## Higher-Order Factor Models (CFA using MLR; then IFA using WLSMV) in Mplus version 8.4

Example data: 1,336 college students self-reporting on 49 items (measuring five factors) assessing childhood maltreatment. Items are answered on a 1–5 scale: 1=Strongly Disagree, 2=Disagree, 3=Neutral, 4=Agree, 5=Strongly Agree. The item responses are NOT normally distributed, so we'll use both CFA with MLR and IFA with WLSMV as two options to examine the fit of these models (as an example of how to do each, but NOT to compare between estimators).

- 1. Spurning: Verbal and nonverbal caregiver acts that reject and degrade a child
- 2. Terrorizing: Caregiver behaviors that threaten or are likely to physically hurt, kill, abandon, or place the child or the child's loved ones or objects in recognizably dangerous situations.
- 3. Isolating: Caregiver acts that consistently deny the child opportunities to meet needs for interacting or communicating with peers or adults inside or outside the home.
- 4. Corrupting: Caregiver acts that encourage the child to develop inappropriate behaviors (self-destructive, antisocial, criminal, deviant, or other maladaptive behaviors).
- 5. Ignoring: Emotional unresponsiveness includes caregiver acts that ignore the child's attempts and needs to interact (failing to express affection, caring, and love for the child) and show no emotion in interactions with the child

### Here are the results from fitting the 5 factors separately to ensure their individual fit FIRST (see Mplus output files for details):

			A	SESSMENT O	F MODEL FIT	USING MLR						
Model	# Items	# Possible Parms	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	Chi-Square DF	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower Cl	RMSEA Higher Cl	RMSEA p-value
MLR Spurning	12	90	36	224.797	1.4009	54	<.0001	0.959	0.049	0.042	0.055	0.619
MLR Terror	9	54	27	189.815	1.5876	27	<.0001	0.918	0.067	0.058	0.076	0.001
MLR Isolate	6	27	18	80.356	1.4944	9	<.0001	0.916	0.077	0.062	0.093	0.002
MLR Corrupt	7	35	21	54.964	1.9075	14	<.0001	0.934	0.047	0.034	0.060	0.633
MLR Ignore	15	135	45	484.291	1.7921	90	<.0001	0.932	0.057	0.052	0.062	0.008
MLR 1 factor only	49	1274	147	6,183.986	1.4874	1127	<.0001	0.766	0.058	0.057	0.059	<.0001
MLR 5 correlated factors	49	1274	157	4,424.701	1.4645	1117	<.0001	0.847	0.047	0.046	0.049	1.000
MLR 5 factors + higher order	49	1274	152	4,486.381	1.4681	1122	<.0001	0.844	0.047	0.046	0.049	0.999
MLR 5 factors + HO + 2 res cov	49	1274	154	4,422.556	1.4669	1120	<.0001	0.847	0.047	0.046	0.048	1.000
			ASE	SSMENT OF	MODEL FIT U	SING WLSM	v					
Model	# Items	# Possible Parms	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	Chi-Square DF	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower Cl	RMSEA Higher Cl	RMSEA p-value
WLSMV Spurning	12	126	60	294.706		54	<.0001	0.983	0.058	0.051	0.064	0.023
WLSMV Terror	9	81	45	263.155		27	<.0001	0.966	0.081	0.072	0.090	<.0001
WLSMV Isolate	6	45	30	129.828		9	<.0001	0.962	0.100	0.085	0.116	<.0001
WLSMV Corrupt	7	56	35	87.487		14	<.0001	0.976	0.063	0.050	0.076	0.044
WLSMV Ignore	15	180	75	897.689		90	<.0001	0.976	0.082	0.077	0.087	<.0001
WLSMV 1 factor only	49	1421	245	7,563.407		1127	<.0001	0.903	0.065	0.064	0.067	<.0001
WLSMV 5 correlated factors	49	1421	255	5,934.136		1117	<.0001	0.927	0.057	0.055	0.058	<.0001
WLSMV 5 factors + higher order	49	1421	250	5,941.909		1122	<.0001	0.927	0.057	0.055	0.058	<.0001
WLSMV 5 factors + HO + 2 res cov	49	1421	252	5,853.773		1122	<.0001	0.928	0.056	0.055	0.058	<.0001

Here are the standardized factor loadings for each item under each estimation method. Note that the WLSMV factor loadings are higher in this case—probably because of range restriction in the original data and thus the implausibility of a linear model.

MLR	WLSMV	MLR	WLSMV	MLR	WLSMV		MLR	WLSMV		MLR	WLSMV
Spurning	Spurning	Terror	Terror	Isolate	Isolate	_	Corrupt	Corrupt	_	Ignore	Ignore
0.599	0.660	0.512	0.617	0.521	0.695		0.589	0.739	-	0.672	0.813
0.457	0.528	0.673	0.771	0.550	0.630		0.545	0.713		0.654	0.749
0.769	0.837	0.451	0.713	0.545	0.685		0.375	0.523		0.657	0.748
0.526	0.597	0.612	0.721	0.540	0.629		0.545	0.854		0.724	0.801
0.607	0.677	0.571	0.787	0.563	0.726		0.631	0.826		0.445	0.540
0.816	0.865	0.554	0.617	0.752	0.822		0.580	0.708		0.745	0.833
0.835	0.907	0.685	0.805				0.646	0.840		0.847	0.913
0.465	0.538	0.643	0.743							0.713	0.813
0.516	0.728	0.732	0.815							0.808	0.891
0.655	0.744									0.749	0.845
0.674	0.756									0.656	0.795
0.610	0.680									0.830	0.904
										0.712	0.806
										0.739	0.815
										0.825	0.918

# Strawman model: Syntax for single-factor CFA model estimated using MLR through 5 PERFECTLY correlated factors

DATA: FILE = abuse.csv; ! Don't need path if in same folder as input	THE MODEL ESTIMATION TERMINATED NORMALLY
TYPE = INDIVIDUAL; FORMAT = FREE; ! Defaults	THE MODEL ESTIMATION TERMINATED NORMALLI
VARIABLE:	Decourse the factor equation on some fixed to 4 years will and the
NAMES = ID ! All variables in DATA SET	Because the factor covariances were fixed to 1, you will see the
p01 p02 p03 p04 p05 p06 p07 p08 p09 p10	message below. In THIS CONTEXT ONLY, you can ignore it.
p11 p12 p13 p14 p15 p16 p17 p18 p19 p20	
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30	WARNING: THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE
p31 p32 p33 p34 p35 p36 p37 p38 p39 p40	DEFINITE. THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE
p41 p42 p43 p44 p45 p46 p47 p48 p49 p50	FOR A LATENT VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN
p51 p52 p53 p54 p55 p56 p57;	TWO LATENT VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO
psi psi psi psi psi psi psi psi,	LATENT VARIABLES. CHECK THE TECH4 OUTPUT FOR MORE INFORMATION.
USEVARIABLES = ! All variables in MODEL	PROBLEM INVOLVING VARIABLE TERROR.
p01 p02 p03 p04 p06 p07 p09 p10	
p11 p12 p13 p14 p16 p17 p18 p19 p20	MODEL FIT INFORMATION
p11 p12 p13 p14 p16 p17 p16 p19 p20 p21 p22 p23 p24 p25 p26 p27 p28 p29 p30	Number of Free Parameters 147
p31 p33 p35 p36 p37 p39 p40	Loglikelihood
p43 p44 p45 p46 p47 p48 p49 p50	H0 Value -70386.526
p51 p52 p53 p54 p55 p56 p57;	HO Scaling Correction Factor 2.3983
IDVARIABLE = ID; ! Person ID variable	for MLR
MISSING = ALL (99999); ! Missing data value used	H1 Value -65787.405
	H1 Scaling Correction Factor 1.5925
ANALYSIS: ESTIMATOR = MLR; ! For non-normal continuous items	for MLR
OUTPUT: STDYX ! Standardized solution	
MODINDICES (3.84) ! Voodoo for fixing the model	Information Criteria
RESIDUAL ! Local fit info	Akaike (AIC) 141067.051
TECH4; ! Factor correlation matrix	Bayesian (BIC) 141831.074
!SAVEDATA: SAVE = FSCORES; ! Save factor scores	Sample-Size Adjusted BIC 141364.120
! FILE = Abuse Thetas.dat; ! File of factor scores	$n^* = (n + 2) / 24$
! MISSFLAG = 99; ! Indicate missing values	
PLOT: TYPE = PLOT1 PLOT2 PLOT3; ! For pictures	Chi-Square Test of Model Fit
	Value 6183.986*
MODEL: ! (To be changed below for each model)	Degrees of Freedom 1127
	P-Value 0.0000
! 5 Factors (loadings for first item are estimated)	Scaling Correction Factor 1.4874
! 12-Item Spurning	for MLR
Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;	
! 9-Item Terrorizing	RMSEA (Root Mean Square Error Of Approximation)
Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;	Estimate 0.058
! 6-Item Isolating	90 Percent C.I. 0.057 0.059
Isolate BY p01* p18* p19* p23* p39* p43*;	Probability RMSEA <= .05 0.000
! 7-Item Corrupting	
Corrupt BY p09* p12* p16* p20* p28* p47* p50*;	CFI/TLI
! 15-Item Ignoring	CFI 0.766
Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*	TLI 0.756
p45* p46* p51* p52* p57*;	
	SRMR (Standardized Root Mean Square Residual)
! Factor Variances (all must be fixed to 1 for identification)	Value 0.062
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;	
! Factor Means (all fixed = 0 by default)	#free parameters = 147 = 49 loadings + 49 intercepts + 49 residuals
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];	+ 0 factor variances + 0 factor covariances = 147 parameters USED
! Factor Covariance (all fixed to 1 to create 1-factor model)	
Spurn Terror Isolate Corrupt Ignore WITH	
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;	Possible = $49*50/2 + 49 = 1274$
	DF =1117 calculation: 1274 – 147 = 1127

# Syntax for CFA model with MLR including all 5 non-perfectly correlated factors ("saturated structural model") for comparison:

MODEL: ! (To	be change	d below fo	r each mode	1)	NOTE: With respect to fit of the structural	model, letting the 5 factors just be
					correlated is as good as it gets. This satur	
! 5 Factors		for first	item are es	timated)	"larger model" baseline with which to com	
! 12-Item Sp		144 054	074 004 0			
9-Item Ter		14* p25* p	2/* p29* p3	3* p35* p48* p49* p53* p54*	account for these correlations via a higher	r-order factor ( smaller models ).
		n13* n17*	n24* n26* r	36* p55* p56*;	Number of Free Parameters	157
! 6-Item Isc		P10 P1/	p=1 p=0 p		Loglikelihood	137
		×23¢ *19	p39* p43*;		H0 Value	-69027.431
! 7-Item Cor			1 1 ,		HO Scaling Correction Factor	
Corrupt BY	p09* p12*	p16* p20*	p28* p47*	p50*;	for MLR	
! 15-Item Ig					H1 Value	-65787.405
				31* p37* p40* p44*	H1 Scaling Correction Factor	1.5925
	p45* p46*	p51* p52*	p57*;		for MLR	
				for identification)	Information Criteria	
-			rupt@1 Ignc	re@1;	Akaike (AIC)	138368.862
! Factor Mea					Bayesian (BIC)	139184.860
			rrupt@0 Igr	oreguj; w 5-factor model)	Sample-Size Adjusted BIC	138686.140
			ted to allo t Ignore	•	$(n^* = (n + 2) / 24)$	
			t* Ignore*;		Chi-Square Test of Model Fit	
opuin ici	101 10010	ce corrup	,e ignore ,		Value	4424.701*
So do we ha	ve one fact	or or five fa	actors?		Degrees of Freedom	1117
					P-Value	0.0000
A					Scaling Correction Factor	1.4645
				ive to the previous single-	for MLR	
factor model:	: −2ΔLL (10	) = 671.68	9, <i>p</i> < .0001			
					RMSEA (Root Mean Square Error Of Appro	
Therefore, or	ne factor do	es not cap	ture the cov	ariances for these 49 items.	Estimate	0.047
				ntly better job.	90 Percent C.I. Probability RMSEA <= .05	0.046 0.049 1.000
	(	0.200.) 0.00	e a eiginiea		FIODADILITY MISER <05	1.000
Here are the	correlation	s among th	e latent fact	ors we are now trying to	CFI/TLI	
				th a higher-order factor.	CFI	0.847
	with mode				TLI	0.839
					SRMR (Standardized Root Mean Square Re	sidual)
	Coturoted	. E Eastar	All Covariar		Value	0.057
	SPURN	TERROR	ISOLATE	CORRUPT IGNORE	#free parameters = 157 = 49 loadings + 4	9 intercepts + 49 residuals
SPURN	1.000				+ 0 factor variances + 10 factor covaria	
		1.000				
TERROR	.929	1.000			Possible = 49*50/2 + 49 = 1274	
ISOLATE	.898	.876	1.000			
CORRUPT	.689	.792	.658	1.000	DF =1117 calculation: 1274 – 157 = 1117	
IGNORE	.830	.767	.828	.630 1.000	Now we can test the fit of a constrained st	
					higher-order "General Abuse" factor to ac	count for the correlations among
					these 5 latent factors (shown on the left fr	

# Syntax for CFA model with MLR and a higher-order factor instead of correlations among 5 factors ("smaller/bigger model" for comparison):

MODEL: ! (To be changed be	elow for each m	odel)		NOTE: With respect to fit of the structural model, we are now fitting a				
! 5 Lower-Order Factors (	loadings for fi	rst item NOW FIXED	=1)		her-order factor INSTEAD OF co			
! 12-Item Spurning			-/					
Spurn BY p06@1 p10* p14	* 25* 27* 29	* 533* 535* 548* 54	19* 53* 551* .	Number of	Free Parameters	152		
! 9-Item Terrorizing	» pzj» pzi» pzj	• poor poor pier pi	49° p33° p34°,	Loglikeli	hood			
	2+ -17+ -24+ -2	6+			HO Value	-69080.656		
	Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;				H0 Scaling Correction Factor	2.5109		
! 6-Item Isolating	101 001 001	10 h			for MLR			
Isolate BY p01@1 p18* p	19* p23* p39* p	43*;			H1 Value	-65787.405		
! 7-Item Corrupting					H1 Scaling Correction Factor			
Corrupt BY p09@1 p12* p	16* p20* p28* p	47* p50*;			for MLR			
! 15-Item Ignoring								
Ignore BY p02@1 p03* p0		0* p31* p37* p40* <u>p</u>	p44*	Trformati	on Criteria			
p45* p46* p5	1* p52* p57*;			IIIOIIIIau		138465.313		
					Akaike (AIC)			
! Factor Variances (all m	ust be free - N	W ARE "DISTURBANCE	ES")		Bayesian (BIC)	139255.323		
Spurn* Terror* Isolate*					Sample-Size Adjusted BIC	138772.486		
! Factor Means (all fixed					$(n^* = (n + 2) / 24)$			
[Spurn@0 Terror@0 Isola								
	Colt Collapsed	-9		Chi-Squar	re Test of Model Fit			
! Higher-Order Factor (es	timate bigher-o	rder factor loading	ae)		Value	4486.381*	*	
Abuse BY Spurn* Terror*	-		95)		Degrees of Freedom	1122		
-					P-Value	0.0000		
! Fix higher-order factor	mean=0 & varia	nce=1			Scaling Correction Factor	1.4681		
[Abuse@0]; Abuse@1;					for MLR			
model against the saturated correlations. This higher-ord 5 higher-order loadings rep	We can use a $-2\Delta LL$ scaled difference to test the fit of the higher-order factor model against the saturated structural model with all possible factor correlations. This higher-order factor model uses 5 fewer parameters: 5 higher-order loadings replace the 10 covariances among the factors. The $-2\Delta LL$ scaled difference is $-2\Delta LL$ (5) = 46.848, $p < .0001$ .				oot Mean Square Error Of Appro Estimate 90 Percent C.I. Probability RMSEA <= .05 CFI TLI	0.047	0.049	
So trying to reproduce the 5	5 factor covarian	res with a single hig	her-order					
				SRMR (Sta	andardized Root Mean Square Re	sidual)		
factor results in a significan		, ,			Value	0.058		
the lower-order factors have	e large (enough)	standardized loadin	igs					
STDYX Standardization Two-Tailed					ameters = 152 = 44 loadings + 4 ctor variances + 5 higher-order lo			
Estim ABUSE BY (HIGHER-ORDER			140	Possible	= 49*50/2 + 49 = 1274			
	971 0.010		000					
			000		7 calculation: 1274 – 152 = 1122			
	952 0.011							
	933 0.016		000					
	745 0.027		000					
IGNORE 0.	846 0.018	48.111 0.0	000					

## Higher-Order Factor Model Output; Comparison of Saturated versus Higher-Order Factor Model predicted correlations:

MODEL MOI	DIFICA	ATION INDICES				
Minimum M	3.840					
			M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
TERROR	WITH	SPURN	4.111	0.011	0.454	0.454
CORRUPT	WITH	SPURN	18.864	-0.018	-0.451	-0.451
CORRUPT	WITH	TERROR	44.080	0.021	0.595	0.595
CORRUPT	WITH	ISOLATE	4.628	-0.006	-0.193	-0.193
IGNORE	WITH	SPURN	4.800	0.010	0.248	0.248
IGNORE	WITH	TERROR	31.774	-0.018	-0.510	-0.510
IGNORE	WITH	ISOLATE	14.098	0.010	0.317	0.317

Based on the modification indices (which are picking up on the discrepancies between the saturated model and higher-order factor model in the factor correlations), it appears we need to allow two more relationships among the factor disturbances, as follows:

#### MODEL: ! (To be changed below for each model)

- ! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)
- ! 12-Item Spurning
- Spurn BY p06@1 p10\* p14\* p25\* p27\* p29\* p33\* p35\* p48\* p49\* p53\* p54\*; ! 9-Item Terrorizing
- Terror BY p07@1 p11\* p13\* p17\* p24\* p26\* p36\* p55\* p56\*;
- ! 6-Item Isolating
- Isolate BY p01@1 p18\* p19\* p23\* p39\* p43\*;
- ! 7-Item Corrupting Corrupt BY p09@1 p12\* p16\* p20\* p28\* p47\* p50\*;
- ! 15-Item Ignoring
  - Ignore BY p02@1 p03\* p04\* p21\* p22\* p30\* p31\* p37\* p40\* p44\* p45\* p46\* p51\* p52\* p57\*;
- ! Factor Variances (all must be free NOW ARE "DISTURBANCES")
  Spurn\* Terror\* Isolate\* Corrupt\* Ignore\*;
- ! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
- ! Higher-Order Factor (estimate higher-order factor loadings) Abuse BY Spurn\* Terror\* Isolate\* Corrupt\* Ignore\*;
- ! Fix higher-order factor mean=0 & variance=1
  [Abuse@0]; Abuse@1;
- ! Add disturbance covariances suggested by voo-doo Corrupt WITH Terror\*; Ignore WITH Terror\*;

		MLR So	olutions							
	Saturated	l: 5-Factor A	All Covarian	ces Model						
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE					
	1.000									
۲	.929	1.000								
Ξ	.898	.876	1.000							
PT	.689	.792	.658	1.000						
	.830	.767	.828	.630	1.000					
Predicted 1: 5-Factor + Higher-Order Factor Model										
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE					
	1.000									
8	.925	1.000								
Ξ	.906	.889	1.000							
PT	.724	.710	.696	1.000						
	.821	.806	.790	.631	1.000					
	Discrep	bancy: Satu	rated - Pred	licted 1						
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE					
{	.004									
Ξ	008	013								

SPURN TERROR ISOLATE CORRUPT IGNORE

SPURN

TERROR

ISOLATE

IGNORE

SPURN

TERROR

CORRUPT

PSQF 6249 Example 8 page 6

ISOLATE	008	013			
CORRUPT	035	.082	038		
IGNORE	.009	039	.038	001	

#### Predicted 2: 5-Factor + Higher-Order Factor + 2 Fact Cov Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE				
SPURN	1.000								
TERROR	.923	1.000							
ISOLATE	.898	.894	1.000						
CORRUPT	.690	.794	.668	1.000					
IGNORE	.838	.766	.812	.623	1.000				

Discrepancy: Saturated - Predicted 2								
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE			
SPURN								
TERROR	.006							
ISOLATE	.000	018						
CORRUPT	001	002	010					
IGNORE	008	.001	.016	.007				

					PSQF 6249 Example 8 page 7
MODEL FIT INFORMAT					Two comparisons are relevant.
Number of Free Par	ameters		154		
Loglikelihood					First, did we help the higher-order factor model by adding two covariances among the
HO Value			-69031.180		factor disturbances? $-2\Delta LL(2) = 46.378$ , $p < .0001$ , so yes, model fit is better.
	.ng Correction	Factor	2.5060		Tactor disturbances: $2\Delta EE(2) = 40.576$ , $p < .0007$ , 30 yes, model in is better.
for ML	-	raccor	2.0000		
H1 Value			-65787.405		Second, does the revised higher-order factor model fit nonsignificantly worse than the
	ng Correction	Factor	1.5925		saturated structural model with all 10 correlations among the 5 factors? $-2\Delta LL$ (3) =
for ML	-				3.171, $p = .3660$ , so yes, our revised model captures those 10 correlations using 3
-					fewer parameters (5 loadings + 2 covariances).
Information Criter	ia				iewei parameters (5 loadings + 2 covanances).
Akaike (	AIC)		138370.360		
Bayesian	(BIC)		139170.765		Example results section for CFA using MLR:
Sample-S	ize Adjusted	BIC	138681.575		
(n* =	(n + 2) / 24)				After examining the fit of each of the five factors individually, as described previously,
Chi-Square Test of	Model Fit				a combined model was estimated in which all five factors were fit simultaneously with
Value			4422.556*		covariances estimated freely among them. A total of 49 items were thus included.
-	of Freedom		1120		Robust maximum likelihood (MLR) estimation was used to estimate all models using
P-Value			0.0000		Mplus v. 8.4 (Muthén & Muthén, 1998–2017), and differences in fit between nested
_	Correction Fa	ctor	1.4669		models were evaluated using $-2^*$ rescaled difference in the model log-likelihood
for MI	ιR				
		C	to a fact a sex y		values. The fit of each model referenced below is shown in Table 1.
RMSEA (Root Mean S		i Approx			
Estimate 90 Perce			0.047	0.048	We first established the need for 5 factors by showing a significant decrease in fit for
	ity RMSEA <=	05	1.000	0.040	a single-factor model relative to that of the 5-factor model, $-2\Delta LL$ (10) = 671.689, p <
TIODADII	LICY MIDDA (-	.05	1.000		.0001. As shown in Table 1, the fit of the model with five correlated factors was
CFI/TLI					
CFI			0.847		acceptable by the RMSEA (.047), but not by the CFI (.847). Standardized model
TLI			0.840		parameters (loadings, intercepts, and residual variances) are shown in Table 2.
					Correlations of .6 or higher were found amongst the five factors, suggesting evidence
SRMR (Standardized	l Root Mean Sq	uare Res	idual)		that the five factors may indicate a single higher-order factor. This idea was testing by
Value	-		0.057		
					eliminating the covariances among the factors and instead estimating loadings for the
					five factors from a single higher-order factor (whose variance was fixed to 1).
STDYX Standardizat	ion				Although the fit of the higher-order factor model remained marginal (see Table 1), a
				Two-Tailed	nested model comparison revealed a significant decrease in fit, $-2\Delta LL(5) = 46.848$ , p
	Estimate		Est./S.E.	P-Value	< .0001, indicating that a single factor did not appear adequate to describe the pattern
	R-ORDER STAND				
SPURN	0.963	0.011		0.000	of correlation amongst the five factors. Inspection of the discrepancy between the
TERROR	0.958	0.012		0.000	factor correlations from the 5-factor model and those predicted by the higher-order
ISOLATE	0.933	0.016		0.000	factor indicated two sources of misfit—the correlation between Corrupt and Terror
CORRUPT	0.716	0.028		0.000	was under-estimated, whereas the correlation between Ignore and Terror was over-
IGNORE	0.870	0.019	45.845	0.000	estimated. These discrepancies were captured via two additional covariances among
CORRUPT WITH TERROR	0.540	0.097	5.550	0.000	those lower-order factor disturbances, resulting in a significant improvement in fit,
IGNORE WITH	0.340	0.097	5.550	0.000	$-2\Delta LL(2) = 46.378$ , $p < .0001$ . Further, the revised model successfully accounted for
TERROR	-0.483	0.172	-2.811	0.005	the pattern of correlation among the 5 factors, as indicated by a nonsignificant
1210.010	0.100	V • ± / Z	2.011	0.000	decrease in model fit relative to the model with all 10 factor correlations estimated
					directly, $-2\Delta LL(3) = 3.171$ , $p = .3660$ .
					$\mu_{\text{HCOHy}}$ , $2\Delta LL(3) = 3.171, \mu = .3000.$
Next we will dup	licate these	analyse	s using WI	SMV, which	
requires starting	with the big	iyest m	odel illist		

Syntax for IFA model with WLSMV including all 5 non-perfectly correlated factors ("saturated structural model") for comparison:

```
DATA: FILE = abuse.csv; ! Don't need path if in same folder as input
      TYPE = INDIVIDUAL; FORMAT = FREE; ! Defaults
                                                                      MODEL: ! (To be changed below for each model)
VARIABLE:
NAMES = ID ! All variables in DATA SET
                                                                      ! 5 Factors (loadings for first item are estimated)
 p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
                                                                      ! 12-Item Spurning
 p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
                                                                       Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
 p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
                                                                      ! 9-Item Terrorizing
 p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
                                                                        Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;
 p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
                                                                      ! 6-Item Isolating
                                                                       Isolate BY p01* p18* p19* p23* p39* p43*;
 p51 p52 p53 p54 p55 p56 p57;
                                                                      ! 7-Item Corrupting
USEVARIABLES = ! All variables in MODEL
                                                                        Corrupt BY p09* p12* p16* p20* p28* p47* p50*;
                   p06 p07
 p01 p02 p03 p04
                                 01g 60g
                                                                      ! 15-Item Ignoring
                                                                       Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
 p11 p12 p13 p14 p16 p17 p18 p19 p20
                                                                                  p45* p46* p51* p52* p57*;
 p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
 p31 p33 p35 p36 p37 p39 p40
         p43 p44 p45 p46 p47 p48 p49 p50
                                                                      ! Factor Variances (all must be fixed to 1 for identification)
                                                                        Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
 p51 p52 p53 p54 p55 p56 p57;
                                                                      ! Factor Means (all fixed = 0 by default)
IDVARIABLE = ID;
                         ! Person ID variable
                                                                        [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
MISSING = ALL (99999);
                        ! Missing data value used
                                                                      ! Factor Covariances (all estimated to allow 5-factor model)
                                                                        Spurn Terror Isolate Corrupt Ignore WITH
CATEGORICAL = ! All ordinal outcomes for IFA
                                                                        Spurn* Terror* Isolate* Corrupt* Ignore*;
 p01 p02 p03 p04 p06 p07
                                 p09 p10
                  p16 p17 p18 p19 p20
p11 p12 p13 p14
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
                                                                      NOTE: With respect to fit of the structural model, letting the 5 factors just be
      p33 p35 p36 p37 p39 p40
p31
        p43 p44 p45 p46 p47 p48 p49 p50
                                                                      correlated is as good as it gets. This saturated structural model will be our
p51 p52 p53 p54 p55 p56 p57;
                                                                      "larger model" baseline with which to compare the fit of models that try to
                                                                      account for these correlations via a higher-order factor ("smaller models").
OUTPUT:
                              ! Standardized solution
            STDYX
           MODINDICES(3.84) ! Voodoo for fixing the model
                                                                      MODEL FIT INFORMATION
           RESIDUAL
                              ! Local fit info
                                                                      Number of Free Parameters
                                                                                                                     255
            TECH4:
                              ! Factor correlation matrix
PLOT:
           TYPE = PLOT1 PLOT2 PLOT3; ! For pictures
                                                                      Chi-Square Test of Model Fit
                                                                                                               5934.136*
                                                                                Value
ANALYSIS: ESTIMATOR = WLSMV;
                                      ! Limited-info in probits
                                                                                Degrees of Freedom
                                                                                                                  1117
            PARAMETERIZATION = THETA;
                                                                                P-Value
                                                                                                                  0.0000
            CONVERGENCE = 0.0000001; ! For OS comparability
                                                                      RMSEA (Root Mean Square Error Of Approximation)
SAVEDATA: DIFFTEST=5factor.dat:
                                      ! Save fit of 5-factor model
                                                                               Estimate
                                                                                                                  0.057
                                                                                                                  0.055 0.058
                                                                                90 Percent C.I.
                                                                                Probability RMSEA <= .05
                                                                                                                 0.000
#free parameters = 255 = 49 loadings + 49*4=196 thresholds
                                                                      CFI/TLI
   + 0 factor variances + 10 factor covariances = 255 parameters USED
                                                                                CFT
                                                                                                                   0.927
                                                                                TLI
                                                                                                                   0.923
Possible = 49*50/2 + 49*4 = 1421
                                                                      SRMR (Standardized Root Mean Square Residual)
DF =1117 calculation: 1421 - 255 - 49 "residual variances" = 1117
                                                                                Value
                                                                                                                   0.056
```

# Strawman model: Syntax for single-factor IFA model estimated using WLSMV through 5 PERFECTLY correlated factors

ANALYSIS: DIFFTEST=5factor.dat; ! Test fit against 5-factor model	THE MODEL ESTIMATION TERMINATED NORMAL	LLY			
! (no SAVEDATA needed)	Because the factor covariances we	ere fixed to 1, you will see the			
MODEL: ! (To be changed below for each model)	message below. In THIS CONTEXT ONLY, you can ignore it.				
! 5 Factors (loadings for first item are estimated)	WARNING: THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE DEFINITE. THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE				
! 12-Item Spurning	FOR A LATENT VARIABLE, A CORRELATION (				
Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;	TWO LATENT VARIABLES, OR A LINEAR DEPH				
<pre>! 9-Item Terrorizing Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;</pre>	LATENT VARIABLES. CHECK THE TECH4 OUT				
! 6-Item Isolating	PROBLEM INVOLVING VARIABLE TERROR.				
Isolate BY p01* p18* p19* p23* p39* p43*;					
! 7-Item Corrupting	MODEL FIT INFORMATION				
Corrupt BY p09* p12* p16* p20* p28* p47* p50*;	Number of Free Parameters	245			
! 15-Item Ignoring					
Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*	Chi-Square Test of Model Fit				
p45* p46* p51* p52* p57*;	Value	7563.407*			
	Degrees of Freedom	1127			
! Factor Variances (all must be fixed to 1 for identification)	P-Value	0.0000			
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;					
! Factor Means (all fixed = 0 by default)	Chi-Square Test for Difference Testing				
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];	Value	769.755*			
! Factor Covariance (all fixed to 1 to create 1-factor model)	Degrees of Freedom P-Value	10 0.0000			
Spurn Terror Isolate Corrupt Ignore WITH	r-value	0.0000			
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;	RMSEA (Root Mean Square Error Of Appro	oximation)			
	Estimate	0.065			
#free percenters 245 40 leadings + 40*4 406 thresholds	90 Percent C.I.	0.064 0.067			
#free parameters = 245 = 49 loadings + 49*4=196 thresholds	Probability RMSEA <= .05	0.000			
+ 0 factor variances + 0 factor covariances = 245 parameters USED	_				
	CFI/TLI				
Possible = 49*50/2 + 49*4 = 1421	CFI	0.903			
DF =1117 calculation: 1421 – 245 – 49 "residual variances" = 1127	TLI	0.898			
	SRMR (Standardized Root Mean Square Re	esidual)			
	Value	0.068			
	Do we have one factor or five factors?				
	According to the DIFFTEST relative to the previous 5-factor model: $\chi^2(10) = 769.755$ , $p < .0001$				
	Therefore, one factor does not capture th Five factors (as hypothesized) does a sig				

# Syntax for IFA model with WLSMV and a higher-order factor instead of correlations among 5 factors ("smaller/bigger model" for comparison):

ANALYSIS: DIFFTEST=5factor.dat; ! Test fit against 5-factor model	NOTE: With respect to fit of the struct	, <b>U</b>	
SAVEDATA: DIFFTEST=HigherOrder.dat; ! Save fit of higher-order model	single higher-order factor INSTEAD	OF covariances among the 5 factors.	
MODEL: ! (To be changed below for each model)	MODEL FIT INFORMATION		
MODEL: ! (To be changed below for each model)	Number of Free Parameters	250	
! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)			
! 12-Item Spurning	Chi-Square Test of Model Fit	5044 0001	
Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;	Value	5941.909*	
! 9-Item Terrorizing	Degrees of Freedom	1122	
Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;	P-Value	0.0000	
! 6-Item Isolating			
Isolate BY p01@1 p18* p19* p23* p39* p43*;	Chi-Square Test for Difference Tes	-	
! 7-Item Corrupting	Value Degrees of Freedom	92.048*	
Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;	P-Value	0.0000	
! 15-Item Ignoring	P-value	0.0000	
Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*	RMSEA (Root Mean Square Error Of A	(norowimation)	
p45* p46* p51* p52* p57*;	Estimate	0.057	
	90 Percent C.I.	0.055 0.058	
! Factor Variances (all must be free - NOW ARE "DISTURBANCES")	Probability RMSEA <= .05		
Spurn* Terror* Isolate* Corrupt* Ignore*;	FIODADILICY RMSEA <03	0.000	
! Factor Means (all fixed = 0 by default)	CFI/TLI		
<pre>[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];</pre>	CFI	0.927	
	TLI	0.924	
! Higher-Order Factor (estimate higher-order factor loadings)	111	0.924	
Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;	SRMR (Standardized Root Mean Squar	Peridual)	
! Fix higher-order factor mean=0 & variance=1	Value 0.057		
[Abuse@0]; Abuse@1;	Varue	0.037	
	#free parameters = 250 = 44 loading	a + 40*4 - 106 througholds	
	+ 5 factor variances + 5 higher-ol	rder loadings = 250 parameters USED	
We can use DIFFTEST to test the fit of the higher-order factor model against			
the saturated structural model with all possible factor correlations. This higher-	Possible = 49*50/2 + 49*4 = 1421		
order factor model uses 5 fewer parameters: 5 higher-order loadings replace	DF =1117 calculation: 1421 – 250 –	40 "residual variances" - 1122	
	D1 = 1117 calculation. 1421 = 250 = 7	49 Tesidual valiances – 1122	
the 10 covariances among the factors. The difference is $\chi^2(5) = 92.048$ , <i>p</i> <			
.0001.	STDYX Standardization		
		Two-Tailed	
So trying to reproduce the 5 factor covariances with a single higher-order	Estimate	S.E. Est./S.E. P-Value	
So trying to reproduce the 5 factor covariances with a single higher-order	ABUSE BY (HIGHER-ORDER STANDARD	DIZED LOADINGS)	
factor results in a significant decrease in fit. Why might this be the case? All	ABUSE BY (HIGHER-ORDER STANDARD SPURN 0.990	DIZED LOADINGS) 0.005 204.056 0.000	
	ABUSE BY (HIGHER-ORDER STANDARD SPURN 0.990 TERROR 0.948	DIZED LOADINGS) 0.005 204.056 0.000 0.007 139.928 0.000	
factor results in a significant decrease in fit. Why might this be the case? All	ABUSE BY (HIGHER-ORDER STANDARD SPURN 0.990 TERROR 0.948 ISOLATE 0.951	DIZED LOADINGS) 0.005 204.056 0.000 0.007 139.928 0.000 0.009 106.595 0.000	
factor results in a significant decrease in fit. Why might this be the case? All	ABUSE BY (HIGHER-ORDER STANDARD SPURN 0.990 TERROR 0.948 ISOLATE 0.951 CORRUPT 0.835	DIZED LOADINGS) 0.005 204.056 0.000 0.007 139.928 0.000	

## Higher-Order Factor Model Output; Comparison of Saturated versus Higher-Order Factor Model predicted correlations:

MODEL MODIFICATION INDICES Minimum M.I. value for printing the modification index						3.840
			M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
TERROR	WITH	SPURN	8.776	0.018	0.558	0.558
ISOLATE	WITH	SPURN	11.743	-0.025	-0.742	-0.742
ISOLATE	WITH	TERROR	5.966	-0.022	-0.256	-0.256
CORRUPT	WITH	SPURN	39.197	-0.056	-0.762	-0.762
CORRUPT	WITH	TERROR	122.583	0.116	0.627	0.627
IGNORE	WITH	SPURN	25.058	0.050	0.596	0.596
IGNORE	WITH	TERROR	82.830	-0.100	-0.471	-0.471
IGNORE	WITH	ISOLATE	42.440	0.080	0.372	0.372
IGNORE	WITH	CORRUPT	6.035	-0.036	-0.077	-0.077

Based on the modification indices (which are picking up on the discrepancies between the saturated model and higher-order factor model in the factor correlations, it appears we need to allow two more relationships among the factor disturbances, as follows:

ANALYSIS: DIFFTEST=5factor.dat; ! Test fit against 5-factor model
SAVEDATA: DIFFTEST=HigherOrder2.dat; ! Save fit of higher-order2 model
MODEL: ! (To be changed below for each model)
! 5 Lower-Order Factors (loadings for first item NOW FIXED =1) ! 12-Item Spurning
Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing Terror BY p0701 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating Isolate BY p0101 p18* p19* p23* p39* p43*;
! 7-Item Corrupting Corrupt BY p0901 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
p45* p46* p51* p52* p57*;
<pre>! Factor Variances (all must be free - NOW ARE "DISTURBANCES") Spurn* Terror* Isolate* Corrupt* Ignore*;</pre>
! Factor Means (all fixed = 0 by default)
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
! Higher-Order Factor (estimate higher-order factor loadings)
Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*; ! Fix higher-order factor mean=0 & variance=1
[Abuse@0]; Abuse@1;
! Add disturbance covariances suggested by voo-doo Corrupt WITH Terror*;
Ignore WITH Terror*;

Saturated: 5-Factor All Covariances Model							
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE		
SPURN	1.000						
TERROR	.947	1.000					
ISOLATE	.925	.885	1.000				
CORRUPT	.791	.866	.776	1.000			
IGNORE	.882	.817	.863	.729	1.000		
Prec	licted 1: 5-	Factor + Hi		r Factor Mo	odel		
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE		
SPURN	1.000						
TERROR	.938	1.000					
ISOLATE	.941	.902	1.000				
CORRUPT	.826	.791	.794	1.000			
IGNORE	.876	.839	.841	.738	1.000		
	Discrepa	ancy: Satur	ated - Pre	dicted 1			
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE		
SPURN							
TERROR	.009						
ISOLATE	016	017					
CORRUPT	035	.075	018				
IGNORE	.006	022	.022	009			
Predicted	2: 5-Factor	+ Higher-C	Order Facto	or + 2 Fact (	Cov Model		
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE		
SPURN	1.000						
TERROR	.939	1.000					
ISOLATE	.927	.907	1.000				
CORRUPT	.792	.866	.765	1.000			
IGNORE	.885	.817	.855	.730	1.000		
	Discrepa	ancv: Satur	ated - Pre	dicted 2			
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE		
SPURN							
TERROR	.008						
ISOLATE	002	022					
CORRUPT	001	.000	.011				
IGNORE	003	.000	.008	001			
IGNORE	005	.000	.008	001			

	INFORMATION				
Number of	Free Paramet	ers		252	
Chi-Square	e Test of Mod	el Fit			
	Value			5853.773*	
	Degrees of F	reedom		1120	
	P-Value			0.0000	
Chi-Square	e Test for Di	fference	Testing		
-	Value		2	8.483*	
	Degrees of F	reedom		3	
	P-Value			0.0370	
RMSEA (Roo	ot Mean Squar	e Error O	f Approxi	mation)	
	Estimate			0.056	
	90 Percent C	.I.		0.055	0.058
	Probability	RMSEA <=	.05	0.000	
CFI/TLI					
011/111	CFI			0.928	
	TLI			0.925	
ODMD (Char	daudi-ad Daa	+ Maan Ca	Deed		
SRMR (Star	ndardized Roo Value	t Mean Sq	uare kesi	0.056	
	value			0.058	
STDYX Star	ndardization				m., m./l.i
	Π	*****	0 5		Two-Tailed
ABUSE F	ES BY (HIGHER-OR	timate		Est./S.E.	P-Value
SPURN				173.657	0.000
TERROF	२	0.959	0.007		
ISOLAT	ſE	0.946	0.009		
CORRUE	PT	0.809	0.015		0.000
IGNORE	Ξ	0.903	0.009	97.384	0.000
CORRUPT	WITH				
TERROF		0.544	0.068	7.984	0.000
IGNORE					
TERROF	ર	-0.406	0.102	-3.991	0.000
MODEL MODI	IFICATION IND	TOPO			
			the modi	fication in	dex 3.840
		1			
		M.I.			StdYX E.P.C.
	VITH SPURN	13.421	0.031		0.757
	VITH TERROR	13.421	-0.036		
	VITH SPURN	5.964	-0.029 0.039		-0.271 0.186
IGNORE V	VITH ISOLATE	8.175	0.039	0.100	0.100
It looks li	ike we could	d add a 1	-2 more	covarianc	es to ensure
					ons) model,
but whic	n should be	added s	seems so	omewhat a	rbitrary so

I'm calling it done.

PSQF 6249 Example 8 page 12 Two comparisons are relevant. First, did we help the higher-order factor model by adding two covariances among the factor disturbances? This comparison is not shown here (had to re-run model without them and compare against model with them), but yes,  $\chi^2(2) = 88.343$ , p < .0001, so yes, model fit is better. Second, does the revised higher-order factor model fit nonsignificantly worse than the saturated structural model with all 10 correlations among the 5 factors? Almost:  $\chi^2(3)$ = 8.483, p = .0370. So our revised model almost captures those 10 correlations using 3 fewer parameters (5 loadings + 2 covariances).

### Example results section for IFA using WLSMV:

After examining the fit of each of the five factors individually, as described previously, a combined model was estimated in which all five factors were fit simultaneously with covariances estimated freely among them. A total of 49 items were thus included. WLSMV estimation (i.e., diagonally weighted least squares) in Mplus v 8.4 including a probit link and the THETA parameterization (such that all item residual variances were constrained to 1) was used to estimate all models (Muthén & Muthén, 1998–2017). Thus, model fit statistics describe the fit of the item factor model to the polychoric correlation matrix among the items. The fit of each model referenced below is shown in Table 1. Nested model comparisons were conducted using the Mplus DIFFTEST procedure.

We first established the need for 5 factors by showing a significant decrease in fit for a single-factor model relative to that of the 5-factor model,  $\chi^2(10) = 769.755$ , p < 100.0001. As shown in Table 1, the fit of the model with five correlated factors was marginally acceptable by both the RMSEA (.057) and the CFI (.927). Standardized model parameters (loadings, intercepts, and residual variances) are shown in Table 2. Correlations of .7 or higher were found amongst the five factors, suggesting evidence that the five factors may indicate a single higher-order factor. This idea was testing by eliminating the covariances among the factors and instead estimating loadings for the five factors from a single higher-order factor (whose variance was fixed to 1). Although the fit of the higher-order factor model remained marginal (see Table 1), a nested model comparison revealed a significant decrease in fit,  $\chi^2(5) = 92.048$ , p < .0001, indicating that a single factor did not appear adequate to describe the pattern of correlation amongst the five factors. Inspection of the discrepancy between the factor correlations from the 5-factor model and those predicted by the higher-order factor indicated two sources of misfit—the correlation between Corrupt and Terror was under-estimated, whereas the correlation between Ignore and Terror was overestimated. These discrepancies were captured via two additional covariances among those lower-order factor disturbances, resulting in a significant improvement in fit,  $\chi^2$ (2) = 88.343, p < .0001. However, the revised model did not completely account for the pattern of correlation among the 5 factors, as indicated by a significant decrease in model fit relative to the model with all 10 factor correlations estimated directly,  $\chi^2(3)$ = 8.483, p = .0370.