Example 8: Higher-Order Factor Models (CFA using MLR; then IFA using WLSMV) in M*plus* version 8.8
(Mplus syntax and output are available electronically; a partial Lavaan version of this example is available here)

Example data: 1,336 college students self-reporting on 49 items (measuring five factors) assessing childhood maltreatment on a 1–5 scale: 1=*Strongly Disagree*, 2=*Disagree*, 3=*Neutral*, 4=*Agree*, 5=*Strongly Agree*. The item responses are NOT normally distributed, so we'll use both CFA (with MLR) and IFA (with WLSMV) as two options to examine the fit of these models (i.e., only as an example of how to do each, NOT to compare between the model types).

1. Spurning: Verbal and nonverbal caregiver acts that reject and degrade a child
2. Terrorizing: Caregiver behaviors that threaten or are likely to physically hurt, kill, abandon, or place the child or the child's loved ones or objects in recognizably dangerous situations.
3. Isolating: Caregiver acts that consistently deny the child opportunities to meet needs for interacting or communicating with peers or adults inside or outside the home.
4. Corrupting: Caregiver acts that encourage the child to develop inappropriate behaviors (self-destructive, antisocial, criminal, deviant, or other maladaptive behaviors).
5. Ignoring: Emotional unresponsiveness includes caregiver acts that ignore the child's attempts and needs to interact (failing to express affection, caring, and love for the child) and show no emotion in interactions with the child

Here are the results from fitting the 5 factors separately to ensure their individual fit FIRST (see Mplus output files for details):

ASESSMENT OF MODEL FIT USING MLR

Model	# Items	# Possible Parms	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	Chi-Square DF	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower CI	RMSEA Higher CI	RMSEA p-value
MLR Spurning	12	90	36	224.797	1.4009	54	<.0001	0.959	0.049	0.042	0.055	0.619
MLR Terror	9	54	27	189.815	1.5876	27	<.0001	0.918	0.067	0.058	0.076	0.001
MLR Isolate	6	27	18	80.356	1.4944	9	<.0001	0.916	0.077	0.062	0.093	0.002
MLR Corrupt	7	35	21	54.964	1.9075	14	<.0001	0.934	0.047	0.034	0.060	0.633
MLR Ignore	15	135	45	484.291	1.7921	90	<.0001	0.932	0.057	0.052	0.062	0.008
MLR 1 factor only	49	1274	147	6,183.986	1.4874	1127	<.0001	0.766	0.058	0.057	0.059	<.0001
MLR 5 correlated factors	49	1274	157	4,424.701	1.4645	1117	<.0001	0.847	0.047	0.046	0.049	1.000
MLR 5 factors + higher order	49	1274	152	4,486.381	1.4681	1122	<.0001	0.844	0.047	0.046	0.049	0.999
MLR 5 factors + HO + 2 res cov	49	1274	154	4,422.556	1.4669	1120	<.0001	0.847	0.047	0.046	0.048	1.000

ASESSMENT OF MODEL FIT USING WLSMV

Model	# Items	# Possible Parms	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	Chi-Square DF	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower CI	RMSEA Higher CI	RMSEA p-value
WLSMV Spurning	12	126	60	294.706		54	<.0001	0.983	0.058	0.051	0.064	0.023
WLSMV Terror	9	81	45	263.155		27	<.0001	0.966	0.081	0.072	0.090	<.0001
WLSMV Isolate	6	45	30	129.828		9	<.0001	0.962	0.100	0.085	0.116	<.0001
WLSMV Corrupt	7	56	35	87.487		14	<.0001	0.976	0.063	0.050	0.076	0.044
WLSMV Ignore	15	180	75	897.689		90	<.0001	0.976	0.082	0.077	0.087	<.0001
WLSMV 1 factor only	49	1421	245	7,563.407		1127	<.0001	0.903	0.065	0.064	0.067	<.0001
WLSMV 5 correlated factors	49	1421	255	5,934.136		1117	<.0001	0.927	0.057	0.055	0.058	<.0001
WLSMV 5 factors + higher order	49	1421	250	5,941.909		1122	<.0001	0.927	0.057	0.055	0.058	<.0001
WLSMV 5 factors + HO + 2 res cov	49	1421	252	5,853.773		1122	<.0001	0.928	0.056	0.055	0.058	<.0001

Here are the standardized factor loadings for each item under each estimation method. Note that the WLSMV factor loadings are higher in this case—probably because of positive skewness in the original data (and thus the implausibility of a linear model).

MLR Spurning	WLSMV Spurning	MLR Terror	WLSMV Terror	MLR Isolate	WLSMV Isolate	MLR Corrupt	WLSMV Corrupt	MLR Ignore	WLSMV Ignore
0.599	0.660	0.512	0.617	0.521	0.695	0.589	0.739	0.672	0.813
0.457	0.528	0.673	0.771	0.550	0.630	0.545	0.713	0.654	0.749
0.769	0.837	0.451	0.713	0.545	0.685	0.375	0.523	0.657	0.748
0.526	0.597	0.612	0.721	0.540	0.629	0.545	0.854	0.724	0.801
0.607	0.677	0.571	0.787	0.563	0.726	0.631	0.826	0.445	0.540
0.816	0.865	0.554	0.617	0.752	0.822	0.580	0.708	0.745	0.833
0.835	0.907	0.685	0.805			0.646	0.840	0.847	0.913
0.465	0.538	0.643	0.743					0.713	0.813
0.516	0.728	0.732	0.815					0.808	0.891
0.655	0.744							0.749	0.845
0.674	0.756							0.656	0.795
0.610	0.680							0.830	0.904
								0.712	0.806
								0.739	0.815
								0.825	0.918

Strawman model: Syntax for single-factor CFA model estimated using MLR through 5 PERFECTLY correlated factors

```
DATA:    FILE = abuse.csv; ! Don't need path if in same folder as input
         TYPE = INDIVIDUAL; FORMAT = FREE; ! Defaults
VARIABLE:
NAMES = ID    ! All variables in DATA SET
  p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
  p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
  p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
  p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
  p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
  p51 p52 p53 p54 p55 p56 p57;

USEVARIABLES =  ! All variables in MODEL
  p01 p02 p03 p04     p06 p07     p09 p10
  p11 p12 p13 p14     p16 p17 p18 p19 p20
  p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
  p31     p33     p35 p36 p37 p39 p40
          p43 p44 p45 p46 p47 p48 p49 p50
  p51 p52 p53 p54 p55 p56 p57;
IDVARIABLE = ID;          ! Person ID variable
MISSING = ALL (99999);    ! Missing data value used

ANALYSIS:   ESTIMATOR = MLR;    ! For non-normal continuous items
OUTPUT:     STDYX               ! Standardized solution
            MODINDICES(3.84)    ! Voodoo for fixing the model
            RESIDUAL            ! Local fit info
            TECH4;              ! Factor correlation matrix
!SAVEDATA:   SAVE = FSCORES;            ! Save factor scores
!            FILE = Abuse_Thetas.dat;   ! File of factor scores
!            MISSFLAG = 99;             ! Indicate missing values
!PLOT:       TYPE = PLOT1 PLOT2 PLOT3;  ! For pictures

MODEL: ! (To be changed below for each model)

! 5 Factors (loadings for first item are estimated)
! 12-Item Spurning
  Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01* p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09* p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45* p46* p51* p52* p57*;

! Factor Variances (all must be fixed to 1 for identification)
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
! Factor Covariance (all fixed to 1 to create 1-factor model)
  Spurn   Terror   Isolate   Corrupt   Ignore   WITH
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
```

```
THE MODEL ESTIMATION TERMINATED NORMALLY
```

Because the factor covariances were fixed to 1, you will see the message below. In THIS CONTEXT ONLY, you can ignore it.

```
WARNING:  THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE
DEFINITE.  THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE
FOR A LATENT VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN
TWO LATENT VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO
LATENT VARIABLES. CHECK THE TECH4 OUTPUT FOR MORE INFORMATION.
PROBLEM INVOLVING VARIABLE TERROR.

MODEL FIT INFORMATION
Number of Free Parameters                      147
Loglikelihood
          H0 Value                       -70386.526
          H0 Scaling Correction Factor      2.3983
            for MLR
          H1 Value                       -65787.405
          H1 Scaling Correction Factor      1.5925
            for MLR

Information Criteria
          Akaike (AIC)                   141067.051
          Bayesian (BIC)                 141831.074
          Sample-Size Adjusted BIC       141364.120
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                           6183.986*
          Degrees of Freedom                 1127
          P-Value                          0.0000
          Scaling Correction Factor        1.4874
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.058
          90 Percent C.I.                    0.057  0.059
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.766
          TLI                                0.756

SRMR (Standardized Root Mean Square Residual)
          Value                              0.062
```

#free parameters = 147 = 49 loadings + 49 intercepts + 49 residuals + 0 factor variances + 0 factor covariances = 147 parameters USED

Possible = 49*50/2 + 49 = 1274
DF =1117 calculation: 1274 – 147 = 1127

Syntax for CFA model with MLR including all 5 non-perfectly correlated factors ("saturated structural model") for comparison:

```
MODEL: ! (To be changed below for each model)

! 5 Factors (loadings for first item are estimated)
! 12-Item Spurning
  Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01* p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09* p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45* p46* p51* p52* p57*;

! Factor Variances (all must be fixed to 1 for identification)
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
! Factor Covariances (all estimated to allow 5-factor model)
  Spurn  Terror  Isolate  Corrupt  Ignore  WITH
  Spurn* Terror* Isolate* Corrupt* Ignore*;
```

So do we have one factor or five factors?

According to the −2ΔLL scaled difference relative to the previous single-factor model: −2ΔLL(10) = 671.689, $p < .0001$

Therefore, one factor does not capture the covariances for these 49 items. Five factors (as hypothesized) does a significantly better job.

Here are the correlations among the latent factors we are now trying to account for—with models that replace them with a higher-order factor.

Saturated: 5-Factor All Covariances Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.929	1.000			
ISOLATE	.898	.876	1.000		
CORRUPT	.689	.792	.658	1.000	
IGNORE	.830	.767	.828	.630	1.000

NOTE: With respect to fit of the structural model, letting the 5 factors just be correlated is as good as it gets. This saturated structural model will be our "larger model" baseline with which to compare the fit of models that try to account for these correlations via a higher-order factor ("smaller models").

```
Number of Free Parameters                      157
Loglikelihood
          H0 Value                       -69027.431
          H0 Scaling Correction Factor        2.5033
            for MLR
          H1 Value                       -65787.405
          H1 Scaling Correction Factor        1.5925
            for MLR

Information Criteria
          Akaike (AIC)                   138368.862
          Bayesian (BIC)                 139184.860
          Sample-Size Adjusted BIC       138686.140
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            4424.701*
          Degrees of Freedom                   1117
          P-Value                            0.0000
          Scaling Correction Factor          1.4645
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.047
          90 Percent C.I.                    0.046  0.049
          Probability RMSEA <= .05           1.000

CFI/TLI
          CFI                                0.847
          TLI                                0.839

SRMR (Standardized Root Mean Square Residual)
          Value                              0.057
```

#free parameters = 157 = 49 loadings + 49 intercepts + 49 residuals
+ 0 factor variances + 10 factor covariances = 157 parameters USED

Possible = 49*50/2 + 49 = 1274
DF =1117 calculation: 1274 – 157 = 1117

Now we can test the fit of a constrained structural model that posits a single higher-order "General Abuse" factor to account for the correlations among these 5 latent factors (shown on the left from TECH 4).

Syntax for CFA model with MLR and a higher-order factor instead of correlations among 5 factors ("smaller/bigger model" for comparison):

```
MODEL: ! (To be changed below for each model)

! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)
! 12-Item Spurning
  Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45*  p46* p51* p52* p57*;

! Factor Variances (all must be free - NOW ARE "DISTURBANCES")
  Spurn* Terror* Isolate* Corrupt* Ignore*;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];

! Higher-Order Factor (estimate higher-order factor loadings)
  Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
! Fix higher-order factor mean=0 & variance=1
  [Abuse@0]; Abuse@1;
```

We can use a −2ΔLL scaled difference to test the fit of the higher-order factor model against the saturated structural model with all possible factor correlations. This higher-order factor model uses 5 fewer parameters: 5 higher-order loadings replace the 10 covariances among the factors. The −2ΔLL scaled difference is −2ΔLL(5) = 46.848, $p < .0001$.

So trying to reproduce the 5 factor covariances with a single higher-order factor results in a significant decrease in fit. Why might this be the case? All the lower-order factors have large (enough) standardized loadings…

```
STDYX Standardization
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
ABUSE    BY (HIGHER-ORDER STANDARDIZED LOADINGS)
    SPURN              0.971      0.010    101.941      0.000
    TERROR             0.952      0.011     88.191      0.000
    ISOLATE            0.933      0.016     59.159      0.000
    CORRUPT            0.745      0.027     27.312      0.000
    IGNORE             0.846      0.018     48.111      0.000
```

NOTE: With respect to fit of the structural model, we are now fitting a single higher-order factor INSTEAD OF covariances among the 5 factors.

```
Number of Free Parameters                      152
Loglikelihood
          H0 Value                       -69080.656
          H0 Scaling Correction Factor        2.5109
            for MLR
          H1 Value                       -65787.405
          H1 Scaling Correction Factor        1.5925
            for MLR

Information Criteria
          Akaike (AIC)                   138465.313
          Bayesian (BIC)                 139255.323
          Sample-Size Adjusted BIC       138772.486
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            4486.381*
          Degrees of Freedom                  1122
          P-Value                           0.0000
          Scaling Correction Factor         1.4681
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.047
          90 Percent C.I.                    0.046  0.049
          Probability RMSEA <= .05           0.999

CFI/TLI
          CFI                                0.844
          TLI                                0.837

SRMR (Standardized Root Mean Square Residual)
          Value                              0.058
```

#free parameters = 152 = 44 loadings + 49 intercepts + 49 residuals + 5 factor variances + 5 higher-order loadings = 152 parameters USED

Possible = 49*50/2 + 49 = 1274
DF =1117 calculation: 1274 – 152 = 1122

Higher-Order Factor Model Output; Comparison of Saturated versus Higher-Order Factor Model predicted correlations:

```
MODEL MODIFICATION INDICES
Minimum M.I. value for printing the modification index      3.840

                                 M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
TERROR   WITH SPURN             4.111      0.011      0.454         0.454
CORRUPT  WITH SPURN            18.864     -0.018     -0.451        -0.451
CORRUPT  WITH TERROR           44.080      0.021      0.595         0.595
CORRUPT  WITH ISOLATE           4.628     -0.006     -0.193        -0.193
IGNORE   WITH SPURN             4.800      0.010      0.248         0.248
IGNORE   WITH TERROR           31.774     -0.018     -0.510        -0.510
IGNORE   WITH ISOLATE          14.098      0.010      0.317         0.317
```

Based on the modification indices (which are picking up on the discrepancies between the saturated model and higher-order factor model in the factor correlations), it appears we need to allow two more relationships among the factor disturbances, as follows:

```
MODEL: ! (To be changed below for each model)

! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)
! 12-Item Spurning
  Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45*  p46* p51* p52* p57*;

! Factor Variances (all must be free - NOW ARE "DISTURBANCES")
  Spurn* Terror* Isolate* Corrupt* Ignore*;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];

! Higher-Order Factor (estimate higher-order factor loadings)
  Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
! Fix higher-order factor mean=0 & variance=1
  [Abuse@0]; Abuse@1;

! Add disturbance covariances suggested by mod indices
  Corrupt WITH Terror*;
  Ignore  WITH Terror*;
```

MLR Solutions

Saturated: 5-Factor All Covariances Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.929	1.000			
ISOLATE	.898	.876	1.000		
CORRUPT	.689	.792	.658	1.000	
IGNORE	.830	.767	.828	.630	1.000

Predicted 1: 5-Factor + Higher-Order Factor Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.925	1.000			
ISOLATE	.906	.889	1.000		
CORRUPT	.724	.710	.696	1.000	
IGNORE	.821	.806	.790	.631	1.000

Discrepancy: Saturated - Predicted 1

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN					
TERROR	.004				
ISOLATE	-.008	-.013			
CORRUPT	-.035	**.082**	-.038		
IGNORE	.009	**-.039**	.038	-.001	

Predicted 2: 5-Factor + Higher-Order Factor + 2 Fact Cov Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.923	1.000			
ISOLATE	.898	.894	1.000		
CORRUPT	.690	.794	.668	1.000	
IGNORE	.838	.766	.812	.623	1.000

Discrepancy: Saturated - Predicted 2

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN					
TERROR	.006				
ISOLATE	.000	-.018			
CORRUPT	-.001	-.002	-.010		
IGNORE	-.008	.001	.016	.007	

```
MODEL FIT INFORMATION
Number of Free Parameters                       154

Loglikelihood
          H0 Value                       -69031.180
          H0 Scaling Correction Factor      2.5060
            for MLR
          H1 Value                       -65787.405
          H1 Scaling Correction Factor      1.5925
            for MLR

Information Criteria
          Akaike (AIC)                   138370.360
          Bayesian (BIC)                 139170.765
          Sample-Size Adjusted BIC       138681.575
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                           4422.556*
          Degrees of Freedom                 1120
          P-Value                          0.0000
          Scaling Correction Factor        1.4669
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                         0.047
          90 Percent C.I.                  0.046  0.048
          Probability RMSEA <= .05         1.000

CFI/TLI
          CFI                              0.847
          TLI                              0.840

SRMR (Standardized Root Mean Square Residual)
          Value                            0.057


STDYX Standardization
                                                        Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
ABUSE    BY (HIGHER-ORDER STANDARDIZED LOADINGS)
    SPURN              0.963      0.011     87.824      0.000
    TERROR             0.958      0.012     77.198      0.000
    ISOLATE            0.933      0.016     58.212      0.000
    CORRUPT            0.716      0.028     25.150      0.000
    IGNORE             0.870      0.019     45.845      0.000

 CORRUPT  WITH
    TERROR             0.540      0.097      5.550      0.000
 IGNORE   WITH
    TERROR            -0.483      0.172     -2.811      0.005
```

Next we will duplicate these analyses using WLSMV, which requires starting with the biggest model first…

Two comparisons are relevant.

First, did we help the higher-order factor model by adding two covariances among the factor disturbances? $-2\Delta LL(2) = 46.378$, $p < .0001$, so yes, model fit is better.

Second, does the revised higher-order factor model fit nonsignificantly worse than the saturated structural model with all 10 correlations among the 5 factors? $-2\Delta LL(3) = 3.171$, $p = .3660$, so yes, our revised model captures those 10 correlations using 3 fewer parameters (5 loadings + 2 covariances).

Example results section for CFA using MLR:

After examining the fit of each of the five factors individually, as described previously, a combined model was estimated in which all five factors were fit simultaneously with covariances estimated freely among them. A total of 49 items were thus included. Robust maximum likelihood (MLR) estimation was used to estimate all models using Mplus v. 8.8 (Muthén & Muthén, 1998–2017), and differences in fit between nested models were evaluated using −2* rescaled difference in the model log-likelihood values. The fit of each model referenced below is shown in Table 1.

We first established the need for five factors by showing a significant decrease in fit for a single-factor model relative to that of the five-factor model, $-2\Delta LL(10) = 671.689$, $p < .0001$. As shown in Table 1, the fit of the model with five correlated factors was acceptable by the RMSEA (.047), but not by the CFI (.847). Standardized model parameters (loadings, intercepts, and residual variances) are shown in Table 2. Correlations ≥ .6 were found amongst the five factors, suggesting evidence that the five factors may indicate a single higher-order factor. This idea was tested by eliminating the covariances among the factors and instead estimating loadings for the five factors from a single higher-order factor (whose variance was fixed to 1). Although the fit of the higher-order factor model remained marginal (see Table 1), a nested model comparison revealed a significant decrease in fit, $-2\Delta LL(5) = 46.848$, $p < .0001$, indicating that a single factor did not appear adequate to describe the pattern of correlation amongst the five factors. Inspection of the discrepancy between the factor correlations from the saturated five-factor model and those predicted by the higher-order factor indicated two sources of misfit—the correlation between Corrupt and Terror was under-estimated, whereas the correlation between Ignore and Terror was over-estimated. These discrepancies were captured via two additional covariances among those lower-order factor disturbances, resulting in a significant improvement in fit, $-2\Delta LL(2) = 46.378$, $p < .0001$. Further, the revised model successfully accounted for the pattern of correlation among the five factors, as indicated by a nonsignificant decrease in model fit relative to the model with all 10 factor correlations estimated directly, $-2\Delta LL(3) = 3.171$, $p = .3660$.

Syntax for IFA model with WLSMV including all 5 non-perfectly correlated factors ("saturated structural model") for comparison:

```
DATA: FILE = abuse.csv; ! Don't need path if in same folder as input
      TYPE = INDIVIDUAL; FORMAT = FREE; ! Defaults
VARIABLE:
NAMES = ID   ! All variables in DATA SET
  p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
  p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
  p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
  p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
  p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
  p51 p52 p53 p54 p55 p56 p57;

USEVARIABLES =  ! All variables in MODEL
  p01 p02 p03 p04     p06 p07     p09 p10
  p11 p12 p13 p14     p16 p17 p18 p19 p20
  p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
  p31     p33     p35 p36 p37 p39 p40
          p43 p44 p45 p46 p47 p48 p49 p50
  p51 p52 p53 p54 p55 p56 p57;

IDVARIABLE = ID;          ! Person ID variable
MISSING = ALL (99999);    ! Missing data value used

CATEGORICAL =  ! All ordinal outcomes for IFA
  p01 p02 p03 p04     p06 p07     p09 p10
 p11 p12 p13 p14     p16 p17 p18 p19 p20
 p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
 p31     p33     p35 p36 p37 p39 p40
         p43 p44 p45 p46 p47 p48 p49 p50
 p51 p52 p53 p54 p55 p56 p57;

OUTPUT:     STDYX              ! Standardized solution
            MODINDICES(3.84)   ! Voodoo for fixing the model
            RESIDUAL           ! Local fit info
            TECH4;             ! Factor correlation matrix
PLOT:       TYPE = PLOT1 PLOT2 PLOT3; ! For pictures

ANALYSIS:   ESTIMATOR = WLSMV;         ! Limited-info in probits
            PARAMETERIZATION = THETA;
            CONVERGENCE = 0.0000001;   ! For OS comparability

SAVEDATA:   DIFFTEST=5factor.dat;      ! Save fit of 5-factor model
```

#free parameters = 255 = 49 loadings + 49*4=196 thresholds
+ 0 factor variances + 10 factor covariances = 255 parameters USED

Possible = 49*50/2 + 49*4 = 1421
DF =1117 calculation: 1421 – 255 – 49 "residual variances" = 1117

```
MODEL: ! (To be changed below for each model)

! 5 Factors (loadings for first item are estimated)
! 12-Item Spurning
  Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01* p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09* p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45* p46* p51* p52* p57*;

! Factor Variances (all must be fixed to 1 for identification)
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
! Factor Covariances (all estimated to allow 5-factor model)
  Spurn  Terror  Isolate  Corrupt  Ignore  WITH
  Spurn* Terror* Isolate* Corrupt* Ignore*;
```

NOTE: With respect to fit of the structural model, letting the 5 factors just be correlated is as good as it gets. This saturated structural model will be our "larger model" baseline with which to compare the fit of models that try to account for these correlations via a higher-order factor ("smaller models").

```
MODEL FIT INFORMATION
Number of Free Parameters                      255

Chi-Square Test of Model Fit
          Value                          5934.136*
          Degrees of Freedom                1117
          P-Value                         0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                         0.057
          90 Percent C.I.                  0.055  0.058
          Probability RMSEA <= .05         0.000

CFI/TLI
          CFI                              0.927
          TLI                              0.923

SRMR (Standardized Root Mean Square Residual)
          Value                            0.056
```

Strawman model: Syntax for single-factor IFA model estimated using WLSMV through 5 PERFECTLY correlated factors

```
ANALYSIS:   DIFFTEST=5factor.dat;   ! Test fit against 5-factor model

! (no SAVEDATA needed)

MODEL: ! (To be changed below for each model)

! 5 Factors (loadings for first item are estimated)
! 12-Item Spurning
  Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01* p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09* p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45* p46* p51* p52* p57*;

! Factor Variances (all must be fixed to 1 for identification)
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
! Factor Covariance (all fixed to 1 to create 1-factor model)
  Spurn   Terror   Isolate   Corrupt   Ignore   WITH
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
```

#free parameters = 245 = 49 loadings + 49*4=196 thresholds
+ 0 factor variances + 0 factor covariances = 245 parameters USED

Possible = 49*50/2 + 49*4 = 1421
DF =1117 calculation: 1421 – 245 – 49 “residual variances” = 1127

```
THE MODEL ESTIMATION TERMINATED NORMALLY
```

Because the factor covariances were fixed to 1, you will see the message below. In THIS CONTEXT ONLY, you can ignore it.

```
WARNING:  THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE
DEFINITE.  THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE
FOR A LATENT VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN
TWO LATENT VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO
LATENT VARIABLES. CHECK THE TECH4 OUTPUT FOR MORE INFORMATION.
PROBLEM INVOLVING VARIABLE TERROR.

MODEL FIT INFORMATION
Number of Free Parameters                      245

Chi-Square Test of Model Fit
          Value                             7563.407*
          Degrees of Freedom                   1127
          P-Value                            0.0000

Chi-Square Test for Difference Testing
          Value                              769.755*
          Degrees of Freedom                      10
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.065
          90 Percent C.I.                    0.064  0.067
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.903
          TLI                                0.898

SRMR (Standardized Root Mean Square Residual)
          Value                              0.068
```

Do we have one factor or five factors?

According to the DIFFTEST relative to the previous 5-factor model:
$\chi^2(10) = 769.755$, $p < .0001$

Therefore, one factor does not capture the covariances for these 49 items. Five factors (as hypothesized) does a significantly better job.

Syntax for IFA model with WLSMV and a higher-order factor instead of correlations among 5 factors ("smaller/bigger model" for comparison):

```
ANALYSIS:   DIFFTEST=5factor.dat;      ! Test fit against 5-factor model

SAVEDATA:   DIFFTEST=HigherOrder.dat;  ! Save fit of higher-order model

MODEL: ! (To be changed below for each model)

! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)
! 12-Item Spurning
  Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45*  p46* p51* p52* p57*;

! Factor Variances (all must be free - NOW ARE "DISTURBANCES")
  Spurn* Terror* Isolate* Corrupt* Ignore*;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];

! Higher-Order Factor (estimate higher-order factor loadings)
  Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
! Fix higher-order factor mean=0 & variance=1
  [Abuse@0]; Abuse@1;
```

We can use DIFFTEST to test the fit of the higher-order factor model against the saturated structural model with all possible factor correlations. This higher-order factor model uses 5 fewer parameters: 5 higher-order loadings replace the 10 covariances among the factors. The difference is $\chi^2(5) = 92.048$, $p < .0001$.

So trying to reproduce the 5 factor covariances with a single higher-order factor results in a significant decrease in fit. Why might this be the case? All the lower-order factors have large (enough) standardized loadings…

NOTE: With respect to fit of the structural model, we are now fitting a single higher-order factor INSTEAD OF covariances among the 5 factors.

```
MODEL FIT INFORMATION
Number of Free Parameters                      250

Chi-Square Test of Model Fit
          Value                            5941.909*
          Degrees of Freedom                  1122
          P-Value                           0.0000

Chi-Square Test for Difference Testing
          Value                              92.048*
          Degrees of Freedom                      5
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.057
          90 Percent C.I.                    0.055  0.058
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.927
          TLI                                0.924

SRMR (Standardized Root Mean Square Residual)
          Value                              0.057
```

#free parameters = 250 = 44 loadings + 49*4=196 thresholds
+ 5 factor variances + 5 higher-order loadings = 250 parameters USED

Possible = 49*50/2 + 49*4 = 1421
DF =1117 calculation: 1421 – 250 – 49 "residual variances" = 1122

```
STDYX Standardization
                                                        Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
ABUSE    BY (HIGHER-ORDER STANDARDIZED LOADINGS)
    SPURN              0.990      0.005    204.056      0.000
    TERROR             0.948      0.007    139.928      0.000
    ISOLATE            0.951      0.009    106.595      0.000
    CORRUPT            0.835      0.014     60.998      0.000
    IGNORE             0.885      0.009     93.999      0.000
```

Higher-Order Factor Model Output; Comparison of Saturated versus Higher-Order Factor Model predicted correlations:

```
MODEL MODIFICATION INDICES
Minimum M.I. value for printing the modification index     3.840

                                  M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
TERROR   WITH SPURN              8.776      0.018       0.558         0.558
ISOLATE  WITH SPURN             11.743     -0.025      -0.742        -0.742
ISOLATE  WITH TERROR             5.966     -0.022      -0.256        -0.256
CORRUPT  WITH SPURN             39.197     -0.056      -0.762        -0.762
CORRUPT  WITH TERROR           122.583      0.116       0.627         0.627
IGNORE   WITH SPURN             25.058      0.050       0.596         0.596
IGNORE   WITH TERROR            82.830     -0.100      -0.471        -0.471
IGNORE   WITH ISOLATE           42.440      0.080       0.372         0.372
IGNORE   WITH CORRUPT            6.035     -0.036      -0.077        -0.077
```

Based on the modification indices (which are picking up on the discrepancies between the saturated model and higher-order factor model in the factor correlations, it appears we need to allow two more relationships among the factor disturbances, as follows:

```
ANALYSIS:    DIFFTEST=5factor.dat;      ! Test fit against 5-factor model

SAVEDATA:    DIFFTEST=HigherOrder2.dat;  ! Save fit of higher-order2 model

MODEL: ! (To be changed below for each model)

! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)
! 12-Item Spurning
  Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45*  p46* p51* p52* p57*;

! Factor Variances (all must be free - NOW ARE "DISTURBANCES")
  Spurn* Terror* Isolate* Corrupt* Ignore*;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];

! Higher-Order Factor (estimate higher-order factor loadings)
  Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
! Fix higher-order factor mean=0 & variance=1
  [Abuse@0]; Abuse@1;

! Add disturbance covariances suggested by mod indices
  Corrupt WITH Terror*;
  Ignore  WITH Terror*;
```

WLSMV Solutions

Saturated: 5-Factor All Covariances Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.947	1.000			
ISOLATE	.925	.885	1.000		
CORRUPT	.791	.866	.776	1.000	
IGNORE	.882	.817	.863	.729	1.000

Predicted 1: 5-Factor + Higher-Order Factor Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.938	1.000			
ISOLATE	.941	.902	1.000		
CORRUPT	.826	.791	.794	1.000	
IGNORE	.876	.839	.841	.738	1.000

Discrepancy: Saturated - Predicted 1

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN					
TERROR	.009				
ISOLATE	-.016	-.017			
CORRUPT	-.035	**.075**	-.018		
IGNORE	.006	**-.022**	.022	-.009	

Predicted 2: 5-Factor + Higher-Order Factor + 2 Fact Cov Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.939	1.000			
ISOLATE	.927	.907	1.000		
CORRUPT	.792	.866	.765	1.000	
IGNORE	.885	.817	.855	.730	1.000

Discrepancy: Saturated - Predicted 2

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN					
TERROR	**.008**				
ISOLATE	-.002	**-.022**			
CORRUPT	-.001	.000	.011		
IGNORE	-.003	.000	.008	-.001	

```
MODEL FIT INFORMATION
Number of Free Parameters                       252

Chi-Square Test of Model Fit
          Value                             5853.773*
          Degrees of Freedom                 1120
          P-Value                           0.0000

Chi-Square Test for Difference Testing
          Value                             8.483*
          Degrees of Freedom                   3
          P-Value                           0.0370

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.056
          90 Percent C.I.                    0.055  0.058
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.928
          TLI                                0.925

SRMR (Standardized Root Mean Square Residual)
          Value                              0.056


STDYX Standardization
                                                        Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
ABUSE    BY (HIGHER-ORDER STANDARDIZED LOADINGS)
    SPURN              0.980      0.006    173.657      0.000
    TERROR             0.959      0.007    130.093      0.000
    ISOLATE            0.946      0.009    105.339      0.000
    CORRUPT            0.809      0.015     54.055      0.000
    IGNORE             0.903      0.009     97.384      0.000

 CORRUPT  WITH
    TERROR             0.544      0.068      7.984      0.000
 IGNORE   WITH
    TERROR            -0.406      0.102     -3.991      0.000

MODEL MODIFICATION INDICES
Minimum M.I. value for printing the modification index     3.840

                         M.I.     E.P.C.  Std E.P.C.  StdYX
E.P.C.
TERROR   WITH SPURN    13.421      0.031       0.757      0.757
ISOLATE  WITH TERROR   13.421     -0.036      -0.454     -0.454
IGNORE   WITH SPURN     5.964     -0.029      -0.271     -0.271
IGNORE   WITH ISOLATE   8.175      0.039       0.186      0.186
```

It looks like we could add a 1–2 more covariances to ensure not worse fit than the saturated (all 10 correlations) model, but which should be added seems somewhat arbitrary… so I'm calling it done.

Two comparisons are relevant. First, did we help the higher-order factor model by adding two covariances among the factor disturbances? This comparison is not shown here (had to re-estimate the model without them and compare against the model with them), but yes, $\chi^2(2) = 88.343$, $p < .0001$, so yes, model fit is better. Second, does the revised higher-order factor model fit nonsignificantly worse than the saturated structural model with all 10 correlations among the 5 factors? Almost: $\chi^2(3) = 8.483$, $p = .0370$. So our revised model almost captures those 10 correlations using 3 fewer parameters (5 loadings + 2 covariances).

Example results section for IFA using WLSMV:

After examining the fit of each of the five factors individually, as described previously, a combined model was estimated in which all five factors were fit simultaneously with covariances estimated freely among them. A total of 49 items were thus included. WLSMV estimation (i.e., diagonally weighted least squares) in Mplus v 8.8 including a probit link and the THETA parameterization (in which all item residual variances were constrained to 1 for identification) was used to estimate all models (Muthén & Muthén, 1998–2017). Thus, model fit statistics describe the fit of the item factor model to the polychoric correlation matrix among the items. The fit of each model referenced below is shown in Table 1. Nested model comparisons were conducted using the Mplus DIFFTEST procedure.

We first established the need for 5 factors by showing a significant decrease in fit for a single-factor model relative to that of the five-factor model, $\chi^2(10) = 769.755$, $p < .0001$. As shown in Table 1, the fit of the model with five correlated factors was marginally acceptable by both the RMSEA (.057) and the CFI (.927). Standardized model parameters (loadings, intercepts, and residual variances) are shown in Table 2. Correlations ≥ .7 were found amongst the five factors, suggesting evidence that the five factors may indicate a single higher-order factor. This idea was tested by eliminating the covariances among the factors and instead estimating loadings for the five factors from a single higher-order factor (whose variance was fixed to 1). Although the fit of the higher-order factor model remained marginal (see Table 1), a nested model comparison revealed a significant decrease in fit, $\chi^2(5) = 92.048$, $p < .0001$, indicating that a single factor did not appear adequate to describe the pattern of correlation amongst the five factors. Inspection of the discrepancy between the factor correlations from the 5-factor model and those predicted by the higher-order factor indicated two sources of misfit—the correlation between Corrupt and Terror was under-estimated, whereas the correlation between Ignore and Terror was over-estimated. These discrepancies were captured via two additional covariances among those lower-order factor disturbances, resulting in a significant improvement in fit, $\chi^2(2) = 88.343$, $p < .0001$. However, the revised model did not completely account for the pattern of correlation among the 5 factors, as indicated by a significant decrease in model fit relative to the model with all 10 factor correlations estimated directly, $\chi^2(3) = 8.483$, $p = .0370$.