PSQF 6249 Example 8 page 1 Example 8: Higher-Order Factor Models (CFA using MLR; then IFA using WLSMV) in Mplus version 8.8 (Mplus syntax and output are available electronically; a partial Lavaan version of this example is available here)

Example data: 1,336 college students self-reporting on 49 items (measuring five factors) assessing <u>childhood maltreatment</u> on a 1–5 scale: 1=*Strongly Disagree*, 2=*Disagree*, 3=*Neutral*, 4=*Agree*, 5=*Strongly Agree*. The item responses are NOT normally distributed, so we'll use both CFA (with MLR) and IFA (with WLSMV) as two options to examine the fit of these models (i.e., only as an example of how to do each, NOT to compare between the model types).

- 1. Spurning: Verbal and nonverbal caregiver acts that reject and degrade a child
- 2. Terrorizing: Caregiver behaviors that threaten or are likely to physically hurt, kill, abandon, or place the child or the child's loved ones or objects in recognizably dangerous situations.
- 3. Isolating: Caregiver acts that consistently deny the child opportunities to meet needs for interacting or communicating with peers or adults inside or outside the home.
- 4. Corrupting: Caregiver acts that encourage the child to develop inappropriate behaviors (self-destructive, antisocial, criminal, deviant, or other maladaptive behaviors).
- 5. Ignoring: Emotional unresponsiveness includes caregiver acts that ignore the child's attempts and needs to interact (failing to express affection, caring, and love for the child) and show no emotion in interactions with the child

Here are the results from fitting the 5 factors separately to ensure their individual fit FIRST (see Mplus output files for details):

ASESSMENT OF MODEL FIT USING MLR													
Model	# Items	# Possible Parms	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	· ·	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower Cl	RMSEA Higher Cl	RMSEA p-value	
MLR Spurning	12	90	36	224.797	1.4009	54	<.0001	0.959	0.049	0.042	0.055	0.619	
MLR Terror	9	54	27	189.815	1.5876	27	<.0001	0.918	0.067	0.058	0.076	0.001	
MLR Isolate	6	27	18	80.356	1.4944	9	<.0001	0.916	0.077	0.062	0.093	0.002	
MLR Corrupt	7	35	21	54.964	1.9075	14	<.0001	0.934	0.047	0.034	0.060	0.633	
MLR Ignore	15	135	45	484.291	1.7921	90	<.0001	0.932	0.057	0.052	0.062	0.008	
MLR 1 factor only	49	1274	147	6,183.986	1.4874	1127	<.0001	0.766	0.058	0.057	0.059	<.0001	
MLR 5 correlated factors	49	1274	157	4,424.701	1.4645	1117	<.0001	0.847	0.047	0.046	0.049	1.000	
MLR 5 factors + higher order	49	1274	152	4,486.381	1.4681	1122	<.0001	0.844	0.047	0.046	0.049	0.999	
MLR 5 factors + HO + 2 res cov	49	1274	154	4,422.556	1.4669	1120	<.0001	0.847	0.047	0.046	0.048	1.000	

ASESSMENT OF MODEL FIT USING WLSMV													
Model	# Items	# Possible Parms	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	· ·	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower Cl	RMSEA Higher Cl	RMSEA p-value	
WLSMV Spurning	12	126	60	294.706		54	<.0001	0.983	0.058	0.051	0.064	0.023	
WLSMV Terror	9	81	45	263.155		27	<.0001	0.966	0.081	0.072	0.090	<.0001	
WLSMV Isolate	6	45	30	129.828		9	<.0001	0.962	0.100	0.085	0.116	<.0001	
WLSMV Corrupt	7	56	35	87.487		14	<.0001	0.976	0.063	0.050	0.076	0.044	
WLSMV Ignore	15	180	75	897.689		90	<.0001	0.976	0.082	0.077	0.087	<.0001	
WLSMV 1 factor only	49	1421	245	7,563.407		1127	<.0001	0.903	0.065	0.064	0.067	<.0001	
WLSMV 5 correlated factors	49	1421	255	5,934.136		1117	<.0001	0.927	0.057	0.055	0.058	<.0001	
WLSMV 5 factors + higher order	49	1421	250	5,941.909		1122	<.0001	0.927	0.057	0.055	0.058	<.0001	
WLSMV 5 factors + HO + 2 res cov	49	1421	252	5,853.773		1122	<.0001	0.928	0.056	0.055	0.058	<.0001	

Here are the standardized factor loadings for each item under each estimation method. Note that the WLSMV factor loadings are higher in this case—probably because of positive skewness in the original data (and thus the implausibility of a linear model).

MLR	WLSMV	MLR	WLSMV	MLR	WLSMV	MLR	WLSMV		MLR	WLSMV
Spurning	Spurning	Terror	Terror	Isolate	Isolate	Corrupt	Corrupt	_	Ignore	Ignore
0.599	0.660	0.512	0.617	0.521	0.695	 0.589	0.739	-	0.672	0.813
0.457	0.528	0.673	0.771	0.550	0.630	0.545	0.713		0.654	0.749
0.769	0.837	0.451	0.713	0.545	0.685	0.375	0.523		0.657	0.748
0.526	0.597	0.612	0.721	0.540	0.629	0.545	0.854		0.724	0.801
0.607	0.677	0.571	0.787	0.563	0.726	0.631	0.826		0.445	0.540
0.816	0.865	0.554	0.617	0.752	0.822	0.580	0.708		0.745	0.833
0.835	0.907	0.685	0.805			0.646	0.840		0.847	0.913
0.465	0.538	0.643	0.743						0.713	0.813
0.516	0.728	0.732	0.815						0.808	0.891
0.655	0.744								0.749	0.845
0.674	0.756								0.656	0.795
0.610	0.680								0.830	0.904
									0.712	0.806
									0.739	0.815
									0.825	0.918

Strawman model: Syntax for single-factor CFA model estimated using MLR through 5 PERFECTLY correlated factors

Dama, ETTER abuse served Dep/A productly if is some folder i	THE MODEL ESTIMATION TERMINATED NORMALLY
DATA: FILE = abuse.csv; ! Don't need path if in same folder as input TYPE = INDIVIDUAL; FORMAT = FREE; ! Defaults	ILE MODEL FOLIWATION LERWINALED NORWATTA
VARIABLE:	
	Because the factor covariances were fixed to 1, you will see the
NAMES = ID ! All variables in DATA SET p01 p02 p03 p04 p05 p06 p07 p08 p09 p10	message below. In THIS CONTEXT ONLY, you can ignore it.
p11 p12 p13 p14 p15 p16 p17 p18 p19 p20	WARNING: THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30	DEFINITE. THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE
p31 p32 p33 p34 p35 p36 p37 p38 p39 p40	FOR A LATENT VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN
p41 p42 p43 p44 p45 p46 p47 p48 p49 p50	TWO LATENT VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO
p51 p52 p53 p54 p55 p56 p57;	LATENT VARIABLES, OK A LINEAR DELEMBENCI AMONG MORE THAN TWO LATENT VARIABLES. CHECK THE TECH4 OUTPUT FOR MORE INFORMATION.
	PROBLEM INVOLVING VARIABLE TERROR.
USEVARIABLES = ! All variables in MODEL	TROBLEM INVOLVING VARIABLE TERROR.
p01 p02 p03 p04 p06 p07 p09 p10	MODEL FIT INFORMATION
p11 p12 p13 p14 p16 p17 p18 p19 p20	Number of Free Parameters 147
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30	Loglikelihood
p31 p33 p35 p36 p37 p39 p40	5
p43 p44 p45 p46 p47 p48 p49 p50	HO Value -70386.526 HO Scaling Correction Factor 2.3983
p51 p52 p53 p54 p55 p56 p57;	for MLR
IDVARIABLE = ID; ! Person ID variable	
MISSING = ALL (99999); ! Missing data value used	H1 Value -65787.405
	H1 Scaling Correction Factor 1.5925
ANALYSIS: ESTIMATOR = MLR; ! For non-normal continuous items	for MLR
OUTPUT: STDYX ! Standardized solution	
MODINDICES(3.84) ! Voodoo for fixing the model	Information Criteria
RESIDUAL ! Local fit info	Akaike (AIC) 141067.051
TECH4; ! Factor correlation matrix	Bayesian (BIC) 141831.074
!SAVEDATA: SAVE = FSCORES; ! Save factor scores	Sample-Size Adjusted BIC 141364.120
! FILE = Abuse_Thetas.dat; ! File of factor scores	$(n^* = (n + 2) / 24)$
! MISSFLAG = 99; ! Indicate missing values	
PLOT: TYPE = PLOT1 PLOT2 PLOT3; ! For pictures	Chi-Square Test of Model Fit
	Value 6183.986*
MODEL: ! (To be changed below for each model)	Degrees of Freedom 1127
	P-Value 0.0000
! 5 Factors (loadings for first item are estimated)	Scaling Correction Factor 1.4874
! 12-Item Spurning	for MLR
Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;	
9-Item Terrorizing	RMSEA (Root Mean Square Error Of Approximation)
Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;	Estimate 0.058
! 6-Item Isolating	90 Percent C.I. 0.057 0.059
Isolate BY p01* p18* p19* p23* p39* p43*;	Probability RMSEA <= .05 0.000
! 7-Item Corrupting	
Corrupt BY p09* p12* p16* p20* p28* p47* p50*;	CFI/TLI
! 15-Item Ignoring	CFI 0.766
Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*	TLI 0.756
p45* p46* p51* p52* p57*;	
	SRMR (Standardized Root Mean Square Residual)
! Factor Variances (all must be fixed to 1 for identification)	Value 0.062
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;	
! Factor Means (all fixed = 0 by default)	#free parameters = 147 = 49 loadings + 49 intercepts + 49 residuals
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];	+ 0 factor variances + 0 factor covariances = 147 parameters USED
! Factor Covariance (all fixed to 1 to create 1-factor model)	$+$ 0 ractor variances \pm 0 ractor covariances $=$ 147 parameters USED
Spurn Terror Isolate Corrupt Ignore WITH	
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;	Possible = $49*50/2 + 49 = 1274$
· · · · · · · · · · · · · · · · · · ·	DF =1117 calculation: 1274 – 147 = 1127

Syntax for CFA model with MLR including all 5 non-perfectly correlated factors ("saturated structural model") for comparison:

MODEL: ! (To be changed below for each model)	NOTE: With respect to fit of the structural model, letting the 5 factors just be
! 5 Factors (loadings for first item are estimated)	correlated is as good as it gets. This saturated structural model will be our
! 12-Item Spurning	"larger model" baseline with which to compare the fit of models that try to
Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54	*; account for these correlations via a higher-order factor ("smaller models").
! 9-Item Terrorizing	
Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;	Number of Free Parameters 157
! 6-Item Isolating Isolate BY p01* p18* p19* p23* p39* p43*;	Loglikelihood H0 Value -69027.431
<pre>! 7-Item Corrupting</pre>	HO Value -69027.431 HO Scaling Correction Factor 2.5033
Corrupt BY p09* p12* p16* p20* p28* p47* p50*;	for MLR
! 15-Item Ignoring	H1 Value -65787.405
Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*	H1 Scaling Correction Factor 1.5925
p45* p46* p51* p52* p57*;	for MLR
! Factor Variances (all must be fixed to 1 for identification)	Information Criteria
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;	Akaike (AIC) 138368.862
! Factor Means (all fixed = 0 by default)	Bayesian (BIC) 139184.860
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];	Sample-Size Adjusted BIC 138686.140
! Factor Covariances (all estimated to allow 5-factor model)	$(n^* = (n + 2) / 24)$
Spurn Terror Isolate Corrupt Ignore WITH Spurn* Terror* Isolate* Corrupt* Ignore*;	Chi-Square Test of Model Fit
spuin" leiloi" isolate" collupt" ignole",	Value 4424.701*
So do we have one factor or five factors?	Degrees of Freedom 1117
	P-Value 0.0000
According to the 2011 cooled difference relative to the provinue single	Scaling Correction Factor 1.4645
According to the $-2\Delta LL$ scaled difference relative to the previous single-	for MLR
factor model: $-2\Delta LL(10) = 671.689$, $p < .0001$	DNGEA (Deet Meen Orners Enner Of Annualization)
	RMSEA (Root Mean Square Error Of Approximation) Estimate 0.047
Therefore, one factor does not capture the covariances for these 49 items.	90 Percent C.I. 0.046 0.049
Five factors (as hypothesized) does a significantly better job.	Probability RMSEA <= .05 1.000
Here are the correlations among the latent factors we are now trying to	CFI/TLI CFI 0.847
account for—with models that replace them with a higher-order factor.	TLI 0.839
	SRMR (Standardized Root Mean Square Residual)
Saturated: 5-Factor All Covariances Model	Value 0.057
SPURN TERROR ISOLATE CORRUPT IGNORE	
	#free parameters = 157 = 49 loadings + 49 intercepts + 49 residuals
SPURN 1.000	+ 0 factor variances + 10 factor covariances = 157 parameters USED
TERROR .929 1.000	
ISOLATE .898 .876 1.000	Possible = $49*50/2 + 49 = 1274$
	DF =1117 calculation: 1274 – 157 = 1117
CORRUPT .689 .792 .658 1.000	
IGNORE .830 .767 .828 .630 1.000	Now we can test the fit of a constrained structural model that posits a single
	higher-order "General Abuse" factor to account for the correlations among
	these 5 latent factors (shown on the left from TECH 4).

Syntax for CFA model with MLR and a higher-order factor instead of correlations among 5 factors ("smaller/bigger model" for comparison):

MODEL: ! (To be changed below for each model)	NOTE: With respect to fit of the structural model, we are now fitting a
! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)	single higher-order factor INSTEAD OF covariances among the 5 factors.
<pre>! 12-Item Spurning Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*; ! 9-Item Terrorizing</pre>	Number of Free Parameters 152 Loglikelihood
Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*; ! 6-Item Isolating	HO Value -69080.656 HO Scaling Correction Factor 2.5109
Isolate BY p0101 p18* p19* p23* p39* p43*; ! 7-Item Corrupting	for MLR H1 Value -65787.405
Corrupt BY p0901 p12* p16* p20* p28* p47* p50*; ! 15-Item Ignoring	H1 Scaling Correction Factor 1.5925 for MLR
Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44* p45* p46* p51* p52* p57*;	Information Criteria Akaike (AIC) 138465.313
<pre>! Factor Variances (all must be free - NOW ARE "DISTURBANCES") Spurn* Terror* Isolate* Corrupt* Ignore*; ! Factor Means (all fixed = 0 by default) [Course00 Terrore00 Terrore00 Terrore00];</pre>	Bayesian (BIC) 139255.323 Sample-Size Adjusted BIC 138772.486 (n* = (n + 2) / 24)
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];	Chi-Square Test of Model Fit Value 4486.381*
<pre>! Higher-Order Factor (estimate higher-order factor loadings) Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*; ! Fix higher-order factor mean=0 & variance=1 [Abuse@0]; Abuse@1;</pre>	Degrees of Freedom 1122 P-Value 0.0000 Scaling Correction Factor 1.4681 for MLR
We can use a -2Δ LL scaled difference to test the fit of the higher-order factor model against the saturated structural model with all possible factor correlations. This higher-order factor model uses 5 fewer parameters: 5 higher-order loadings replace the 10 covariances among the factors. The -2Δ LL scaled difference is -2Δ LL(5) = 46.848, <i>p</i> < .0001.	RMSEA (Root Mean Square Error Of Approximation) Estimate 0.047 90 Percent C.I. 0.046 0.049 Probability RMSEA <= .05 0.999 CFI/TLI CFI 0.844 TLI 0.837
So trying to reproduce the 5 factor covariances with a single higher-order factor results in a significant decrease in fit. Why might this be the case? All the lower-order factors have large (enough) standardized loadings	SRMR (Standardized Root Mean Square Residual) Value 0.058
STDYX Standardization Two-Tailed Estimate S.E. Est./S.E. P-Value	<pre>#free parameters = 152 = 44 loadings + 49 intercepts + 49 residuals + 5 factor variances + 5 higher-order loadings = 152 parameters USED</pre>
ABUSE BY (HIGHER-ORDER STANDARDIZED LOADINGS)	Possible = 49*50/2 + 49 = 1274
SPURN0.9710.010101.9410.000TERROR0.9520.01188.1910.000ISOLATE0.9330.01659.1590.000	DF =1117 calculation: 1274 – 152 = 1122
CORRUPT 0.745 0.027 27.312 0.000 IGNORE 0.846 0.018 48.111 0.000	

Higher-Order Factor Model Output; Comparison of Saturated versus Higher-Order Factor Model predicted correlations:

MODEL MODI	IFICATION INDICES				
Minimum M.	3.840				
		M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
TERROR W	WITH SPURN	4.111	0.011	0.454	0.454
CORRUPT W	WITH SPURN	18.864	-0.018	-0.451	-0.451
CORRUPT W	WITH TERROR	44.080	0.021	0.595	0.595
CORRUPT W	NITH ISOLATE	4.628	-0.006	-0.193	-0.193
IGNORE W	WITH SPURN	4.800	0.010	0.248	0.248
IGNORE W	WITH TERROR	31.774	-0.018	-0.510	-0.510
IGNORE W	NITH ISOLATE	14.098	0.010	0.317	0.317

Based on the modification indices (which are picking up on the discrepancies between the saturated model and higher-order factor model in the factor correlations), it appears we need to allow two more relationships among the factor disturbances, as follows:

MODEL: ! (To be changed below for each model)

- ! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)
- ! 12-Item Spurning
- Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*; ! 9-Item Terrorizing
- Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
- ! 6-Item Isolating
- Isolate BY p01@1 p18* p19* p23* p39* p43*;
- ! 7-Item Corrupting Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;
- ! 15-Item Ignoring
 - Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44* p45* p46* p51* p52* p57*;
- ! Factor Variances (all must be free NOW ARE "DISTURBANCES")
 Spurn* Terror* Isolate* Corrupt* Ignore*;
- ! Factor Means (all fixed = 0 by default)
 [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
- ! Higher-Order Factor (estimate higher-order factor loadings) Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
- ! Fix higher-order factor mean=0 & variance=1
 [Abuse@0]; Abuse@1;
- ! Add disturbance covariances suggested by mod indices Corrupt WITH Terror*; Ignore WITH Terror*;

MLR Solutions												
Saturated: 5-Factor All Covariances Model												
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE							
SPURN	1.000											
TERROR	.929	1.000										
ISOLATE	.898	.876	1.000									
CORRUPT	.689	.792	.658	1.000								
IGNORE	.830	.767	.828	.630	1.000							
Predicted 1: 5-Factor + Higher-Order Factor Model												
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE							
SPURN	1.000											
TERROR	.925	1.000										
ISOLATE	.906	.889	1.000									
CORRUPT	.724	.710	.696	1.000								
IGNORE	.821	.806	.790	.631	1.000							
Discrepancy: Saturated - Predicted 1												
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE							
SPURN												
TERROR	.004											
ISOLATE	008	013										
CORRUPT	035	.082	038									
IGNORE	.009	039	.038	001								
Predicte	ed 2: 5-Facto	r + Higher-O		r + 2 Fact Co	ov Model							
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE							
SPURN	1.000											
TERROR	.923	1.000										
ISOLATE	.898	.894	1.000									
CORRUPT	.690	.794	.668	1.000								
IGNORE	.838	.766	.812	.623	1.000							
Discrepancy: Saturated - Predicted 2												
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE							
SPURN												
TERROR	.006											
ISOLATE	.000	018										
CORRUPT	001	002	010									

IGNORE

-.008

.001

.016

.007

					PSQF 6249 Example 8 page 7
MODEL FIT INFORMAT	'ION				Two comparisons are relevant.
Number of Free Par	ameters		154		
					First did we hale the birther ender factor we delive adding two severing areas the
Loglikelihood					First, did we help the higher-order factor model by adding two covariances among the
H0 Value	1		-69031.180		factor disturbances? $-2\Delta LL(2) = 46.378$, $p < .0001$, so yes, model fit is better.
HO Scali	ng Correction H	Factor	2.5060		
for ML					Second, does the revised higher-order factor model fit nonsignificantly worse than the
H1 Value			-65787.405		
	ng Correction H	Factor	1.5925		saturated structural model with all 10 correlations among the 5 factors? $-2\Delta LL(3) =$
for ML	.R				3.171, $p = .3660$, so yes, our revised model captures those 10 correlations using 3
					fewer parameters (5 loadings + 2 covariances).
Information Criter					
Akaike (,		138370.360		Example results section for CEA using MLP.
Bayesian			139170.765		Example results section for CFA using MLR:
-	ize Adjusted Bl	IC	138681.575		
(n* =	(n + 2) / 24)				After examining the fit of each of the five factors individually, as described previously,
					a combined model was estimated in which all five factors were fit simultaneously with
Chi-Square Test of	Model Fit				
Value	- ·		4422.556*		covariances estimated freely among them. A total of 49 items were thus included.
_	of Freedom		1120		Robust maximum likelihood (MLR) estimation was used to estimate all models using
P-Value	· · · · ·		0.0000		Mplus v. 8.8 (Muthén & Muthén, 1998–2017), and differences in fit between nested
-	Correction Fact	tor	1.4669		models were evaluated using -2* rescaled difference in the model log-likelihood
for ML	'K				
		7	to a fact a sex y		values. The fit of each model referenced below is shown in Table 1.
RMSEA (Root Mean S	-	Approx			
Estimate			0.047 0.046	0.048	We first established the need for five factors by showing a significant decrease in fit
90 Perce	ity RMSEA <= .(0.5	1.000	0.040	for a single-factor model relative to that of the five-factor model, $-2\Delta LL(10) = 671.689$,
FIODADII	ILY AMODA <(00	1.000		
CFI/TLI					p < .0001. As shown in Table 1, the fit of the model with five correlated factors was
CFI			0.847		acceptable by the RMSEA (.047), but not by the CFI (.847). Standardized model
TLI			0.847		parameters (loadings, intercepts, and residual variances) are shown in Table 2.
			0.040		Correlations \geq .6 were found amongst the five factors, suggesting evidence that the
SRMR (Standardized	Root Mean Sour	are Res	idual)		o o o o
Value	i nooc nean oqui	are nes	0.057		five factors may indicate a single higher-order factor. This idea was tested by
Vaiue			0.007		eliminating the covariances among the factors and instead estimating loadings for the
					five factors from a single higher-order factor (whose variance was fixed to 1).
STDYX Standardizat	ion				Although the fit of the higher-order factor model remained marginal (see Table 1), a
Sibin Scandardizae	.1011			Two-Tailed	
	Estimate	S.E.	Est./S.E.	P-Value	nested model comparison revealed a significant decrease in fit, $-2\Delta LL(5) = 46.848$, p
ABUSE BY (HIGHE	R-ORDER STANDAR				< .0001, indicating that a single factor did not appear adequate to describe the pattern
SPURN	0.963	0.011		0.000	of correlation amongst the five factors. Inspection of the discrepancy between the
TERROR	0.958	0.012		0.000	factor correlations from the saturated five-factor model and those predicted by the
ISOLATE	0.933	0.016		0.000	
CORRUPT	0.716	0.028		0.000	higher-order factor indicated two sources of misfit—the correlation between Corrupt
IGNORE	0.870	0.019	45.845	0.000	and Terror was under-estimated, whereas the correlation between Ignore and Terror
					was over-estimated. These discrepancies were captured via two additional
CORRUPT WITH					covariances among those lower-order factor disturbances, resulting in a significant
TERROR	0.540	0.097	5.550	0.000	improvement in fit, $-2\Delta LL(2) = 46.378$, $p < .0001$. Further, the revised model
IGNORE WITH					
TERROR	-0.483	0.172	-2.811	0.005	successfully accounted for the pattern of correlation among the five factors, as
					indicated by a nonsignificant decrease in model fit relative to the model with all 10
					factor correlations estimated directly, $-2\Delta LL(3) = 3.171$, $p = .3660$.
		-	_		
Next we will dup	licate these a	nalyse	es using WI	_SMV, which	
requires starting	with the bigg	iest m	odel first		
		,			

Syntax for IFA model with WLSMV including all 5 non-perfectly correlated factors ("saturated structural model") for comparison:

```
DATA: FILE = abuse.csv; ! Don't need path if in same folder as input
      TYPE = INDIVIDUAL; FORMAT = FREE; ! Defaults
                                                                      MODEL: ! (To be changed below for each model)
VARIABLE:
NAMES = ID ! All variables in DATA SET
                                                                      ! 5 Factors (loadings for first item are estimated)
 p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
                                                                      ! 12-Item Spurning
 p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
                                                                       Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
 p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
                                                                      ! 9-Item Terrorizing
 p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
                                                                        Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;
 p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
                                                                      ! 6-Item Isolating
                                                                       Isolate BY p01* p18* p19* p23* p39* p43*;
 p51 p52 p53 p54 p55 p56 p57;
                                                                      ! 7-Item Corrupting
USEVARIABLES = ! All variables in MODEL
                                                                        Corrupt BY p09* p12* p16* p20* p28* p47* p50*;
                   p06 p07
 p01 p02 p03 p04
                                 01g 60g
                                                                      ! 15-Item Ignoring
                                                                       Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
 p11 p12 p13 p14 p16 p17 p18 p19 p20
                                                                                  p45* p46* p51* p52* p57*;
 p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
 p31 p33 p35 p36 p37 p39 p40
         p43 p44 p45 p46 p47 p48 p49 p50
                                                                      ! Factor Variances (all must be fixed to 1 for identification)
                                                                        Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
 p51 p52 p53 p54 p55 p56 p57;
                                                                      ! Factor Means (all fixed = 0 by default)
IDVARIABLE = ID;
                         ! Person ID variable
                                                                        [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
MISSING = ALL (99999);
                        ! Missing data value used
                                                                      ! Factor Covariances (all estimated to allow 5-factor model)
                                                                        Spurn Terror Isolate Corrupt Ignore WITH
CATEGORICAL = ! All ordinal outcomes for IFA
                                                                        Spurn* Terror* Isolate* Corrupt* Ignore*;
 p01 p02 p03 p04 p06 p07
                                 p09 p10
                  p16 p17 p18 p19 p20
p11 p12 p13 p14
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
                                                                      NOTE: With respect to fit of the structural model, letting the 5 factors just be
      p33 p35 p36 p37 p39 p40
p31
        p43 p44 p45 p46 p47 p48 p49 p50
                                                                      correlated is as good as it gets. This saturated structural model will be our
p51 p52 p53 p54 p55 p56 p57;
                                                                      "larger model" baseline with which to compare the fit of models that try to
                                                                      account for these correlations via a higher-order factor ("smaller models").
OUTPUT:
                              ! Standardized solution
            STDYX
           MODINDICES(3.84) ! Voodoo for fixing the model
                                                                      MODEL FIT INFORMATION
           RESIDUAL
                              ! Local fit info
                                                                      Number of Free Parameters
                                                                                                                     255
            TECH4:
                              ! Factor correlation matrix
PLOT:
           TYPE = PLOT1 PLOT2 PLOT3; ! For pictures
                                                                      Chi-Square Test of Model Fit
                                                                                                               5934.136*
                                                                                Value
ANALYSIS: ESTIMATOR = WLSMV;
                                      ! Limited-info in probits
                                                                                Degrees of Freedom
                                                                                                                  1117
            PARAMETERIZATION = THETA;
                                                                                P-Value
                                                                                                                  0.0000
            CONVERGENCE = 0.0000001; ! For OS comparability
                                                                      RMSEA (Root Mean Square Error Of Approximation)
SAVEDATA: DIFFTEST=5factor.dat:
                                      ! Save fit of 5-factor model
                                                                               Estimate
                                                                                                                  0.057
                                                                                                                  0.055 0.058
                                                                                90 Percent C.I.
                                                                                Probability RMSEA <= .05
                                                                                                                 0.000
#free parameters = 255 = 49 loadings + 49*4=196 thresholds
                                                                      CFI/TLI
   + 0 factor variances + 10 factor covariances = 255 parameters USED
                                                                                CFT
                                                                                                                   0.927
                                                                                TLI
                                                                                                                   0.923
Possible = 49*50/2 + 49*4 = 1421
                                                                      SRMR (Standardized Root Mean Square Residual)
DF =1117 calculation: 1421 - 255 - 49 "residual variances" = 1117
                                                                                Value
                                                                                                                   0.056
```

Strawman model: Syntax for single-factor IFA model estimated using WLSMV through 5 PERFECTLY correlated factors

ANALYSIS: DIFFTEST=5factor.dat; ! Test fit against 5-factor model	THE MODEL ESTIMATION TERMINATED NORMAI	ЪТХ				
! (no SAVEDATA needed)	Because the factor covariances were fixed to 1, you will see the					
MODEL: ! (To be changed below for each model)	message below. In THIS CONTEXT ONLY, you can ignore it.					
! 5 Factors (loadings for first item are estimated) ! 12-Item Spurning	WARNING: THE LATENT VARIABLE COVARIAN DEFINITE. THIS COULD INDICATE A NEGAT					
Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;	FOR A LATENT VARIABLE, A CORRELATION G					
<pre>! 9-Item Terrorizing Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;</pre>	TWO LATENT VARIABLES, OR A LINEAR DEPE LATENT VARIABLES. CHECK THE TECH4 OUTE					
! 6-Item Isolating	PROBLEM INVOLVING VARIABLE TERROR.					
Isolate BY p01* p18* p19* p23* p39* p43*;	MODEL FIT INFORMATION					
<pre>! 7-Item Corrupting Corrupt BY p09* p12* p16* p20* p28* p47* p50*;</pre>	Number of Free Parameters	245				
! 15-Item Ignoring Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*	Chi-Square Test of Model Fit					
p45* p46* p51* p52* p57*;	Value	7563.407*				
FIG FIG FGT FGT ,	Degrees of Freedom	1127				
! Factor Variances (all must be fixed to 1 for identification) Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;	P-Value	0.0000				
! Factor Means (all fixed = 0 by default)	Chi-Square Test for Difference Testing					
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];	Value	, 769.755*				
! Factor Covariance (all fixed to 1 to create 1-factor model)	Degrees of Freedom	10				
Spurn Terror Isolate Corrupt Ignore WITH	P-Value	0.0000				
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;						
	RMSEA (Root Mean Square Error Of Appro					
	Estimate	0.065				
#free parameters = 245 = 49 loadings + 49*4=196 thresholds	90 Percent C.I. Probability RMSEA <= .05	0.064 0.067 0.000				
+ 0 factor variances + 0 factor covariances = 245 parameters USED	FIODADIIICY RMSEA <05	0.000				
	CFI/TLI					
Possible = 49*50/2 + 49*4 = 1421	CFI	0.903				
DF =1117 calculation: $1421 - 245 - 49$ "residual variances" = 1127	TLI	0.898				
	SRMR (Standardized Root Mean Square Re	esidual)				
	Value	0.068				
	Do we have one factor or five factors?					
	According to the DIFFTEST relative to the previous 5-factor model: $\chi^2(10) = 769.755$, $p < .0001$					
	Therefore, one factor does not capture the covariances for these 49 items. Five factors (as hypothesized) does a significantly better job.					

Syntax for IFA model with WLSMV and a higher-order factor instead of correlations among 5 factors ("smaller/bigger model" for comparison):

ANALYSIS: DIFFTEST=5factor.dat; ! Test fit against 5-factor model	NOTE: With respect to fit of the structu	, .
SAVEDATA: DIFFTEST=HigherOrder.dat; ! Save fit of higher-order model	single higher-order factor INSTEAD O	F covariances among the 5 factors.
MODEL: ! (To be changed below for each model)	MODEL FIT INFORMATION	
MODEL: (10 be changed below for each model)	Number of Free Parameters	250
! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)		
! 12-Item Spurning	Chi-Square Test of Model Fit	5044 0001
Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;	Value	5941.909*
! 9-Item Terrorizing	Degrees of Freedom	1122
Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;	P-Value	0.0000
! 6-Item Isolating	Chi Omenne Mart fan Differense Mart	:
Isolate BY p01@1 p18* p19* p23* p39* p43*;	Chi-Square Test for Difference Test	-
! 7-Item Corrupting	Value Degrees of Freedom	92.048*
Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;	P-Value	0.0000
! 15-Item Ignoring	P-value	0.0000
Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*	RMSEA (Root Mean Square Error Of Ap	provimation)
p45* p46* p51* p52* p57*;	Estimate	0.057
	90 Percent C.I.	0.055 0.058
! Factor Variances (all must be free - NOW ARE "DISTURBANCES")	Probability RMSEA <= .05	0.000
Spurn* Terror* Isolate* Corrupt* Ignore*;	FIODADIIICY RMSEA <05	0.000
! Factor Means (all fixed = 0 by default)	CFI/TLI	
<pre>[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];</pre>	CFI	0.927
	TLI	0.924
! Higher-Order Factor (estimate higher-order factor loadings)		0.924
Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;	SRMR (Standardized Root Mean Square	Peridual)
! Fix higher-order factor mean=0 & variance=1	Value	0.057
[Abuse@0]; Abuse@1;	Value	0.037
	#free parameters = 250 = 44 loadings	10*1-106 througholds
	+ 5 factor variances + 5 higher-ord	er loadings = 250 parameters USED
We can use DIFFTEST to test the fit of the higher-order factor model against		
the saturated structural model with all possible factor correlations. This higher-	Possible = 49*50/2 + 49*4 = 1421	
order factor model uses 5 fewer parameters: 5 higher-order loadings replace	DF =1117 calculation: $1421 - 250 - 49$	"residual variances" - 1122
	DF = 1117 calculation. 1421 - 250 - 43	e residual valiances – 1122
the 10 covariances among the factors. The difference is $\chi^2(5) = 92.048$, <i>p</i> <		
.0001.	STDYX Standardization	
		Two-Tailed
	Estimate	S.E. Est./S.E. P-Value
So trying to reproduce the 5 factor covariances with a single higher-order		
So trying to reproduce the 5 factor covariances with a single higher-order	ABUSE BY (HIGHER-ORDER STANDARDI	
factor results in a significant decrease in fit. Why might this be the case? All	ABUSE BY (HIGHER-ORDER STANDARDI SPURN 0.990 0	.005 204.056 0.000
	ABUSE BY (HIGHER-ORDER STANDARDI SPURN 0.990 0 TERROR 0.948 0	.005 204.056 0.000 .007 139.928 0.000
factor results in a significant decrease in fit. Why might this be the case? All	ABUSE BY (HIGHER-ORDER STANDARDI SPURN 0.990 0 TERROR 0.948 0 ISOLATE 0.951 0	.005204.0560.000.007139.9280.000.009106.5950.000
factor results in a significant decrease in fit. Why might this be the case? All	ABUSEBY(HIGHER-ORDER STANDARDISPURN0.9900TERROR0.9480ISOLATE0.9510CORRUPT0.8350	.005 204.056 0.000 .007 139.928 0.000

Higher-Order Factor Model Output; Comparison of Saturated versus Higher-Order Factor Model predicted correlations:

MODEL MO Minimum	3.840					
			M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
TERROR	WITH	SPURN	8.776	0.018	0.558	0.558
ISOLATE	WITH	SPURN	11.743	-0.025	-0.742	-0.742
ISOLATE	WITH	TERROR	5.966	-0.022	-0.256	-0.256
CORRUPT	WITH	SPURN	39.197	-0.056	-0.762	-0.762
CORRUPT	WITH	TERROR	122.583	0.116	0.627	0.627
IGNORE	WITH	SPURN	25.058	0.050	0.596	0.596
IGNORE	WITH	TERROR	82.830	-0.100	-0.471	-0.471
IGNORE	WITH	ISOLATE	42.440	0.080	0.372	0.372
IGNORE	WITH	CORRUPT	6.035	-0.036	-0.077	-0.077

Based on the modification indices (which are picking up on the discrepancies between the saturated model and higher-order factor model in the factor correlations, it appears we need to allow two more relationships among the factor disturbances, as follows:

ANALYSIS: DIFFTEST=5factor.dat; ! Test fit against 5-factor model							
SAVEDATA: DIFFTEST=HigherOrder2.dat; ! Save fit of higher-order2 model							
MODEL: ! (To be changed below for each model)							
! 5 Lower-Order Factors (loadings for first item NOW FIXED =1) ! 12-Item Spurning							
Spurn BY p0601 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*; 9-Item Terrorizing							
Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*; ! 6-Item Isolating							
Isolate BY p0101 p18* p19* p23* p39* p43*;							
! 7-Item Corrupting Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;							
! 15-Item Ignoring Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44* p45* p46* p51* p52* p57*;							
! Factor Variances (all must be free - NOW ARE "DISTURBANCES") Spurn* Terror* Isolate* Corrupt* Ignore*;							
<pre>! Factor Means (all fixed = 0 by default)</pre>							
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];							
! Higher-Order Factor (estimate higher-order factor loadings)							
Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*; ! Fix higher-order factor mean=0 & variance=1							
[Abuse@0]; Abuse@1;							
<pre>! Add disturbance covariances suggested by mod indices Corrupt WITH Terror*; Ignore WITH Terror*;</pre>							

rder Factor Model predicted correlations:								
Saturated: 5-Factor All Covariances Model								
SPURN	TERROR	ISOLATE	CORRUPT	IGNORE				
1.000								
.947	1.000							
.925	.885	1.000						
.791	.866	.776	1.000					
.882	.817	.863	.729	1.000				
Predicted 1: 5-Factor + Higher-Order Factor Model								
SPURN	TERROR	ISOLATE	CORRUPT	IGNORE				
1.000								
.938	1.000							
.941	.902	1.000						
.826	.791	.794	1.000					
.876	.839	.841	.738	1.000				
Discrepa	ancy: Satur	ated - Pred	dicted 1					
SPURN	TERROR	ISOLATE	CORRUPT	IGNORE				
.009								
016	017							
035	.075	018						
.006	022	.022	009					
Predicted 2: 5-Factor + Higher-Order Factor + 2 Fact Cov Model								
SPURN	TERROR	ISOLATE	CORRUPT	IGNORE				
1.000								
.939	1.000							
.927	.907	1.000						
.792	.866	.765	1.000					
.885	.817	.855	.730	1.000				
Discrepancy: Saturated - Predicted 2								
SPURN .	TERROR	ISOLATE	CORRUPT	IGNORE				
.008								
.008 002	022							
	022 .000	.011						
	Saturated: SPURN 1.000 .947 .925 .791 .882 licted 1: 5-I SPURN 1.000 .938 .941 .826 .876 .000 .941 .826 .876 .000 .016 .035 .006 2: 5-Factor SPURN 1.000 .939 .927 .939 .927 .792 .885 .015crepa SPURN	WLSMV S Saturated: 5-Factor A SPURN TERROR 1.000	WLSMV Solutions Saturated: 5-Factor All Covarian SPURN TERROR ISOLATE 1.000	WLSMV Solutions Saturated: 5-Factor All Covariances Model SPURN TERROR ISOLATE CORRUPT 1.000 - - - 947 1.000 - - .947 1.000 - - .925 .885 1.000 - .925 .885 1.000 - .925 .885 1.000 - .925 .885 1.000 - .925 .887 .863 .729 Itoted 1: 5-Factor + Higher-Order Factor Model - - SPURN TERROR ISOLATE CORRUPT 1.000 - 1.000 - .938 1.000 - - .941 .902 1.000 - .826 .791 .794 1.000 .826 .791 .794 1.000 .809 .0017 - - .009 012 .022				

MODEL FIT INFORMATIO	N					
Number of Free Param			252			
Chi-Square Test of M	odel Fit					
Value			5853.773*			
Degrees of	Freedom		1120			
P-Value			0.0000			
Chi-Square Test for	Difference	Testing				
Value	_		8.483*			
Degrees of Freedom			3			
P-Value			0.0370			
RMSEA (Root Mean Squ	aro Error ()f Approvi	mation			
Estimate	are Error (JI APPIOXI	0.056			
90 Percent	СТ			0.058		
	y RMSEA <=	. 0.5	0.000	0.000		
1100001110	y renobin (.00	0.000			
CFI/TLI						
CFI			0.928			
TLI			0.925			
SRMR (Standardized R	oot Mean So	quare Resi	dual)			
Value			0.056			
STDYX Standardizatio	n					
				Two-Tailed		
	Estimate		Est./S.E.	P-Value		
ABUSE BY (HIGHER- SPURN		0.006		0.000		
TERROR	0.959	0.008				
ISOLATE	0.946	0.009		0.000		
CORRUPT	0.809	0.015		0.000		
IGNORE	0.903	0.009		0.000		
CORRUPT WITH						
TERROR	0.544	0.068	7.984	0.000		
IGNORE WITH						
TERROR	-0.406	0.102	-3.991	0.000		
MODEL MODIFICATION I						
Minimum M.I. value f	or printing	g the modi	fication ind	dex 3.840		
	м т			0 + 33737		
E D C	M.I.	E.P.C.	Std E.P.C.	StdYX		
E.P.C. TERROR WITH SPURN	13.421	0.031	0.757	0.757		
ISOLATE WITH TERROR		-0.031		-0.454		
IGNORE WITH SPURN	5.964	-0.029		-0.271		
IGNORE WITH ISOLAT		0.039		0.186		
	_ 0.1,0	0.000	3.100	0.100		
It looks like we could	-1 e bhe b	more co	variances to	onsura not		
worse fit than the saturated (all 10 correlations) model, but which						
should be added see	ems somev	vhat arbitı	rary… so l'n	n calling it		
done.						

PSQF 6249 Example 8 page 12 Two comparisons are relevant. First, did we help the higher-order factor model by adding two covariances among the factor disturbances? This comparison is not shown here (had to re-estimate the model without them and compare against the model with them), but yes, $\chi^2(2) = 88.343$, p < .0001, so yes, model fit is better. Second, does the revised higher-order factor model fit nonsignificantly worse than the saturated structural model with all 10 correlations among the 5 factors? Almost: $\chi^2(3)$ = 8.483. p = .0370. So our revised model almost captures those 10 correlations using 3 fewer parameters (5 loadings + 2 covariances).

Example results section for IFA using WLSMV:

After examining the fit of each of the five factors individually, as described previously, a combined model was estimated in which all five factors were fit simultaneously with covariances estimated freely among them. A total of 49 items were thus included. WLSMV estimation (i.e., diagonally weighted least squares) in Mplus v 8.8 including a probit link and the THETA parameterization (in which all item residual variances were constrained to 1 for identification) was used to estimate all models (Muthén & Muthén, 1998–2017). Thus, model fit statistics describe the fit of the item factor model to the polychoric correlation matrix among the items. The fit of each model referenced below is shown in Table 1. Nested model comparisons were conducted using the Mplus DIFFTEST procedure.

We first established the need for 5 factors by showing a significant decrease in fit for a single-factor model relative to that of the five-factor model, $\chi^2(10) = 769.755$, p < .0001. As shown in Table 1, the fit of the model with five correlated factors was marginally acceptable by both the RMSEA (.057) and the CFI (.927). Standardized model parameters (loadings, intercepts, and residual variances) are shown in Table 2. Correlations \geq .7 were found amongst the five factors, suggesting evidence that the five factors may indicate a single higher-order factor. This idea was tested by eliminating the covariances among the factors and instead estimating loadings for the five factors from a single higher-order factor (whose variance was fixed to 1). Although the fit of the higher-order factor model remained marginal (see Table 1), a nested model comparison revealed a significant decrease in fit, $\chi^2(5) = 92.048$, p < .0001, indicating that a single factor did not appear adequate to describe the pattern of correlation amongst the five factors. Inspection of the discrepancy between the factor correlations from the 5-factor model and those predicted by the higher-order factor indicated two sources of misfit—the correlation between Corrupt and Terror was under-estimated, whereas the correlation between Ignore and Terror was overestimated. These discrepancies were captured via two additional covariances among those lower-order factor disturbances, resulting in a significant improvement in fit, χ^2 (2) = 88.343, p < .0001. However, the revised model did not completely account for the pattern of correlation among the 5 factors, as indicated by a significant decrease in model fit relative to the model with all 10 factor correlations estimated directly, $\chi^2(3)$ = 8.483, p = .0370.