of obs      =        100
Optimization     : ML                         Residual df     =         92
                                              Scale parameter =   2907.643
Deviance         =  267503.1219               (1/df) Deviance =   2907.643
Pearson          =  267503.1219               (1/df) Pearson  =   2907.643 → REML residual variance
Variance function: V(u) = 1                   [Gaussian]
Link function    : g(u) = u                   [Identity]
                                              AIC             =   10.88959
Log likelihood   = -536.4796698               BIC             =   267079.4
------------------------------------------------------------------------------------
                    |                 OIM
              price |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------------------+---------------------------------------------------------------
                new |   59.52165   19.13903     3.11   0.002     22.00984    97.03346 Beta1
             bed3v2 |   14.21484    16.4218     0.87   0.387     -17.9713    46.40098 Beta2
             bed3v4 |   5.813162    16.4301     0.35   0.723    -26.38925    38.01557 Beta3
            bath2v1 |  -6.372286   16.92815    -0.38   0.707    -39.55085    26.80628 Beta4
            bath2v3 |  -14.49037   21.53875    -0.67   0.501    -56.70554    27.72481 Beta5
            sqft150 |   10.02966   1.867685     5.37   0.000     6.369064    13.69026 Beta6
c.sqft150#c.sqft150 |    .149102   .0906363     1.65   0.100    -.0285419    .3267458 Beta7
              _cons |   128.1352   7.544411    16.98   0.000     113.3485     142.922 Beta0
------------------------------------------------------------------------------------

display "-2LL= " e(ll)*-2   // Print -2LL for model
-2LL= 1072.9593
```

```
test (c.new=0) (c.bed3v2=0) (c.bed3v4=0) (c.bath2v1=0) (c.bath2v3=0) ///
     (c.sqft150=0) (c.sqft150#c.sqft150=0) // Multiv Wald test of model
           chi2(  7) =   279.49
         Prob > chi2 =    0.0000

print("R Predict Price using Identity Link, Normal Distribution")
ModelNorm = glm(data=Example4b, family=gaussian(link="identity"), # I(x^2) squares predictor
                formula=price~1+new+bed3v2+bed3v4+bath2v1+bath2v3+sqft150+sqftsq)
print("Print -2LL with results"); -2*logLik(ModelNorm); summary(ModelNorm)

'log Lik.' 1072.9593 (df=9) → -2LL for model

Coefficients:
              Estimate Std. Error z value     Pr(>|z|)
(Intercept) 128.135249   7.544411 16.9841    < 2.2e-16 Beta0
new          59.521653  19.139032  3.1100     0.002491 Beta1
bed3v2       14.214838  16.421801  0.8656     0.388957 Beta2
bed3v4        5.813161  16.430103  0.3538     0.724290 Beta3
bath2v1      -6.372286  16.928150 -0.3764     0.707463 Beta4
bath2v3     -14.490364  21.538751 -0.6728     0.502788 Beta5
sqft150      10.029661   1.867685  5.3701 0.0000005877 Beta6
sqftsq        0.149102   0.090636  1.6451     0.103371 Beta7

(Dispersion parameter for gaussian family taken to be 2907.6426) → REML residual variance

    Null deviance: 1015150  on 99  degrees of freedom
Residual deviance:  267503  on 92  degrees of freedom
AIC: 1090.96

print("Multiv Wald Test of Model")
NormR2 = glht(model=ModelNorm, linfct=c("new=0","bed3v2=0","bed3v4=0","bath2v1=0",
                "bath2v3=0","sqft150=0","sqftsq=0"))  # Couldn't square predictor here
summary(NormR2, test=Chisqtest()) # Joint chi-square test

Global Test:
   Chisq DF Pr(>Chisq)
1 257.13  7 8.4006e-52
```



Residual plots from SAS:

The conditional distribution still has some outliers… it also deviates from normal to some extent (with greater variance due to an outlier with a large negative residual for an expensive house).

Let's see if we can do better…

2a) Predict <u>Log-Transformed</u> Price with Identity Link and Normal Conditional Distribution:

$LogPrice_i \sim Normal(\hat{y}_i, \sigma_e^2)$ → Regular general linear model on log-transformed outcome (ML estimation)

```
display "STATA Predict Log-Transformed Price using Identity Link, Normal Distribution"
glm logprice c.new c.bed3v2 c.bed3v4 c.bath2v1 c.bath2v3 c.sqft150 ///
             c.sqft150#c.sqft150, ml link(identity) family(gaussian) nolog

Generalized linear models                         No. of obs      =        100
Optimization     : ML                             Residual df     =         92
                                                  Scale parameter =   .1180992
Deviance         =  10.86512647                   (1/df) Deviance =   .1180992
Pearson          =  10.86512647                   (1/df) Pearson  =   .1180992 → REML residual variance
Variance function: V(u) = 1                       [Gaussian]
Link function    : g(u) = u                       [Identity]
                                                  AIC             =   .7782651
Log likelihood   = -30.91325673                   BIC             =  -412.8105
------------------------------------------------------------------------------
                    |                 OIM
           logprice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------------------+---------------------------------------------------------
                new |   .2391817   .1219756     1.96   0.050     .0001139    .4782494 Beta1
             bed3v2 |   .1539676   .1046583     1.47   0.141    -.051159     .3590941 Beta2
             bed3v4 |   .0129777   .1047112     0.12   0.901    -.1922525    .2182079 Beta3
            bath2v1 |  -.1455129   .1078853    -1.35   0.177    -.3569643    .0659385 Beta4
            bath2v3 |  -.0561446   .1372693    -0.41   0.683    -.3251876    .2128983 Beta5
            sqft150 |   .0795194    .011903     6.68   0.000     .0561899    .1028488 Beta6
c.sqft150#c.sqft150 |  -.0012611   .0005776    -2.18   0.029    -.0023933   -.000129  Beta7
              _cons |   4.814402   .0480815   100.13   0.000     4.720164     4.90864  Beta0
------------------------------------------------------------------------------

display "-2LL= " e(ll)*-2  // Print -2LL for model
-2LL= 61.826513

test (c.new=0) (c.bed3v2=0) (c.bed3v4=0) (c.bath2v1=0) (c.bath2v3=0) ///
     (c.sqft150=0) (c.sqft150#c.sqft150=0) // Multiv Wald test of model
           chi2(  7) =  172.69
         Prob > chi2 =    0.0000

print("R Predict Log-Transformed Price using Identity Link, Normal Distribution")
ModelLogNorm = glm(data=Example4b, family=gaussian(link="identity"),
                   formula=logprice~1+new+bed3v2+bed3v4+bath2v1+bath2v3+sqft150+sqftsq)
print("Print -2LL with results"); -2*logLik(ModelLogNorm); summary(ModelLogNorm)

'log Lik.' 61.826517 (df=9) → -2LL for model

Coefficients:
               Estimate  Std. Error  z value       Pr(>|z|)
(Intercept)  4.81440211  0.04808153 100.1300      < 2.2e-16 Beta0
new          0.23918164  0.12197559   1.9609        0.05292 Beta1
bed3v2       0.15396753  0.10465832   1.4711        0.14466 Beta2
bed3v4       0.01297764  0.10471123   0.1239        0.90164 Beta3
bath2v1     -0.14551293  0.10788535  -1.3488        0.18072 Beta4
bath2v3     -0.05614470  0.13726932  -0.4090        0.68348 Beta5
sqft150      0.07951937  0.01190301   6.6806 0.000000001786 Beta6
sqftsq      -0.00126111  0.00057764  -2.1832        0.03156 Beta7

(Dispersion parameter for gaussian family taken to be 0.11809921) → REML residual variance

    Null deviance: 31.2597  on 99  degrees of freedom
Residual deviance: 10.8651  on 92  degrees of freedom
AIC: 79.8265

print("Multiv Wald Test of Model")
LogTNormR2 = glht(model=ModelLogNorm, linfct=c("new=0","bed3v2=0","bed3v4=0",
                  "bath2v1=0","bath2v3=0","sqft150=0","sqftsq=0"))
summary(LogTNormR2, test=Chisqtest()) # Joint chi-square test
Global Test:
   Chisq DF Pr(>Chisq)
1 172.69  7 6.7988e-34
```

2b) Predict Price with Identity Link and Lognormal Conditional Distribution:
$Price_i \sim Lognormal(\hat{y}_i, \sigma_e^2)$ → Residuals are expected to follow a lognormal distribution

```
TITLE1 "SAS Predict Price using Identity Link, Log-Normal Distribution";
TITLE2 " Using RSPL=OLS=REML to get SEs that match STATA and R";
PROC GLIMMIX DATA=work.Example4b NAMELEN=100 GRADIENT METHOD=RSPL;
     MODEL price = new bed3v2 bed3v4 bath2v1 bath2v3 sqft150 sqft150*sqft150
                   / SOLUTION DDFM=NONE LINK=IDENTITY DIST=LOGNORMAL;
     CONTRAST "Multiv Wald test of Model" new 1, bed3v2 1, bed3v4 1,
               bath2v1 1, bath2v3 1, sqft150 1, sqft150*sqft150 1 / CHISQ;
RUN; TITLE;

// No Stata regression with a lognormal distribution that I could find
# Could not find lognormal conditional distribution in R that was likelihood-equivalent
```

---

3) Predict Price with Log Link and Gamma Conditional Distribution: $Price_i \sim Gamma(\mu, \phi)$, where $\hat{y}_i = Log(\mu)$ and ϕ is a "scale" multiplier of the variance, such that variance = $\mu^2\phi$ (or at least I think that's right).

Stata's GLM does not give the same LL as in SAS for gamma, but here is an "Lgamma" routine that does:

```
display "STATA: Price using Log Link, Gamma Distribution"
display "Using LGAMMA that does not allow factor variables or interactions"
display "GLM gives different LL and solution for gamma distribution"
lgamma price new bed3v2 bed3v4 bath2v1 bath2v3 sqft150 sqft150sq, nolog

Log-gamma model                                   Number of obs   =        100
                                                  LR chi2(7)      =     117.57
Log likelihood = -517.21898                       Prob > chi2     =     0.0000
------------------------------------------------------------------------------
       price |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         new |    .204721   .1136043     1.80   0.072    -.0179394    .4273814 Beta1
      bed3v2 |   .1728484   .1002319     1.72   0.085    -.0236026    .3692993 Beta2
      bed3v4 |   .0218806   .0952913     0.23   0.818    -.1648869    .2086482 Beta3
     bath2v1 |  -.1323233   .0999321    -1.32   0.185    -.3281866      .06354 Beta4
     bath2v3 |  -.0526695   .1244118    -0.42   0.672    -.2965123    .1911732 Beta5
     sqft150 |   .0752007   .0111396     6.75   0.000     .0533675    .0970339 Beta6
    sqft150sq |  -.0009965   .0005487    -1.82   0.069    -.0020719    .0000789 Beta7
       _cons |   4.854958   .0441468   109.97   0.000     4.768432    4.941484 Beta0
-------------+----------------------------------------------------------------
     /ln_phi |  -2.298655   .1391173   -16.52   0.000     -2.57132    -2.02599 → log(phi)
---------+--------------------------------------------------------------------
         phi |   .1003938   .0139665                      .0764346    .1318632 → phi variance multiplier
------------------------------------------------------------------------------

display "-2LL= " e(ll)*-2  // Print -2LL for model
-2LL= 1034.438

test (c.new=0) (c.bed3v2=0) (c.bed3v4=0) (c.bath2v1=0) (c.bath2v3=0) ///
     (c.sqft150=0) (c.sqft150#c.sqft150=0) // Multiv Wald test of model
           chi2(  7) =  187.18
         Prob > chi2 =    0.0000

display "STATA LGAMMA: Price using Log Link, Gamma Distribution"
display "Get Incident-Rate Ratios as exp(slope)"
lgamma price new bed3v2 bed3v4 bath2v1 bath2v3 sqft150 sqft150sq, eform nolog
------------------------------------------------------------------------------
       price |        IRR   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         new |   1.227183   .1394133     1.80   0.072     .9822205    1.533237 exp(Beta1)
      bed3v2 |   1.188686   .1191443     1.72   0.085     .9766738    1.446721 exp(Beta2)
      bed3v4 |   1.022122   .0973993     0.23   0.818     .8479896    1.232011 exp(Beta3)
     bath2v1 |   .8760577   .0875463    -1.32   0.185     .7202286    1.065602 exp(Beta4)
     bath2v3 |   .9486935   .1180287    -0.42   0.672     .7434065    1.210669 exp(Beta5)
     sqft150 |     1.0781   .0120096     6.75   0.000     1.054817    1.101898 exp(Beta6)
    sqft150sq |    .999004   .0005481    -1.82   0.069     .9979302    1.000079 exp(Beta7)
       _cons |   128.3753   5.667357   109.97   0.000     117.7345    139.9779 exp(Beta0)
-------------+----------------------------------------------------------------
```

```
print("R Predict Price using Log Link, Gamma Distribution")
print("SEs and scale parameter are differ slightly from SAS and STATA")
ModelGamma = glm(data=Example4b, family=Gamma(link="log"), # I(x^2) squares predictor
                 formula=price~1+new+bed3v2+bed3v4+bath2v1+bath2v3+sqft150+sqftsq)
print("Print -2LL, with results"); -2*logLik(ModelGamma); summary(ModelGamma)

'log Lik.' 1034.4521 (df=9) → -2LL for model

Coefficients:
               Estimate  Std. Error  t value              Pr(>|t|)
(Intercept)  4.85495821  0.04559534 106.4793 < 0.00000000000000022 Beta0
new          0.20472068  0.11566850   1.7699               0.08006 Beta1
bed3v2       0.17285544  0.09924667   1.7417               0.08491 Beta2
bed3v4       0.02188128  0.09929685   0.2204               0.82608 Beta3
bath2v1     -0.13232450  0.10230684  -1.2934               0.19911 Beta4
bath2v3     -0.05266582  0.13017143  -0.4046               0.68672 Beta5
sqft150      0.07520161  0.01128753   6.6624        0.000000001942 Beta6
sqftsq      -0.00099659  0.00054777  -1.8194               0.07211 Beta7

(Dispersion parameter for Gamma family taken to be 0.10620167) → phi variance multiplier (close to Stata)

    Null deviance: 31.9401  on 99  degrees of freedom
Residual deviance: 10.2072  on 92  degrees of freedom
AIC: 1052.45

print("Pearson Chi-Square / DF Index of Fit")
sum(residuals(ModelGamma, type="pearson")^2)/(100-8)

[1] 0.10620167 → less variance in residuals than model expects!

print("Multiv Wald Test of Model -- differs from SAS and STATA")
GammaR2 = glht(model=ModelGamma, linfct=c("new=0","bed3v2=0","bed3v4=0",
               "bath2v1=0","bath2v3=0","sqft150=0","sqftsq=0"))
summary(GammaR2, test=Chisqtest()) # Joint chi-square test

Global Test:
   Chisq DF Pr(>Chisq)
1 178.37  7 4.2939e-35 → results differ from SAS or STATA

print("Get incidence rate ratios with 95% CIs")
exp(cbind(IRRR=coefficients(ModelGamma), confint.default(ModelGamma)))

                      IRR         2.5 %      97.5 %
(Intercept) 128.37532692 117.40071335 140.3758469 exp(Beta0)
new           1.22718224   0.97825449   1.5394524 exp(Beta1)
bed3v2        1.18869426   0.97856853   1.4439398 exp(Beta2)
bed3v4        1.02212243   0.84135889   1.2417225 exp(Beta3)
bath2v1       0.87605667   0.71688330   1.0705721 exp(Beta4)
bath2v3       0.94869699   0.73506442   1.2244178 exp(Beta5)
sqft150       1.07810149   1.05451238   1.1022183 exp(Beta6)
sqftsq        0.99900391   0.99793195   1.0000770 exp(Beta7)
```

---

4) Predict Price Median (50th Percentile) instead of Mean using Quantile Regression

Back in intro stat you learned that variables with skewness, outliers, or other kinds of non-normal distributions could be better described using median and interquartile range (i.e., the 50th percentile and the distance from the 25th to 75th percentile) than using the mean and standard deviation. **So why not predict these percentiles instead of the mean using a regression model?** This is the basis of **quantile regression**: the slope estimates are those that minimize a weighted absolute value of the residuals (rather than an unweighted sum of squared residuals as in traditional regression). While the residuals are still assumed to be normal, this is of little consequence because most quantile procedures use some kind of resampling (i.e., bootstrapping in SAS and STATA) to get the standard errors without relying on distributional properties.

```
TITLE "SAS Predict Price 50th Percentile (Median) using Quantile Regression";
PROC QUANTREG DATA=work.Example4b NAMELEN=100 CI=RESAMPLING(NREP=500);
     MODEL price = new bed3v2 bed3v4 bath2v1 bath2v3 sqft150 sqft150*sqft150
                   / QUANTILE=.50 SEED=8675309; * Jenny is my random seed;
     Model: TEST new bed3v2 bed3v4 bath2v1 bath2v3 sqft150 sqft150*sqft150 / WALD;
RUN; TITLE;
```

```
                            Parameter Estimates
                             Standard    95% Confidence
Parameter       DF Estimate    Error         Limits        t Value Pr > |t|
Intercept        1 133.0000   7.2909  118.5197  147.4803    18.24   <.0001 predicted 50th percentile at ref
new              1  32.1650  24.6156  -16.7236   81.0536     1.31   0.1946
bed3v2           1   1.0778  18.4457  -35.5569   37.7125     0.06   0.9535
bed3v4           1 -28.1157  17.6509  -63.1719    6.9404    -1.59   0.1146
bath2v1          1 -13.7301  15.3765  -44.2691   16.8088    -0.89   0.3742
bath2v3          1  -1.2992  29.5853  -60.0581   57.4596    -0.04   0.9651
sqft150          1   8.6648   2.4979    3.7038   13.6258     3.47   0.0008
sqft150*sqft150  1   0.3827   0.1653    0.0545    0.7110     2.32   0.0228
```

For an unknown reason, the bootstrap SEs and multivariate Wald test results differ between SAS and STATA (beyond correcting for F vs. χ^2)

```
               Test Model Results
                       Test       Chi-
Test                Statistic DF  Square Pr > ChiSq
Wald                 109.8928  7  109.89      <.000 → Translates to F = 109.89/7 = 15.70
```

```
display "STATA Predict Price 50th Percentile (Median) using Quantile Regression"
set seed 8675309 // Set Jenny as random seed to get same results each time
sqreg price c.new c.bed3v2 c.bed3v4 c.bath2v1 c.bath2v3 c.sqft150 ///
            c.sqft150#c.sqft150, quantile(.50) reps(500) nolog
```

```
Simultaneous quantile regression                   Number of obs =        100
  bootstrap(500) SEs                                .50 Pseudo R2 =     0.4523
------------------------------------------------------------------------------
                     |              Bootstrap
               price |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------------+--------------------------------------------------------
q50                  |
                 new |   32.16499   29.56973     1.09   0.280    -26.56305    90.89303
              bed3v2 |    1.07779   19.89831     0.05   0.957    -38.44197    40.59755
              bed3v4 |  -28.11573   21.78021    -1.29   0.200    -71.37311    15.14165
             bath2v1 |  -13.73013    14.5145    -0.95   0.347    -42.55717    15.09691
             bath2v3 |  -1.299235   32.61557    -0.04   0.968    -66.07658    63.47811
             sqft150 |   8.664786   2.330797     3.72   0.000     4.035623    13.29395
 c.sqft150#c.sqft150 |   .3827353   .2509158     1.53   0.131    -.1156051    .8810758
               _cons |        133    7.28882    18.25   0.000     118.5238    147.4762 50th percent for ref
------------------------------------------------------------------------------
```

```
test (c.new=0) (c.bed3v2=0) (c.bed3v4=0) (c.bath2v1=0) (c.bath2v3=0) ///
     (c.sqft150=0) (c.sqft150#c.sqft150=0) // Multiv Wald test of model does not match SAS
```

```
       F(  7,     92) =   10.52
            Prob > F =    0.0000
```

```
print("R Predict Price 50th Percentile [Median] using Quantile Regression")
print("Did not figure out how to get same SEs and test statistics as SAS and STATA")
set.seed(8675309) # Jenny is my random seed
ModelQ50 = rq(data=Example4b, tau=.5, formula=price~1+new+bed3v2+bed3v4+bath2v1+bath2v3+sqft150+sqftsq)
summary(ModelQ50)
```

```
Coefficients:
            coefficients lower bd   upper bd
(Intercept) 133.000000   119.479154 139.878004 50th percentile for ref
new          32.164989     3.529067  82.654677
bed3v2        1.077787   -14.270654  32.900320
bed3v4      -28.115733   -44.735514  -2.981709
bath2v1     -13.730133   -35.257264   7.080776
bath2v3      -1.299234   -43.256743  27.989451
sqft150       8.664785     6.543296  13.021328
sqftsq        0.382735    -0.149437   0.491025
```

4) Predict Price 25th and 75th Percentile using Quantile Regression:

Besides “handling” outliers, another use of quantile regression is to answer research questions about differences at other points of a distribution. Here, we predict the 25th percentile to ask, “among (relatively) cheap houses, what predicts sale price?” Likewise, we predict the 75th percentile to ask, “among (relatively) expensive houses, what predicts sale price?” We can also ask for differences in the predictor effects across these quantiles (e.g., is being a new house more important if the house is expensive than if the house is cheap?), which is analogous to an interaction of the predictor with the quantiles.

```
TITLE "SAS Predict Price 25th and 75th Percentile using Quantile Regression";
PROC QUANTREG DATA=work.Example4b NAMELEN=100 CI=RESAMPLING(NREP=500);
     MODEL price = new bed3v2 bed3v4 bath2v1 bath2v3 sqft150 sqft150*sqft150
                  / QUANTILE=.25 .75 SEED=8675309; * Jenny is my random seed;
    * Multiv wald test of Model (provided for each quantile);
      EachModel: TEST new bed3v2 bed3v4 bath2v1 bath2v3 sqft150 sqft150*sqft150 / WALD;
    * Multiv wald test of slope differences between quantiles;
      ModelDiff: TEST new bed3v2 bed3v4 bath2v1 bath2v3 sqft150 sqft150*sqft150 / QINTERACT;
      newDiff:   TEST new / QINTERACT; * How to test single slope diff across quantiles;
RUN; TITLE;
```

Parameter Estimates Predicting 25th percentile

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		t Value	Pr > |t|
Intercept	1	101.1147	7.2839	86.6482	115.5813	13.88	<.0001
new	1	45.6732	26.3641	-6.6881	98.0345	1.73	0.0866
bed3v2	1	4.7000	16.2591	-27.5920	36.9920	0.29	0.7732
bed3v4	1	-0.2206	18.0406	-36.0508	35.6095	-0.01	0.9903
bath2v1	1	-0.7478	16.5383	-33.5943	32.0988	-0.05	0.9640
bath2v3	1	2.3978	39.9465	-76.9394	81.7351	0.06	0.9523
sqft150	1	9.4049	2.4080	4.6225	14.1874	3.91	0.0002
sqft150*sqft150	1	0.1069	0.2230	-0.3360	0.5498	0.48	0.6329

Parameter Estimates Predicting 75th percentile

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		t Value	Pr > |t|
Intercept	1	145.7357	7.5581	130.7246	160.7467	19.28	<.0001
new	1	24.3886	35.5563	-46.2292	95.0065	0.69	0.4945
bed3v2	1	31.5946	19.8498	-7.8288	71.0179	1.59	0.1149
bed3v4	1	-31.6868	38.1827	-107.5210	44.1474	-0.83	0.4088
bath2v1	1	-15.0642	15.3389	-45.5285	15.4001	-0.98	0.3286
bath2v3	1	-1.2579	38.0627	-76.8537	74.3379	-0.03	0.9737
sqft150	1	10.8404	3.2413	4.4028	17.2779	3.34	0.0012
sqft150*sqft150	1	0.3295	0.2020	-0.0718	0.7307	1.63	0.1063

Test EachModel Results

Quantile Level	Test	Test Statistic	DF	Chi-Square	Pr > ChiSq	
0.25	Wald	65.3371	7	65.34	<.0001	→ F= 65.34/7 = 9.33
0.75	Wald	91.5617	7	91.56	<.0001	→ F= 91.56/7 = 13.08

Test ModelDiff Results
Equal Coefficients Across Quantiles

Chi-Square	DF	Pr > ChiSq
4.4799	7	0.7231

Test newDiff Results
Equal Coefficients Across Quantiles

Chi-Square	DF	Pr > ChiSq
0.3636	1	0.5465

STATA Syntax and Output from SQREG—these are the predictor slopes per quantile:

```
display "STATA Predict Price 25th and 75th Percentile using Quantile Regression"
set seed 8675309 // Set Jenny as random seed to get same results each time
sqreg price c.new c.bed3v2 c.bed3v4 c.bath2v1 c.bath2v3 c.sqft150 ///
            c.sqft150#c.sqft150, quantile(.25 .75) reps(500) nolog

Simultaneous quantile regression                    Number of obs =        100
  bootstrap(500) SEs                                .25 Pseudo R2 =     0.3747
                                                    .75 Pseudo R2 =     0.5713
------------------------------------------------------------------------------
                    |              Bootstrap
              price |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
--------------------+---------------------------------------------------------
q25                 |
                new |   45.67319   23.28024     1.96   0.053    -.5633818    91.90976
             bed3v2 |        4.7   16.55032     0.28   0.777    -28.17036    37.57036
             bed3v4 |  -.2206333   22.16177    -0.01   0.992    -44.23583    43.79456
            bath2v1 |  -.7477557   15.38074    -0.05   0.961    -31.29524    29.79972
            bath2v3 |   2.397835   33.72783     0.07   0.943    -64.58855    69.38422
            sqft150 |   9.404941   1.757855     5.35   0.000      5.91369    12.89619
c.sqft150#c.sqft150 |   .1068575   .2572658     0.42   0.679    -.4040946    .6178097
              _cons |   101.1147   7.681166    13.16   0.000     85.85928    116.3702 pred 25th for ref
--------------------+---------------------------------------------------------
q75                 |
                new |   24.38865   37.27569     0.65   0.515    -49.64408    98.42139
             bed3v2 |   31.59456    18.9706     1.67   0.099    -6.082685     69.2718
             bed3v4 |  -31.68683   45.05709    -0.70   0.484    -121.1741    57.80045
            bath2v1 |  -15.06422   13.76459    -1.09   0.277    -42.40189    12.27344
            bath2v3 |  -1.257883   43.82958    -0.03   0.977    -88.30722    85.79145
            sqft150 |   10.84037   3.055926     3.55   0.001     4.771039    16.90971
c.sqft150#c.sqft150 |   .3294847    .201842     1.63   0.106    -.0713909    .7303603
              _cons |   145.7357   5.482533    26.58   0.000     134.8469    156.6244 pred 75th for ref
------------------------------------------------------------------------------

// Multiv Wald test of model at 25th percentile
test ([q25]c.new=0) ([q25]c.bed3v2=0) ([q25]c.bed3v4=0) ([q25]c.bath2v1=0) ///
     ([q25]c.bath2v3=0)([q25]c.sqft150=0)([q25]c.sqft150#c.sqft150=0)
        F(  7,    92) =   12.10
            Prob > F =    0.0000

// Multiv Wald test of model at 75th percentile
test ([q75]c.new=0) ([q75]c.bed3v2=0) ([q75]c.bed3v4=0) ([q75]c.bath2v1=0) ///
     ([q75]c.bath2v3=0)([q75]c.sqft150=0)([q75]c.sqft150#c.sqft150=0)
       F(  7,    92) =   9.48
            Prob > F =    0.0000

// Multiv Wald test of difference in model between 25th and 75th percentile
test ([q25]c.new=[q75]c.new)([q25]c.bed3v2=[q75]c.bed3v2) ///
     ([q25]c.bed3v4=[q75]c.bed3v4)([q25]c.bath2v1=[q75]c.bath2v1) ///
     ([q25]c.bath2v3=[q75]c.bath2v3)([q25]c.sqft150=[q75]c.sqft150) ///
      ([q25]c.sqft150#c.sqft150=[q75]c.sqft150#c.sqft150)
       F(  7,    92) =    0.55
            Prob > F =    0.7918

// How to test single slope diff across quantiles
test ([q25]c.new=[q75]c.new)
       F(  1,    92) =    0.37
            Prob > F =    0.5460
```

For unknown reasons, the multivariate Wald test results continue to differ between SAS and STATA (beyond correcting for F vs. χ^2)

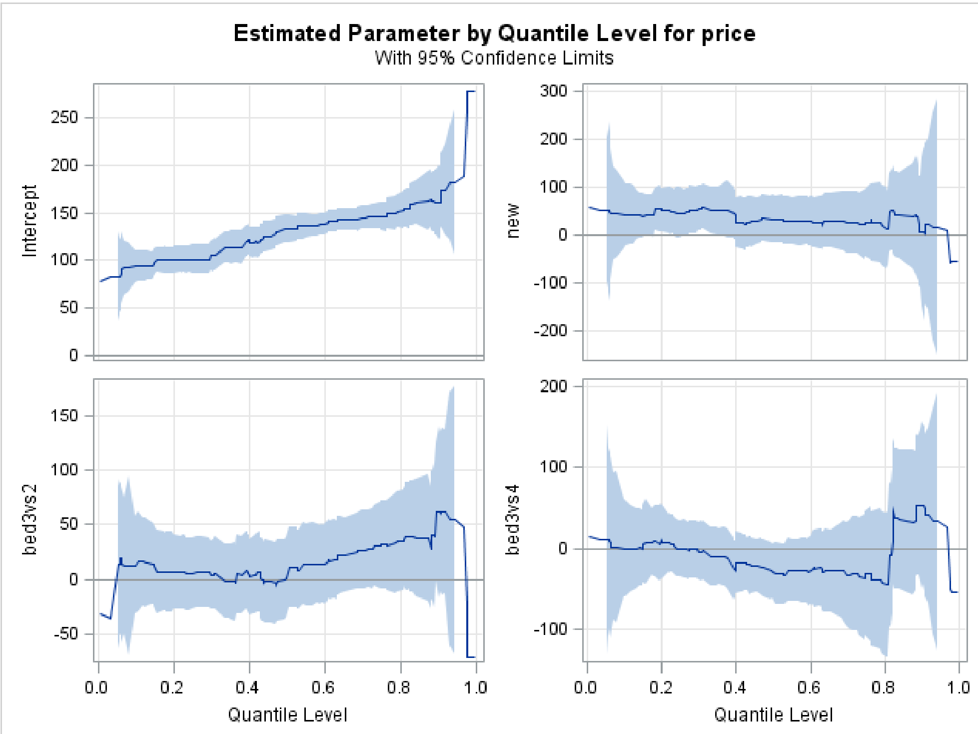
STATA Syntax and Output from IQREG—these are *differences* in predictor slopes between quantiles:

```
display "STATA Predict Price 25-75 Inter-Quantile Regression"
display "Output now directly provides predictor slope differences"
set seed 8675309 // Set Jenny as random seed to get same results each time
iqreg price c.new c.bed3v2 c.bed3v4 c.bath2v1 c.bath2v3 c.sqft150 ///
            c.sqft150#c.sqft150, quantile(.25 .75) reps(500) nolog

.75-.25 Interquantile regression                    Number of obs =        100
  bootstrap(500) SEs                                .75 Pseudo R2 =     0.5713
                                                    .25 Pseudo R2 =     0.3747
------------------------------------------------------------------------------
                    |              Bootstrap
              price |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
--------------------+---------------------------------------------------------
                new |  -21.28454   35.11913    -0.61   0.546    -91.03417    48.46509
             bed3v2 |   26.89456   21.05773     1.28   0.205    -14.92791    68.71703
             bed3v4 |  -31.46619   43.83957    -0.72   0.475    -118.5354    55.60297
            bath2v1 |  -14.31647   16.55987    -0.86   0.390     -47.2058    18.57287
            bath2v3 |  -3.655718   42.55953    -0.09   0.932    -88.18263    80.87119
            sqft150 |   1.435431   2.880917     0.50   0.619    -4.286319    7.157181
c.sqft150#c.sqft150 |   .2226272   .2837418     0.78   0.435    -.3409085    .7861628
              _cons |   44.62092   8.548936     5.22   0.000     27.64199    61.59984
------------------------------------------------------------------------------
test (c.new=0) (c.bed3v2=0) (c.bed3v4=0) (c.bath2v1=0) (c.bath2v3=0) ///
     (c.sqft150=0) (c.sqft150#c.sqft150=0) // Multiv Wald test of differences
       F(  7,    92) =    0.55
            Prob > F =    0.7918

print("R Predict Price 25th and 75th Percentile using Quantile Regression")
print("Did not figure out how to get same SEs and test statistics as SAS and STATA")
set.seed(8675309) # Jenny is my random seed
ModelQ2575 = rq(data=Example4b, tau=c(.25,.75),
               formula=price~1+new+bed3v2+bed3v4+bath2v1+bath2v3+sqft150+sqftsq)
summary(ModelQ2575)

tau: [1] 0.25

Coefficients:
            coefficients lower bd   upper bd
(Intercept) 101.114737    93.093346 113.687477 predicted 25th percentile for ref
new          45.673190    31.445800  92.285814
bed3v2        4.700000   -14.872686  33.256801
bed3v4       -0.220641   -27.352594  19.000892
bath2v1      -0.747755   -18.718106  20.884363
bath2v3       2.397843   -59.449552  37.667577
sqft150       9.404941     6.816233  10.564952
sqftsq        0.106858    -0.258119   0.405855

tau: [1] 0.75

Coefficients:
            coefficients lower bd   upper bd
(Intercept) 145.735654   141.481847 157.961905 predicted 75th percentile for ref
new          24.388649    -0.554536  92.452481
bed3v2       31.594557     4.661877  49.800555
bed3v4      -31.686826   -55.983707  78.477871
bath2v1     -15.064223   -28.281428   3.033738
bath2v3      -1.257882   -47.710414 107.001254
sqft150      10.840372     7.669831  16.773869
sqftsq        0.329485     0.124996   0.816528
```

5) Predict Price All Percentiles using Quantile Regression (couldn't find this in STATA or R):

```
TITLE "SAS Predict Price at All Percentiles using Quantile Regression";
PROC QUANTREG DATA=work.Example4b NAMELEN=100 CI=RESAMPLING(NREP=500);
     MODEL price = new bed3v2 bed3v4 bath2v1 bath2v3 sqft150 sqft150*sqft150
                            / QUANTILE=PROCESS PLOT=QUANTPLOT SEED=8675309;
RUN; TITLE;
```

SAS Output Graphical Summary (lots of voluminous output omitted; is Figure 1 in results section):

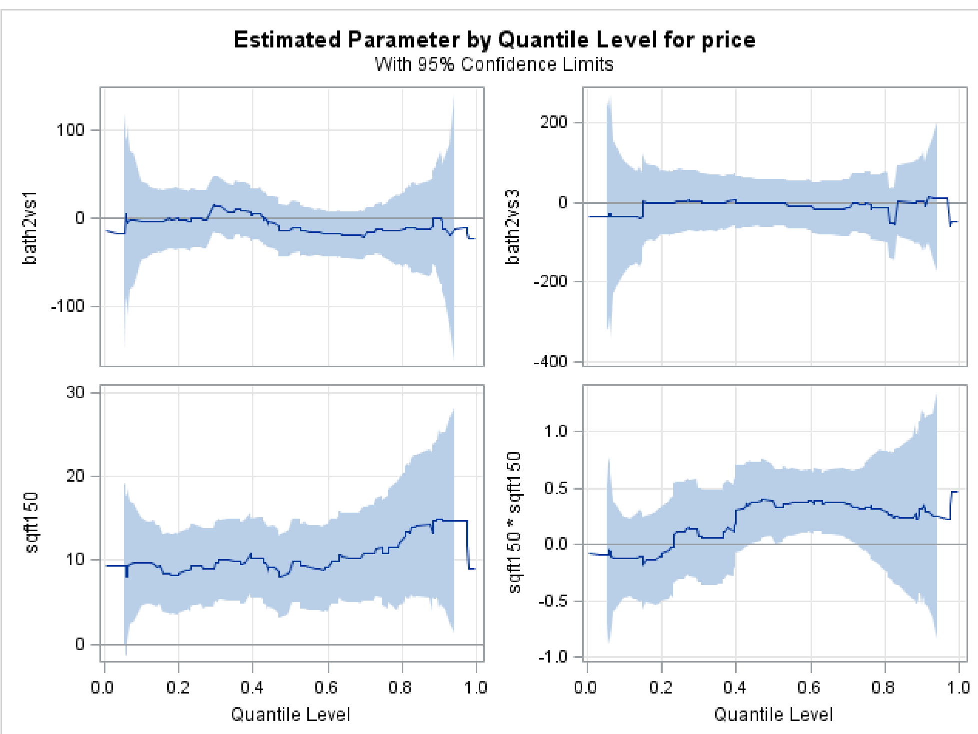


Top left: The intercept increases across percentiles (called "quantiles") as expected.

Top right: The slope for new construction stays just north of 0 until the 40th percentile or so.

Bottom left: The slope for 3 vs 2 bedrooms appears to not be different than 0 through most percentiles, although with an apparent increase in the upper quantiles (with lots of noise).

Bottom right: The slope for 3 vs 4 bedrooms appears to not be different than 0 through most of the percentiles, although with an apparent decrease in the upper percentiles (with lots of noise) until .80 or so, in which it suddenly jumps up to positive (with lots of noise)…?



Top left: The slope for bath 2 vs 1 is 0 with no trend across percentiles.

Top right: The slope for bath 2 vs 3 is 0 with no trend across percentiles.

Bottom left: The slope for the linear effect of square footage (which is the instantaneous slope at 1500 sq ft) is significantly positive across percentiles and looks to grow in strength after .60 or so.

Bottom right: The slope the quadratic effect of square footage is not different than 0 until about .50, at which point it is significantly positive (i.e., an accelerated effect of square footage). Although it stays positive, there is greater noise making it not different than 0 after .70 or so.

Sample results using SAS output:

The present analysis sought to predict the final sale price of 100 homes from four characteristics: whether they were new construction (0=no, 1=yes), linear and quadratic effects of square footage in 100s (centered at 1500), number of bedrooms (2,3, or 4+), and number of bathrooms (1,2, or 3+). Because the observed distribution of home sale prices was positively skewed and contained seven potential outliers, the robustness of the model results to these characteristics was examined using several distinct approaches. All models included the same predictor effects and were estimated using maximum likelihood within SAS GLIMMIX unless otherwise noted. The extent of conditional distribution fit was examined using the Pearson χ^2/DF statistic (in which 1=good fit); all predictor fixed effects were tested univariately using z-distributions without denominator degrees of freedom unless otherwise noted. As expected given the positively skewed distribution of sale prices, the residuals of a model specifying a normal conditional distribution indicated a lack of fit and several outliers.

We then examined two alternative models that were better suited for positively skewed residuals. First, we predicted home sale prices using a lognormal conditional distribution for the residuals, for which distribution fit is not readily available). In the lognormal solution, after controlling for the number of bedrooms and bathrooms, new houses sold for significantly more money (0.24 log $1000 units; *p* = .0499), and sale prices were also uniquely predicted by a quadratic function of square footage. More specifically, the sale price increased significantly by 0.08 log $1000 units per 100 additional square feet as evaluated at 1500 square feet (*p* < .001), but this positive slope of house size became significantly less positive by twice the quadratic coefficient of –0.001 per additional 100 square feet (i.e., the impact of being a bigger house was reduced in bigger houses; *p* = .023). The number of bedrooms or bathrooms did not have significant unique effects. Second, we fit the same predictive model using a log link function and a gamma conditional distribution, which showed evidence for underdispersion given its conditional distribution fit (Pearson χ^2/DF = 0.10). However, the effect of being new construction and the quadratic effect of house size were then nonsignificant (*p*'s ≈ .07).

We then turned to a different modeling approach that would be more robust to outliers—quantile regression, in which one can predict any percentile of the distribution (labeled a "quantile") instead of the mean as in traditional regression. In our quantile regressions, the point estimates for the predictor slopes were found by minimizing a weighted function of the absolute value of the model residuals (in which the weights reflect the chosen percentile). Standard errors were found through 500 bootstrap replications (i.e., in which 500 samples with replacement were generated to capture the empirical sampling distribution of the slope estimates for more valid standard errors). SAS QUANTREG was used to conduct the analyses, and residual denominator degrees of freedom were used to evaluate the significance of the model predictors.

First, in predicting the 50^{th} percentile (i.e., the median home price), no unique predictor effects were significant except square footage, for which significant positive linear and quadratic effects were found. More specifically, the sale price increased by 8.66 $1000 units per 100 additional square feet as evaluated at 1500 square feet (*p* < .001), and this positive slope of house size became significantly more positive by twice the quadratic coefficient of 0.38 per additional 100 square feet (i.e., the price bonus of being a bigger house was magnified in bigger houses; *p* = .023). We repeated this analysis to predict the 25^{th} and 75^{th} percentiles to examine potential differences in prediction for relatively inexpensive or relatively expensive houses, respectively. At the 25^{th} percentile, there was a marginally significant positive effect of new construction (Est = 45.67, *p* = .087), a significant linear effect of house size at 1500 square feet (Est = 9.40 per 100 square feet; *p* < .001), but no significant quadratic effect of house size (Est = 0.107, *p* = .633). At the 75^{th} percentile, there was a nonsignificant effect of new construction (Est = 24.29, *p* = .495), a significant linear effect of house size at 1500 square feet (Est = 10.84 per 100 square feet; *p* = .001), but no significant quadratic effect of house size (Est = 0.33, *p* = .106). Finally, Figure 1 provides the results in examining prediction at 144 distinct values ranging from the 0.004^{th} to 99.6^{th} percentiles, in which the solid line in each image depicts the point estimate for the slope (y-axis) as a function of the percentile (x-axis), and the shading conveys the 95% confidence interval around the slope estimates. The unique effects of number of bedrooms and number of bathrooms did not appear to be significant at any percentile. The effect of new construction appeared marginally significantly positive from approximately the 20^{th} to the 40^{th} percentiles, and nonsignificantly positive otherwise. The linear effect of house size at 1500 square feet was significantly positive at nearly every percentile and appeared to grow in size as home prices increased. The quadratic effect of house size appeared to transition from nonsignificantly negative until the 20^{th} percentile, to nonsignificantly positive until the 40^{th} percentile, to significantly positive until the 70^{th} percentile, after which it remained nonsignificantly positive. Thus, it appears that having a bigger house is even more helpful among midrange houses, but not for inexpensive or very expensive houses.