Example 2a: Predicting Binary Outcomes via STATA LOGIT, R GLM, and SAS GLIMMIX
(complete syntax, data, and output available for STATA, R, and SAS electronically))

The (fake) data for this example demonstrating "logistic regression" (i.e., using a logit link function and Bernoulli conditional response distribution) came from: https://stats.idre.ucla.edu/sas/dae/ordinal-logistic-regression/. In this example we will predict a student's **binary decision** of how likely they are to apply to grad school (0=no, >0=pry) using undergraduate GPA (centered at 3.0), whether at least one of their parents has a graduate degree (0=no, 1=yes), and whether they attended a private university (0=no, 1=yes). I am using STATA LOGIT (along with a user-created command FITSTAT) and R GLM, which do not use denominator degrees of freedom. Consequently, single slopes will be tested using univariate Wald tests (i.e., the z-tests as given directly in the program output), and sets of slopes will be tested using likelihood ratio tests (the difference in −2LL between models, a more general approach that should be better in small samples, and that also gives more consistent results across packages than do multivariate Wald tests). A version of the last model using a probit link function instead is available in the electronic materials. (Syntax and output for SAS GLIMMIX is available in the electronic materials, which I used because it has more helpful options even though these are not mixed-effects models.)

<u>STATA</u> Syntax for Importing and Preparing Data for Analysis:

```
// Defining global variable for file location to be replaced in code below
// \\Client\ precedes path in Virtual Desktop outside H drive
global filesave "C:\Dropbox\23_PSQF6270\PSQF6270_Example2ab"

// Open Example 2 STATA dataset and clear away any existing data
use "$filesave\STATA_Example2.dta", clear // Has converted all variables to lower-case

// Create ID variable
gen PersonID = _n
// Rename ordinal outcome to distinguish as 3-category version
gen apply3=apply

// Create new binary outcome JUST to demonstrate logistic regression
gen apply2=.  // New empty variable
replace apply2=0 if apply==0
replace apply2=1 if apply>0

// Rename and center predictors
gen parD=pared
gen gpa3=gpa-3

// Recode ref for public to create positive slope
gen priv=.
replace priv=0 if public==1
replace priv=1 if public==0

// Label variables
label variable apply3 "apply3: 0=Not, 1=Eh, 2=Very"
label variable apply2 "apply2: 0=No, 1=Pry"
label variable parD   "parD: Parent Has Graduate Degree (0=N,1=Y)"
label variable priv   "priv: Student Attends Private University (0=N,1=Y)"
label variable gpa3   "gpa3: Student GPA (0=3)"

// Filter to only cases complete on all variables to be used below
egen nmiss=rowmiss(apply3 parD priv gpa3)
drop if nmiss>0
```

R Syntax for Importing and Preparing Data for Analysis (after loading packages *readxl*, *TeachingDemos*, *psych*, *multcomp*, *prediction*, and *DescTools* as shown online):

```
# Define variables for working directory and data name
filesave = "C:\\Dropbox/23_PSQF6270/PSQF6270_Example2ab/"
filename = "Excel_Example2.xlsx"
setwd(dir=filesave)

# Import Example2a and Example2b Excel data
Example2 = read_excel(path=paste0(filesave,filename))
# Convert to data frame without labels to use for analysis
Example2 = as.data.frame(Example2)

# Create ID variable
Example2$PersonID <- seq.int(nrow(Example2))

# Rename ordinal outcome to distinguish as 3-category version
Example2$apply3=Example2$apply

# Create new binary outcome JUST to demonstrate logistic regression
Example2$apply2 = NA  # New empty variable
Example2$apply2[which(Example2$apply==0)]=0
Example2$apply2[which(Example2$apply>0)]=1

# Rename and center predictors
Example2$parD=Example2$pared
Example2$gpa3=Example2$gpa-3

# Recode ref for public to create positive slope
Example2$priv = NA  # New empty variable
Example2$priv[which(Example2$public==1)]=0
Example2$priv[which(Example2$public==0)]=1

# Label variables as comments only (not actually added to data)
#apply3= "apply3: 0=Not, 1=Eh, 2=Very"
#apply2= "apply2: 0=No, 1=Pry"
#parD=   "parD: Parent Has Graduate Degree (0=N,1=Y)"
#priv=   "priv: Student Attends Private University (0=N,1=Y)"
#gpa3=   "gpa3: Student GPA (0=3)"

# Filter to only cases complete on all variables to be used below
Example2 = Example2[complete.cases(Example2[ ,5:9]),]
```

Syntax and STATA Output for Descriptive Statistics:

```
display "STATA Descriptive Statistics"
summarize gpa3 parD priv apply2

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
        gpa3 |        400    -.001075    .3979409       -1.1          1
        parD |        400       .1575    .3647277          0          1
        priv |        400       .8575         .35          0          1
      apply2 |        400         .45    .4981168          0          1

print("R Descriptive Statistics")
describe(x=Example2[ , c("gpa3","parD","priv","apply2")])
```

So now we know that **45% of the respondents have apply2=1, and 55% have apply2=0**. This information will come in handy in making sure we understand which value our logistic regression models are predicting!

STATA Syntax and Partial Output for an Empty Model:

$$Log\left(\frac{Apply2_i=1}{Apply2_i=0}\right) = Logit(Apply2_i = 1) = \beta_0 \quad \rightarrow \quad Probability(Apply2_i = 1) = \frac{exp(\beta_0)}{1+exp(\beta_0)}$$

```
display "STATA Empty Model Predicting Binary Apply2"
logit apply2, nolog

Logistic regression                             Number of obs     =        400
                                                LR chi2(0)        =       0.00 → LRT for MODEL
                                                Prob > chi2       =          .
Log likelihood = -275.25553                     Pseudo R2         =     0.0000

------------------------------------------------------------------------------
      apply2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |  -.2006707   .1005038    -2.00   0.046    -.3976545   -.0036869 → Beta0 in logits!
------------------------------------------------------------------------------
display "-2LL= " e(ll)*-2  // Print -2LL for model

-2LL= 550.51105

estat ic, n(400)           // AIC and BIC using N=400

Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |        400 -275.2555  -275.2555       1    552.5111   556.5025
-----------------------------------------------------------------------------
               Note: N=400 used in calculating BIC.

margins                    // Intercept in probability

Expression   : Pr(apply2), predict()
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |        .45   .0248747    18.09   0.000     .4012465    .4987535 → Beta0 in probability!
------------------------------------------------------------------------------
```

STATA gives LL (so you need to *−2)

Probability of $(Apply2_i = 1) = \frac{exp(-0.2007)}{[1+exp(-0.2007)]} = 0.450$

R Syntax and Partial Output for an Empty Model:

$$Log\left(\frac{Apply2_i=1}{Apply2_i=0}\right) = Logit(Apply2_i = 1) = \beta_0 \quad \rightarrow \quad Probability(Apply2_i = 1) = \frac{exp(\beta_0)}{1+exp(\beta_0)}$$

```
print("R Empty Model Predicting Binary Apply2")
Model2Empty = glm(data=Example2, family=binomial(link="logit"), formula=apply2~1)
summary(Model2Empty);

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.20067    0.10050 -1.9967  0.04586 → Beta0 in logits!

    Null deviance: 550.511  on 399  degrees of freedom → Is always empty model -2LL
Residual deviance: 550.511  on 399  degrees of freedom → Is always current model -2LL

AIC: 552.511

BIC(Model2Empty) # Get BIC too

[1] 556.50252

print("Convert logits to probability to check interpretation")
Model2EmptyProb=1/(1+exp(-1*coefficients(Model2Empty))); Model2EmptyProb

       0.45 → Beta0 in probability!
```

STATA Syntax and Partial Output for a Main-Effects Model with 3 Predictors:

$$Logit(Apply2_i = 1) = \beta_0 + \beta_1(GPA_i - 3) + \beta_2(ParentGD_i) + \beta_3(Private_i)$$

This likelihood ratio test (LRT) is analogous to an F-test for the model R^2 in general linear models.

```
display "STATA Main-Effects Model Predicting Binary Apply2"
logit apply2 c.gpa3 c.parD c.priv, nolog
estimates store Main    // Save for next LRT

Logistic regression                               Number of obs     =        400
                                                  LR chi2(3)        =      20.59 → LRT for MODEL
                                                  Prob > chi2       =     0.0001
Log likelihood =  -264.9624 * -2 = -2LL           Pseudo R2         =     0.0374 → McFadden's R2
------------------------------------------------------------------------------
      apply2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        gpa3 |   .5482457   .2724341     2.01   0.044     .0142846    1.082207  Beta1
        parD |   1.059612   .2973854     3.56   0.000     .4767471    1.642476  Beta2
        priv |   .2005571   .3053354     0.66   0.511    -.3978894    .7990035  Beta3
       _cons |  -.5387909    .287416    -1.87   0.061    -1.102116    .0245341  Beta0
------------------------------------------------------------------------------
```

Interpret each fixed effect…

Intercept:

GPA3:

parentGD:

private:

```
display "-2LL= " e(ll)*-2  // Print -2LL for model

-2LL= 529.92481

estat ic, n(400)           // AIC and BIC using N=400

Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |        400 -275.2555  -264.9624       4    537.9248   553.8907
-----------------------------------------------------------------------------
               Note: N=400 used in calculating BIC.

fitstat                    // Additional R2 and fit stats (user-defined function)

Measures of Fit for logit of apply2
Log-Lik Intercept Only:      -275.256     Log-Lik Full Model:          -264.962
D(396):                       529.925     LR(3):                         20.586
                                          Prob > LR:                      0.000
McFadden's R2:                  0.037     McFadden's Adj R2:              0.023
Maximum Likelihood R2:          0.050     Cragg & Uhler's R2:             0.067
McKelvey and Zavoina's R2:      0.063     Efron's R2:                     0.051
Variance of y*:                 3.512     Variance of error:              3.290 → pi^2/3 because logit
Count R2:                       0.608     Adj Count R2:                   0.128
AIC:                            1.345     AIC*n:                        537.925
BIC:                        -1842.695     BIC':                          -2.612
```

Look at how many flavors of Pseudo-R^2 there are! If you choose to use one, make sure to specify which one, how it's computed, and be prepared to defend why you chose that one (in case Reviewer 2 prefers a different one).

```
// For at, (from(by)to) for range of predictors
margins, at(c.gpa3=(-1(1)1) c.parD=0 c.priv=0) predict(xb) // Example yhat in logits
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         _at |
          1  |  -1.087037   .4312041    -2.52   0.012    -1.932181    -.241892
          2  |  -.5387909    .287416    -1.87   0.061    -1.102116    .0245341 → Diff = .5483
          3  |   .0094548   .3573788     0.03   0.979    -.6909948    .7099044 → Diff = .5483
------------------------------------------------------------------------------
```

The difference in the predicted logit outcome for each unit of GPA is $\beta_1 = .548$, which is a **linear** slope in predicting the **logit** outcome.

```
margins, at(c.gpa3=(-1(1)1) c.parD=0 c.priv=0) // Example yhat in probability
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   .2521767    .081318     3.10   0.002     .0927963    .4115571
          2  |   .3684689   .0668816     5.51   0.000     .2373834    .4995544 → Diff = .1163
          3  |   .5023637   .0893427     5.62   0.000     .3272552    .6774722 → Diff = .1339
------------------------------------------------------------------------------
```

If I convert the logit slope = .548 into a probability, I get .389, but that is NOT the expected change in probability per unit GPA!

The difference in the predicted probability outcome for each unit of GPA is NOT constant. This is why **you cannot "unlogit" a slope to compute a slope for change in probability**—it doesn't make sense! The effect of a predictor on probability depends on depends where you are on the probability scale (biggest impact is near probability = .50).

```
// Must re-estimate with 'or' added to first line to get odds ratios
display "STATA Main-Effects Model Predicting Binary Apply2"
display "Get Odds Ratios Instead of Logit Fixed Effects"
logit apply2 c.gpa3 c.parD c.priv, or nolog

------------------------------------------------------------------------------
      apply2 | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        gpa3 |   1.730215   .4713696     2.01   0.044     1.014387    2.951185  exp(Beta1)
        parD |   2.885251   .8580314     3.56   0.000     1.610826    5.167952  exp(Beta2)
        priv |   1.222083   .3731454     0.66   0.511     .6717363    2.223324  exp(Beta3)
       _cons |   .5834533   .1676938    -1.87   0.061     .3321675    1.024838  exp(Beta0)
------------------------------------------------------------------------------
```

R Syntax and Partial Output for a Main-Effects Model with 3 Predictors:

$$Logit(Apply2_i = 1) = \beta_0 + \beta_1(GPA_i - 3) + \beta_2(ParentGD_i) + \beta_3(Private_i)$$

```
print("R Main-Effects Model Predicting Binary Apply2")
Model2Main = glm(data=Example2, family=binomial(link="logit"),
                 formula=apply2~1+gpa3+parD+priv); summary(Model2Main);

Coefficients:
            Estimate Std. Error z value  Pr(>|z|)
(Intercept) -0.53879    0.28742 -1.8746 0.0608473  Beta0
gpa3         0.54825    0.27243  2.0124 0.0441780  Beta1
parD         1.05961    0.29739  3.5631 0.0003665  Beta2
priv         0.20056    0.30534  0.6568 0.5112826  Beta3

    Null deviance: 550.511  on 399  degrees of freedom → Is always empty model -2LL
Residual deviance: 529.925  on 396  degrees of freedom → Is always current model -2LL

AIC: 537.925

BIC(Model2Main) # Get BIC too
[1] 553.89066
```

```
confint.default(Model2Main) # Get 95% CIs for logit parameters
                  2.5 %      97.5 %
(Intercept) -1.102115790 0.024534011
gpa3         0.014284691 1.082206667
parD         0.476747275 1.642476234
priv        -0.397889291 0.799003465

print("Likelihood Ratio Test of Predictors (analogous to F-test for model R2 in general LM")
anova(Model2Empty, Model2Main, test="LRT") # Nested "fewer" model goes first

Analysis of Deviance Table
Model 1: apply2 ~ 1
Model 2: apply2 ~ 1 + gpa3 + parD + priv

  Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
1       399    550.511
2       396    529.925  3  20.5862 0.0001283
```

"Deviance" is the difference in −2LL between nested models, which is treated as a χ^2 test statistic with degrees of freedom equal to the number of new parameters (3 fixed slopes here).

It's written like this: $-2\Delta LL(3) = 20.59, p < .001$
or like this: $\chi^2(3) = 20.59, p < .001$

```
print("Pseudo-R2 values"); PseudoR2(x=Model2Main, which="all")

       McFadden     McFaddenAdj        CoxSnell      Nagelkerke   AldrichNelson
    0.037394791     0.022862839     0.050163690     0.067110121     0.048946550
VeallZimmermann           Efron McKelveyZavoina            Tjur             AIC
    0.084510995     0.050966133     0.063047115     0.050877331   537.924805234
            BIC          logLik         logLik0              G2
  553.890663422 -264.962402617 -275.255525485    20.586245737
```

Look at how many flavors of Pseudo-R^2 there are! If you choose to use one, make sure to specify which one, how it's computed, and be prepared to defend why you chose that one (in case Reviewer 2 prefers a different one).

```
print("Odds ratios with 95% CIs using standard errors")
OddsRatio(x=Model2Main, conf.level=.95, digits=5, use.profile=FALSE)
```

```
# If the OddsRatio function
doesn't work, use this instead
exp(cbind(OR=coef(Model2Main),
    confint(Model2Main)))
```

```
Odds Ratios:
                 or  or.lci  or.uci Pr(>|z|)
(Intercept) 0.58345 0.33217 1.02484   0.0608 exp(Beta0)
gpa3        1.73022 1.01439 2.95118   0.0442 exp(Beta1)
parD        2.88525 1.61083 5.16795 3.67e-04 exp(Beta2)
priv        1.22208 0.67174 2.22332   0.5113 exp(Beta3)

print("Example yhat in logits for specific values of predictors")
Main2Logits = prediction(model=Model2Main, type="link",
                         at=list(gpa3=-1:1, parD=0, priv=0));
summary(Main2Logits)

at(gpa3) at(parD) at(priv) Prediction     SE        z         p   lower    upper
      -1        0        0  -1.087037 0.4312 -2.52093 0.0117044 -1.9322 -0.24189
       0        0        0  -0.538791 0.2874 -1.87460 0.0608473 -1.1021  0.02453 → Diff = .5483
       1        0        0   0.009455 0.3574  0.02646 0.9788937 -0.6910  0.70990 → Diff = .5483
```

The difference in the predicted logit outcome for each unit of GPA is $\beta_1 = .548$, which is a linear slope in predicting the logit outcome.

If I convert the logit slope = .548 into a probability, I get .389, but that is NOT the expected change in probability per unit GPA!

```
print("Example yhat in probability for specific values of predictors")
Main2Probs = prediction(model=Model2Main, type="response",
                        at=list(gpa3=-1:1, parD=0, priv=0));
summary(Main2Probs)

 at(gpa3) at(parD) at(priv) Prediction      SE     z         p  lower  upper
       -1        0        0     0.2522 0.08132 3.101 1.928e-03 0.0928 0.4116
        0        0        0     0.3685 0.06688 5.509 3.603e-08 0.2374 0.4996 → Diff = .1163
        1        0        0     0.5024 0.08934 5.623 1.878e-08 0.3273 0.6775 → Diff = .1339
```

The difference in the predicted probability outcome for each unit of GPA is NOT constant. This is why **you cannot "unlogit" a slope to compute a slope for change in probability**—it doesn't make sense! The effect of a predictor on probability depends on depends where you are on the probability scale (bigger impact near probability = .50).

STATA Syntax and Partial Output adding 2 New Interactions:

$$Logit(Apply2_i = 1) = \beta_0 + \beta_1(GPA_i - 3) + \beta_2(ParentGD_i) + \beta_3(Private_i) + \boldsymbol{\beta_4(GPA_i - 3)(ParentGD_i) + \beta_5(GPA_i - 3)(Private_i)}$$

Model-implied GPA Slope: $\beta_1 + \beta_4(ParentGD_i) + \beta_5(Private_i)$

This likelihood ratio test (LRT) is analogous to an F-test for the model R^2 in general linear models. To assess the contribution of new predictors (analogous to an F-test for the change in R^2) we must do our own LRT (see below).

```
display "STATA Interaction Model Predicting Binary Apply2"
logit apply2 c.gpa3 c.parD c.priv c.gpa3#c.parD c.gpa3#c.priv, nolog
estimates store Interact    // Save for this LRT
display "-2LL= " e(ll)*-2   // Print -2LL for model
estat ic, n(400)            // AIC and BIC using N=400

Logistic regression                             Number of obs     =
                                                LR chi2(5)        =     22.34 → Is LRT for MODEL
                                                Prob > chi2       =    0.0005
Log likelihood = -264.08684 * -2 = -2LL         Pseudo R2         =    0.0406
------------------------------------------------------------------------------
      apply2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        gpa3 |    1.25645   .7736689     1.62   0.104    -.2599127    2.772814  Beta1
        parD |   1.162334   .3196688     3.64   0.000     .5357943    1.788873  Beta2
        priv |   .3197742    .351815     0.91   0.363    -.3697705    1.009319  Beta3
c.gpa3#c.parD |  -.8358888   .7695577    -1.09   0.277    -2.344194    .6724166  Beta4
c.gpa3#c.priv |  -.6821162   .8077466    -0.84   0.398    -2.265271    .9010381  Beta5
       _cons |  -.6594466   .3373972    -1.95   0.051    -1.320733    .0018397  Beta0
------------------------------------------------------------------------------

lrtest Interact Main        // LRT for two new interactions ("fewer" model goes LAST)

Likelihood-ratio test                                 LR chi2(2)  =      1.75
(Assumption: Main nested in Interact)                 Prob > chi2 =    0.4166
```

The two new interaction slopes do not significantly improve the model prediction, $-2\Delta LL(2) = 1.75, p = .417$.

```
// For at, (from(by)to) for range of predictors
margins, at(c.gpa3=(-1(1)1) c.parD=(0(1)1) c.priv=(0(1)1)) predict(xb) // Yhat in logits
margins, at(c.gpa3=(-1(1)1) c.parD=(0(1)1) c.priv=(0(1)1))              // Yhat in probability
lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*0  // GPA slope: Ndeg Pub = B1
lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*1  // GPA slope: Ndeg Pri = B1+B5
lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*0  // GPA slope: Ydeg Pub = B1+B4
lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*1  // GPA slope: Ydeg Pri = B1+B4+B5
```

The rest of the (very long) STATA output for this model is available electronically…

```
// Must re-estimate with 'or' added to first line to get odds ratios
display "STATA Interaction Model Predicting Binary Apply2"
display "Get Odds Ratios Instead of Logit Fixed Effects"
logit apply2 c.gpa3 c.parD c.priv c.gpa3#c.parD c.gpa3#c.priv, or
lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*0, or  // GPA slope: Ndeg Pub = B1
lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*1, or  // GPA slope: Ndeg Pri = B1+B5
lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*0, or  // GPA slope: Ydeg Pub = B1+B4
lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*1, or  // GPA slope: Ydeg Pri = B1+B4+B5

------------------------------------------------------------------------------
      apply2 | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        gpa3 |    3.51293   2.717845     1.62   0.104     .7711189     16.0036  exp(Beta1)
        parD |   3.197386   1.022105     3.64   0.000     1.708805    5.982706  exp(Beta2)
        priv |   1.376817   .4843848     0.91   0.363     .6908928    2.743732  exp(Beta3)
c.gpa3#c.parD |    .433489   .3335948    -1.09   0.277     .0959245    1.958966  exp(Beta4)
c.gpa3#c.priv |    .505546   .4083531    -0.84   0.398     .1038019    2.462158  exp(Beta5)
       _cons |   .5171374   .1744807    -1.95   0.051     .2669396    1.001841  exp(Beta0)
------------------------------------------------------------------------------
```

R Syntax and Partial Output adding 2 New Interactions:

$$Logit(Apply2_i = 1) = \beta_0 + \beta_1(GPA_i - 3) + \beta_2(ParentGD_i) + \beta_3(Private_i)$$
$$+\boldsymbol{\beta_4(GPA_i - 3)(ParentGD_i)} + \boldsymbol{\beta_5(GPA_i - 3)(Private_i)}$$

```
print("R Interaction Model Predicting Binary Apply2")
Model2Int = glm(data=Example2, family=binomial(link="logit"),
                formula=apply2~1+gpa3+parD+priv+gpa3:parD+gpa3:priv)
summary(Model2Int); BIC(Model2Int) # Get BIC too
confint.default(Model2Int) # Get 95% CIs for logit parameters (not shown here)
```

```
Coefficients:
            Estimate Std. Error z value  Pr(>|z|)
(Intercept) -0.65945    0.33740 -1.9545 0.0506406  Beta0
gpa3         1.25645    0.77367  1.6240 0.1043722  Beta1
parD         1.16233    0.31967  3.6361 0.0002768  Beta2
priv         0.31977    0.35181  0.9089 0.3633882  Beta3
gpa3:parD   -0.83589    0.76956 -1.0862 0.2773930  Beta4
gpa3:priv   -0.68212    0.80775 -0.8445 0.3984075  Beta5

    Null deviance: 550.511  on 399  degrees of freedom → Is empty model -2LL
Residual deviance: 528.174  on 394  degrees of freedom → Is current model -2LL

AIC: 540.174
BIC: 564.12247
```

Interpret each simple effect and interaction…

GPA3:

parentGD:

private:

GPA3*parentGD:

GPA3*private:

Model-implied GPA Slope: $\beta_1 + \beta_4(ParentGD_i) + \beta_5(Private_i)$

```
print("Simple slopes for GPA by moderators")
Int2Slopes = (summary(glht(model=Model2Int, linfct=rbind(
        "GPA Slope: Ndeg Pub = B1      " = c(0,1,0,0,0,0),  # in order of fixed effects
        "GPA Slope: Ndeg Pri = B1+B5   " = c(0,1,0,0,0,1),
        "GPA Slope: Ydeg Pub = B1+B4   " = c(0,1,0,0,1,0),
        "GPA Slope: Ydeg Pri = B1+B4+B5" = c(0,1,0,0,1,1))),test=adjusted("none")))
Int2Slopes
```

```
Linear Hypotheses:
                                    Estimate Std. Error z value Pr(>|z|)
GPA Slope: Ndeg Pub = B1       == 0  1.25645    0.77367  1.6240  0.10437
GPA Slope: Ndeg Pri = B1+B5    == 0  0.57433    0.30900  1.8587  0.06307
GPA Slope: Ydeg Pub = B1+B4    == 0  0.42056    0.94762  0.4438  0.65718
GPA Slope: Ydeg Pri = B1+B4+B5 == 0 -0.26155    0.73545 -0.3556  0.72211
(Adjusted p values reported -- none method)
```

```
print("Likelihood Ratio Test of New Interactions (analogous to F-test for R2 change in general LM")
anova(Model2Main, Model2Int, test="LRT") # Nested "fewer" model goes first

Analysis of Deviance Table
Model 1: apply2 ~ 1 + gpa3 + parD + priv
Model 2: apply2 ~ 1 + gpa3 + parD + priv + gpa3:parD + gpa3:priv

  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
1       396    529.925
2       394    528.174  2  1.75113  0.41663
```

The 2 new interaction slopes do not significantly improve the model prediction, $-2\Delta LL(2) = 1.75, p = .417$.

```
# If the OddsRatio function
doesn't work, use this instead
exp(cbind(OR=coef(Model2Int),
    confint(Model2Int)))
```

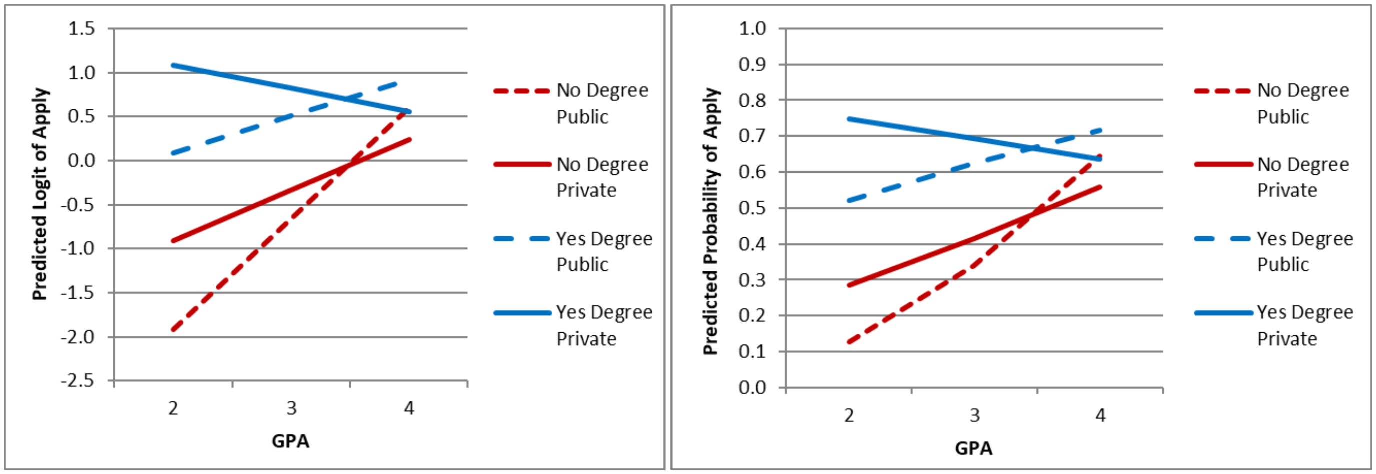
```
print("Odds ratios with 95% CIs using standard errors")
OddsRatio(x=Model2Int, conf.level=.95, digits=5, use.profile=FALSE)

Odds Ratios:
                 or   or.lci   or.uci Pr(>|z|)
(Intercept) 0.51714 0.26694  1.00184   0.0506  exp(Beta0)
gpa3        3.51293 0.77112 16.00359   0.1044  exp(Beta1)
parD        3.19739 1.70881  5.98270 2.77e-04  exp(Beta2)
priv        1.37682 0.69089  2.74373   0.3634  exp(Beta3)
gpa3:parD   0.43349 0.09592  1.95896   0.2774  exp(Beta4)
gpa3:priv   0.50555 0.10380  2.46216   0.3984  exp(Beta5)

print("Odds ratios for simple slopes using GLHT output")
data.frame(OR=exp(Int2Slopes$test$coefficients))
                                     OR
GPA Slope: Ndeg Pub = B1       3.51293093
GPA Slope: Ndeg Pri = B1+B5    1.77594769
GPA Slope: Ydeg Pub = B1+B4    1.52281717
GPA Slope: Ydeg Pri = B1+B4+B5 0.76985392

print("Yhat in logits for specific values of predictors")
Int2Logits = prediction(model=Model2Int, type="link",
                        at=list(gpa3=-1:1,parD=0:1,priv=0:1))
summary(Int2Logits)

at(gpa3) at(parD) at(priv) Prediction     SE        z        p   lower     upper
      -1        0        0   -1.91590 0.9908 -1.93362 0.053160 -3.8579  0.026104
       0        0        0   -0.65945 0.3374 -1.95451 0.050641 -1.3207  0.001839
       1        0        0    0.59700 0.6656  0.89692 0.369762 -0.7076  1.901588
      -1        1        0    0.08233 1.2300  0.06693 0.946638 -2.3285  2.493135
       0        1        0    0.50289 0.4289  1.17249 0.241000 -0.3378  1.343526
       1        1        0    0.92345 0.8068  1.14459 0.252377 -0.6578  2.504730
      -1        0        1   -0.91401 0.3194 -2.86135 0.004218 -1.5401 -0.287932
       0        0        1   -0.33967 0.1187 -2.86067 0.004227 -0.5724 -0.106949
       1        0        1    0.23466 0.3422  0.68567 0.492921 -0.4361  0.905434
      -1        1        1    1.08422 0.8967  1.20917 0.226598 -0.6732  2.841641
       0        1        1    0.82266 0.3034  2.71185 0.006691  0.2281  1.417232
       1        1        1    0.56111 0.6796  0.82566 0.408995 -0.7709  1.893064

print("Yhat in probability for specific values of predictors")
Int2Probs = prediction(model=Model2Int, type="response",
                       at=list(gpa3=-1:1,parD=0:1,priv=0:1))
summary(Int2Probs)
```

For comparison, the last column shows the predicted probabilities using a probit link instead of logit:

```
at(gpa3) at(parD) at(priv) Prediction      SE      z         p    lower  upper     Prediction
      -1        0        0     0.1283 0.11083  1.158 2.469e-01 -0.08890 0.3455         0.1250
       0        0        0     0.3409 0.07580  4.497 6.905e-06  0.19229 0.4894         0.3446
       1        0        0     0.6450 0.15242  4.232 2.320e-05  0.34624 0.9437         0.6370
      -1        1        0     0.5206 0.30699  1.696 8.993e-02 -0.08111 1.1223         0.5338
       0        1        0     0.6231 0.10072  6.187 6.145e-10  0.42572 0.8206         0.6256
       1        1        0     0.7157 0.16415  4.360 1.298e-05  0.39403 1.0375         0.7108
      -1        0        1     0.2862 0.06525  4.386 1.156e-05  0.15829 0.4141         0.2832
       0        0        1     0.4159 0.02884 14.418 3.972e-47  0.35935 0.4724         0.4162
       1        0        1     0.5584 0.08439  6.617 3.673e-11  0.39299 0.7238         0.5598
      -1        1        1     0.7473 0.16933  4.413 1.019e-05  0.41541 1.0792         0.7459
       0        1        1     0.6948 0.06433 10.801 3.407e-27  0.56872 0.8209         0.6945
       1        1        1     0.6367 0.15719  4.050 5.112e-05  0.32861 0.9448         0.6390
```

Illustrating these simple (conditional) slopes for GPA at each combination of degree and school type:

```
Linear Hypotheses:
                                    Estimate Std. Error z value Pr(>|z|)
GPA Slope: Ndeg Pub = B1       == 0  1.25645    0.77367  1.6240  0.10437
GPA Slope: Ndeg Pri = B1+B5    == 0  0.57433    0.30900  1.8587  0.06307
GPA Slope: Ydeg Pub = B1+B4    == 0  0.42056    0.94762  0.4438  0.65718
GPA Slope: Ydeg Pri = B1+B4+B5 == 0 -0.26155    0.73545 -0.3556  0.72211
(Adjusted p values reported -- none method)
```



The **model provides direct tests of the differences in logits** amongst the degree and school conditions, as well as for the simple slopes of GPA for each degree and school type. Model-predicted logit outcomes can then be converted through an inverse link (the "un-logit" back-transformation) into predicted probabilities for ease of interpretation, but the **slopes or mean differences themselves cannot be converted in differences in probabilities, only odds ratios**.

Sample results section:

We examined the extent to which a binary decision to apply to graduate school (55% 0=No, 45% 1=Pry) could be predicted by a student's undergraduate GPA (M = 3.00, SD = 0.40, range = 1.90 to 4.00), whether at least one of their parents has a graduate degree (15.75% 0=No, 84.25% 1=Yes), and whether they attended a private university (14.25% 0=No, 85.75% 1=Yes). Specifically, we estimated generalized linear models using maximum likelihood, in which the conditional probability of applying to graduate school was predicted using a logit link function and a conditional Bernoulli distribution (i.e., logistic regressions). The GPA predictor was centered such that 0 indicated a GPA = 3. Effect sizes are provided using odds ratios (OR), in which OR values between 0 and 1 indicate negative effects, 1 indicates no effect, and values above 1 indicate positive effects. Nested model comparisons were conducted using likelihood ratio tests (i.e., the difference in −2LL between nested models with degrees of freedom equal to the number of new parameters).

The first model examined only the main effects of the three predictors, which together resulted in a significant model, $-2\Delta LL(3) = 20.59$, $p < .001$. GPA had a significantly positive effect, such that for every unit greater GPA, the logit of applying to graduate school was greater by 0.548 (SE = 0.272; OR = 1.730). Likewise, the logit of applying to graduate school was significantly greater for students for whom at least one parent had a graduate degree by 1.060 (SE = 0.297, OR = 2.882). However, the logit of applying to graduate school was nonsignificantly greater for students who attended a private university by 0.201 (SE = 0.305, OR = 1.222).

The second model then included two-way interactions of GPA with parent graduate degree and GPA with university type. This augmented model was not a significant improvement over the main effects model, $-2\Delta LL(2) = 1.75$, $p = .417$. Neither individual interaction term was significant, nor was the simple slope of GPA significant in any of the four subgroups (i.e., formed by parent graduate degree by university type).