

### Binary IFA-IRT Models in *Mplus* v. 8.4

Example data: 635 older adults (age 80–100) self-reporting on 7 items assessing the Instrumental Activities of Daily Living (IADL) as follows:

1. Housework (cleaning and laundry): 1=64%
2. Bedmaking: 1=84%
3. Cooking: 1=77%
4. Everyday shopping: 1=66%
5. Getting to places outside of walking distance: 1=65%
6. Handling banking and other business: 1=73%
7. Using the telephone 1=94%

Two versions of a response format were available:

Binary → 0 = needs help, 1 = does not need help

Categorical → 0 = can't do it, 1 = big problems, 2 = some problems, 3 = no problems

Higher scores indicate greater function. We will look at each response format in turn.

### Binary 2-PL Model Syntax (left) and 1-PL Model Syntax (right) using ML and a logit link:

<pre> <b>TITLE:</b> Assess binary IADL items using 2PL under full-info ML <b>DATA:</b> FILE = Example5.csv; ! Don't need path if in same folder         FORMAT = free; ! Default         TYPE = INDIVIDUAL; ! Default  <b>VARIABLE:</b> NAMES = case dial-dia7 cial-cia7; ! All vars in data         USEVARIABLES = dial-dia7; ! All vars in model         CATEGORICAL = dial-dia7; ! All ordinal outcomes         MISSING = ALL (99999); ! Missing value code         IDVARIABLE = case; ! Person ID variable  <b>ANALYSIS:</b> TYPE = GENERAL; ! Default         ESTIMATOR = ML; LINK = LOGIT; ! Full-info ML in logits         CONVERGENCE = 0.0000001; ! For OS comparability  <b>MODEL:</b> ! Factor loadings all estimated in 2PL IADL BY dial-dia7*; ! Item thresholds all estimated [dial\$1-dia7\$1*]; ! Factor mean=0 and variance=1 for identification [IADL@0]; IADL@1;  <b>OUTPUT:</b> STDYX; ! Standardized solution         RESIDUAL TECH10; ! Local fit info  <b>SAVEDATA:</b> SAVE = FSCORES; ! Save factor scores (thetas)         FILE = IADL_2PLThetas.dat; ! File factor scores saved to         MISSFLAG = 99999; ! Missing data value in file  <b>PLOT:</b> TYPE = PLOT1; ! PLOT1 gets you sample descriptives         TYPE = PLOT2; ! PLOT2 gets you the IRT-relevant curves         TYPE = PLOT3; ! PLOT3 gets you descriptives for theta     </pre>	<pre> <b>TITLE:</b> Assess binary IADL items using 1PL under full-info ML <b>DATA:</b> FILE = Example5.csv; ! Don't need path if in same directory         FORMAT = free; ! Default         TYPE = INDIVIDUAL; ! Default  <b>VARIABLE:</b> NAMES = case dial-dia7 cial-cia7; ! All vars in data         USEVARIABLES = dial-dia7; ! All vars in model         CATEGORICAL = dial-dia7; ! All ordinal outcomes         MISSING = ALL (99999); ! Missing value code         IDVARIABLE = case; ! Person ID variable  <b>ANALYSIS:</b> TYPE = GENERAL; ! Default         ESTIMATOR = ML; LINK = LOGIT; ! Full-info ML in logits         CONVERGENCE = 0.0000001; ! For OS comparability  <b>MODEL:</b> ! Factor loadings all held equal in 1PL IADL BY dial-dia7* (loading); ! Item thresholds all estimated [dial\$1-dia7\$1*]; ! Factor mean=0 and variance=1 for identification [IADL@0]; IADL@1;  <b>OUTPUT:</b> STDYX; ! Standardized solution         RESIDUAL TECH10; ! Local fit info  <b>SAVEDATA:</b> SAVE = FSCORES; ! Save factor scores (thetas)         FILE = IADL_1PLThetas.dat; ! File factor scores saved to         MISSFLAG = 99999; ! Missing data value in file  <b>PLOT:</b> TYPE = PLOT1; ! PLOT1 gets you sample descriptives         TYPE = PLOT2; ! PLOT2 gets you the IRT-relevant curves         TYPE = PLOT3; ! PLOT3 gets you descriptives for theta     </pre>
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**Binary 2-PL Model Fit (left) and 1-PL Model Fit (right) using ML logit:**

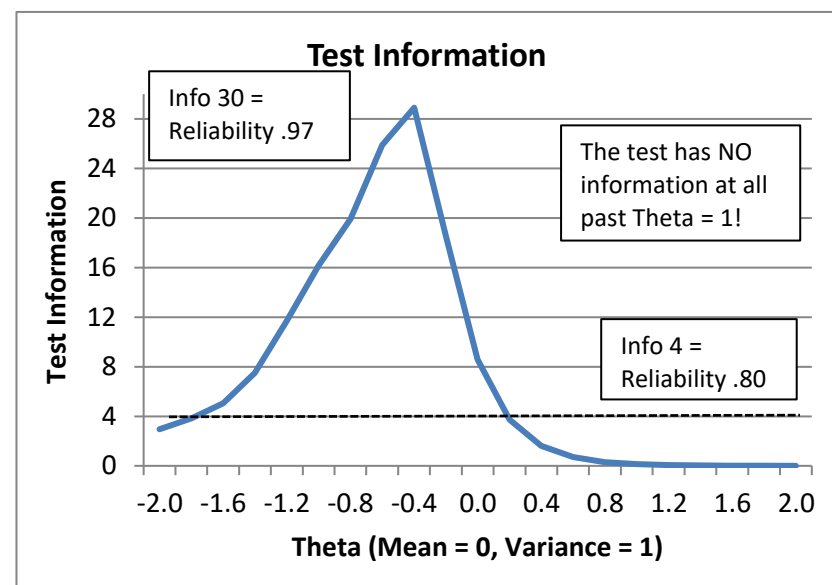
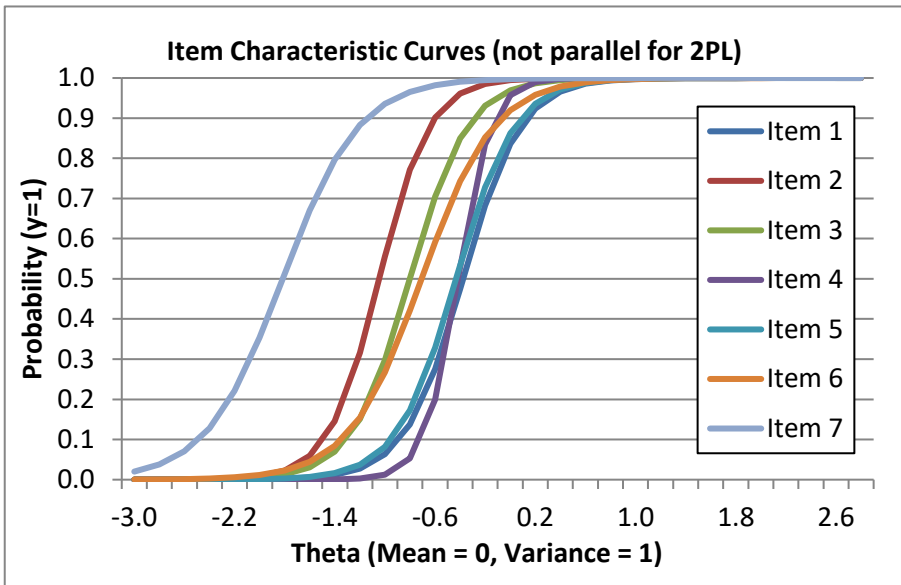
MODEL FIT INFORMATION - 2PL	MODEL FIT INFORMATION - 1 PL
Number of Free Parameters 14	Number of Free Parameters 8
Loglikelihood	Loglikelihood
H0 Value -1454.634	H0 Value -1464.457
Information Criteria	Information Criteria
Akaike (AIC) 2937.268	Akaike (AIC) 2944.915
Bayesian (BIC) 2999.619	Bayesian (BIC) 2980.544
Sample-Size Adjusted BIC 2955.170 (n* = (n + 2) / 24)	Sample-Size Adjusted BIC 2955.144 (n* = (n + 2) / 24)
Chi-Square Test of Model Fit for the Binary and Ordered Categorical (Ordinal) Outcomes	Chi-Square Test of Model Fit for the Binary and Ordered Categorical (Ordinal) Outcomes**
Pearson Chi-Square	Pearson Chi-Square
Value 340.829	Value 296.199
Degrees of Freedom 113	Degrees of Freedom 118
P-Value 0.0000	P-Value 0.0000
Likelihood Ratio Chi-Square	Likelihood Ratio Chi-Square
Value 120.273	Value 126.354
Degrees of Freedom 113	Degrees of Freedom 118
P-Value 0.3023	P-Value 0.2828
Linda Muthén suggests that if these 2 $\chi^2$ values don't match, they should not be used to assess model fit.	** Of the 630 cells in the latent class indicator table, 1 were deleted in the calculation of chi-square due to extreme values.
Further, the possible total df for the $\chi^2$ is calculated based on # possible response patterns. Here, for 7 binary items:	This error message indicates that these 2 sets of chi-squares for the 2-PL and 1-PL are not on the same scale because they are not based on the same data. So we can't compare the chi-squares to test the difference in model fit, but we can still compare LL values.
2PL model: $2^7 = 128$ possible – 7 loadings – 7 thresholds – 1 = 113	
1PL model: $2^7 = 128$ possible – 1 loading – 7 thresholds – 1 = 119	
However, the 1PL only has df=118 because of the deleted cell.	

**Does the 2-PL fit better than the 1-PL?**

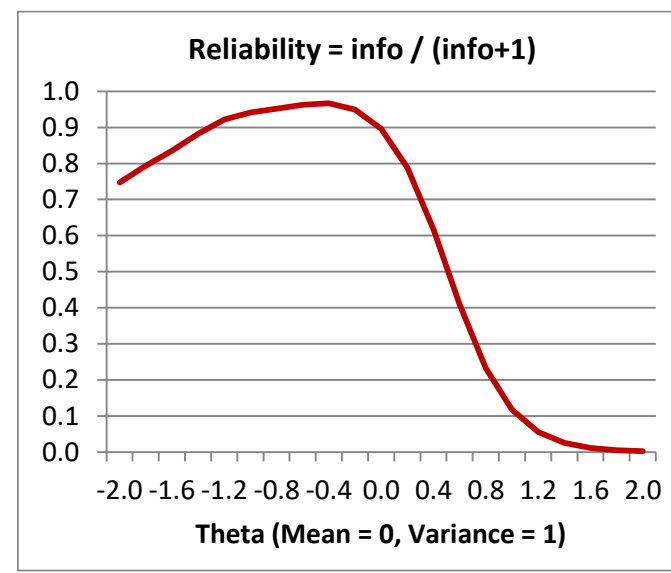
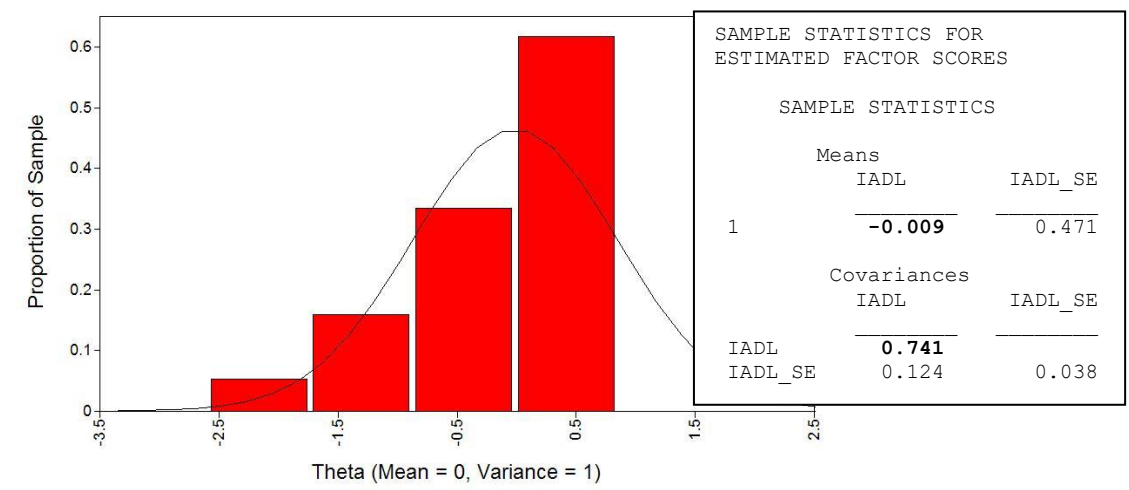
-1454.634\*-2 = 2909.258      -2LL difference = 19.946, df = 6, p = .0032  
 -1464.457\*-2 = 2928.914      AIC (but not BIC) is smaller for 2PL, too

**3 differently scaled 2-PL solutions from ML logit provided by Mplus – all provide the exact same model predictions!**

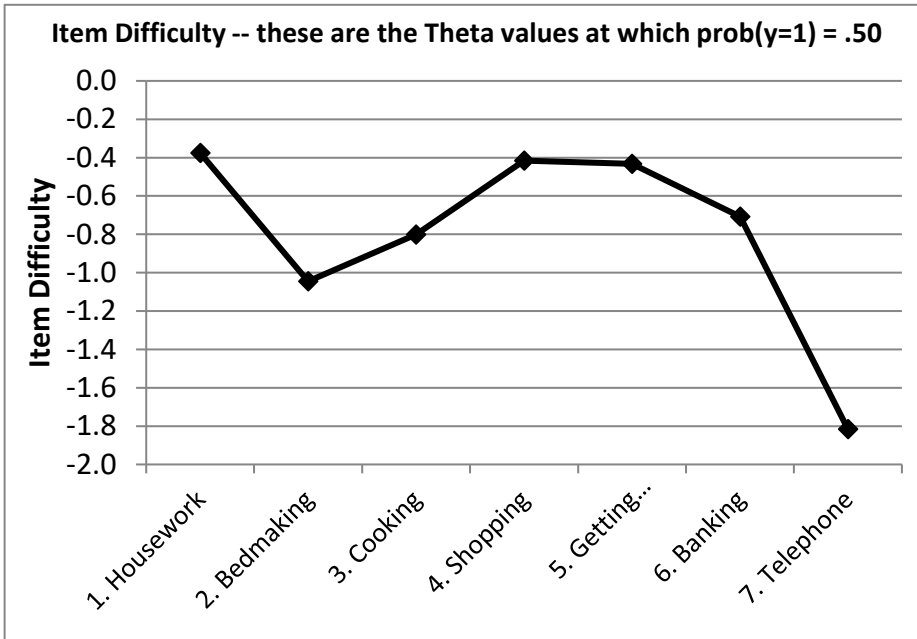
<p><b>UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)</b></p> <p style="text-align: right;">Two-Tailed</p> <p style="text-align: center;">Estimate      S.E.    Est./S.E.    P-Value</p> <p><b>FACTOR LOADINGS = CHANGE IN LOGIT(Y=1) PER UNIT CHANGE IN THETA</b></p> <p>IADL    BY</p> <table border="1"> <tr><td>DIA1</td><td></td><td>4.328</td><td>0.560</td><td>7.725</td><td>0.000</td></tr> <tr><td>DIA2</td><td></td><td>4.978</td><td>0.808</td><td>6.159</td><td>0.000</td></tr> <tr><td>DIA3</td><td></td><td>4.323</td><td>0.570</td><td>7.579</td><td>0.000</td></tr> <tr><td>DIA4</td><td></td><td>7.511</td><td>1.696</td><td>4.429</td><td>0.000</td></tr> <tr><td>DIA5</td><td></td><td>4.248</td><td>0.527</td><td>8.062</td><td>0.000</td></tr> <tr><td>DIA6</td><td></td><td>3.451</td><td>0.401</td><td>8.600</td><td>0.000</td></tr> <tr><td>DIA7</td><td></td><td>3.283</td><td>0.601</td><td>5.467</td><td>0.000</td></tr> </table> <p><b>THRESHOLDS = EXPECTED LOGIT(Y=0) WHEN THETA IS 0</b></p> <table border="1"> <tr><td>DIA1\$1</td><td></td><td>-1.629</td><td>0.295</td><td>-5.516</td><td>0.000</td></tr> <tr><td>DIA2\$1</td><td></td><td>-5.202</td><td>0.770</td><td>-6.754</td><td>0.000</td></tr> <tr><td>DIA3\$1</td><td></td><td>-3.462</td><td>0.441</td><td>-7.842</td><td>0.000</td></tr> <tr><td>DIA4\$1</td><td></td><td>-3.120</td><td>0.744</td><td>-4.193</td><td>0.000</td></tr> <tr><td>DIA5\$1</td><td></td><td>-1.833</td><td>0.298</td><td>-6.158</td><td>0.000</td></tr> <tr><td>DIA6\$1</td><td></td><td>-2.442</td><td>0.292</td><td>-8.368</td><td>0.000</td></tr> <tr><td>DIA7\$1</td><td></td><td>-5.962</td><td>0.858</td><td>-6.951</td><td>0.000</td></tr> </table> <p><b>STDYX MODEL RESULTS (STANDARDIZED IFA MODEL SOLUTION)</b></p> <p style="text-align: right;">Two-Tailed</p> <p style="text-align: center;">Estimate      S.E.    Est./S.E.    P-Value</p> <p><b>FACTOR LOADINGS IN STANDARDIZED METRIC = loading*SD(Theta)/SD(Y)</b></p> <p>IADL    BY</p> <table border="1"> <tr><td>DIA1</td><td></td><td>0.922</td><td>0.018</td><td>51.712</td><td>0.000</td></tr> <tr><td>DIA2</td><td></td><td>0.940</td><td>0.018</td><td>52.557</td><td>0.000</td></tr> <tr><td>DIA3</td><td></td><td>0.922</td><td>0.018</td><td>50.622</td><td>0.000</td></tr> <tr><td>DIA4</td><td></td><td>0.972</td><td>0.012</td><td>80.380</td><td>0.000</td></tr> <tr><td>DIA5</td><td></td><td>0.920</td><td>0.018</td><td>52.291</td><td>0.000</td></tr> <tr><td>DIA6</td><td></td><td>0.885</td><td>0.022</td><td>39.729</td><td>0.000</td></tr> <tr><td>DIA7</td><td></td><td>0.875</td><td>0.037</td><td>23.380</td><td>0.000</td></tr> </table> <p><b>THRESHOLDS IN STANDARDIZED METRIC = threshold/SD(Y)</b></p> <table border="1"> <tr><td>DIA1\$1</td><td></td><td>-0.347</td><td>0.048</td><td>-7.303</td><td>0.000</td></tr> <tr><td>DIA2\$1</td><td></td><td>-0.982</td><td>0.056</td><td>-17.409</td><td>0.000</td></tr> <tr><td>DIA3\$1</td><td></td><td>-0.739</td><td>0.051</td><td>-14.373</td><td>0.000</td></tr> <tr><td>DIA4\$1</td><td></td><td>-0.404</td><td>0.045</td><td>-8.928</td><td>0.000</td></tr> <tr><td>DIA5\$1</td><td></td><td>-0.397</td><td>0.048</td><td>-8.348</td><td>0.000</td></tr> <tr><td>DIA6\$1</td><td></td><td>-0.626</td><td>0.050</td><td>-12.558</td><td>0.000</td></tr> <tr><td>DIA7\$1</td><td></td><td>-1.590</td><td>0.080</td><td>-19.949</td><td>0.000</td></tr> </table> <p><b>R-SQUARE = standardized loading<sup>2</sup></b></p> <table border="1"> <tr><td>DIA1</td><td></td><td>0.851</td><td>0.033</td><td>25.856</td><td>0.000</td></tr> <tr><td>DIA2</td><td></td><td>0.883</td><td>0.034</td><td>26.278</td><td>0.000</td></tr> <tr><td>DIA3</td><td></td><td>0.850</td><td>0.034</td><td>25.311</td><td>0.000</td></tr> <tr><td>DIA4</td><td></td><td>0.945</td><td>0.024</td><td>40.190</td><td>0.000</td></tr> <tr><td>DIA5</td><td></td><td>0.846</td><td>0.032</td><td>26.145</td><td>0.000</td></tr> <tr><td>DIA6</td><td></td><td>0.784</td><td>0.039</td><td>19.865</td><td>0.000</td></tr> <tr><td>DIA7</td><td></td><td>0.766</td><td>0.066</td><td>11.690</td><td>0.000</td></tr> </table>					DIA1		4.328	0.560	7.725	0.000	DIA2		4.978	0.808	6.159	0.000	DIA3		4.323	0.570	7.579	0.000	DIA4		7.511	1.696	4.429	0.000	DIA5		4.248	0.527	8.062	0.000	DIA6		3.451	0.401	8.600	0.000	DIA7		3.283	0.601	5.467	0.000	DIA1\$1		-1.629	0.295	-5.516	0.000	DIA2\$1		-5.202	0.770	-6.754	0.000	DIA3\$1		-3.462	0.441	-7.842	0.000	DIA4\$1		-3.120	0.744	-4.193	0.000	DIA5\$1		-1.833	0.298	-6.158	0.000	DIA6\$1		-2.442	0.292	-8.368	0.000	DIA7\$1		-5.962	0.858	-6.951	0.000	DIA1		0.922	0.018	51.712	0.000	DIA2		0.940	0.018	52.557	0.000	DIA3		0.922	0.018	50.622	0.000	DIA4		0.972	0.012	80.380	0.000	DIA5		0.920	0.018	52.291	0.000	DIA6		0.885	0.022	39.729	0.000	DIA7		0.875	0.037	23.380	0.000	DIA1\$1		-0.347	0.048	-7.303	0.000	DIA2\$1		-0.982	0.056	-17.409	0.000	DIA3\$1		-0.739	0.051	-14.373	0.000	DIA4\$1		-0.404	0.045	-8.928	0.000	DIA5\$1		-0.397	0.048	-8.348	0.000	DIA6\$1		-0.626	0.050	-12.558	0.000	DIA7\$1		-1.590	0.080	-19.949	0.000	DIA1		0.851	0.033	25.856	0.000	DIA2		0.883	0.034	26.278	0.000	DIA3		0.850	0.034	25.311	0.000	DIA4		0.945	0.024	40.190	0.000	DIA5		0.846	0.032	26.145	0.000	DIA6		0.784	0.039	19.865	0.000	DIA7		0.766	0.066	11.690	0.000	<p>(output from same model continued)</p> <p><b>IRT PARAMETERIZATION IN TWO-PARAMETER LOGISTIC METRIC WHERE THE LOGIT IS DISCRIMINATION*(THETA - DIFFICULTY)</b></p> <p><b>Item Discriminations = SLOPE OF ICC AT P=.50</b></p> <p>IADL    BY</p> <table border="1"> <tr><td>DIA1</td><td></td><td>4.328</td><td>0.560</td><td>7.725</td><td>0.000</td></tr> <tr><td>DIA2</td><td></td><td>4.978</td><td>0.808</td><td>6.159</td><td>0.000</td></tr> <tr><td>DIA3</td><td></td><td>4.323</td><td>0.570</td><td>7.579</td><td>0.000</td></tr> <tr><td>DIA4</td><td></td><td>7.511</td><td>1.696</td><td>4.429</td><td>0.000</td></tr> <tr><td>DIA5</td><td></td><td>4.248</td><td>0.527</td><td>8.062</td><td>0.000</td></tr> <tr><td>DIA6</td><td></td><td>3.451</td><td>0.401</td><td>8.600</td><td>0.000</td></tr> <tr><td>DIA7</td><td></td><td>3.283</td><td>0.601</td><td>5.467</td><td>0.000</td></tr> </table> <p><b>Item Difficulties = LOCATION OF ITEM ON LATENT TRAIT at P=.50, LOGIT=0</b></p> <table border="1"> <tr><td>DIA1\$1</td><td></td><td>-0.376</td><td>0.052</td><td>-7.298</td><td>0.000</td></tr> <tr><td>DIA2\$1</td><td></td><td>-1.045</td><td>0.065</td><td>-15.978</td><td>0.000</td></tr> <tr><td>DIA3\$1</td><td></td><td>-0.801</td><td>0.059</td><td>-13.562</td><td>0.000</td></tr> <tr><td>DIA4\$1</td><td></td><td>-0.415</td><td>0.047</td><td>-8.849</td><td>0.000</td></tr> <tr><td>DIA5\$1</td><td></td><td>-0.432</td><td>0.052</td><td>-8.296</td><td>0.000</td></tr> <tr><td>DIA6\$1</td><td></td><td>-0.708</td><td>0.060</td><td>-11.889</td><td>0.000</td></tr> <tr><td>DIA7\$1</td><td></td><td>-1.816</td><td>0.126</td><td>-14.454</td><td>0.000</td></tr> </table> <p><b>USING RESULTS FROM IFA MODEL (LEFT PANEL):</b></p> <p><u>IFA model: Logit(y) = -threshold + loading(Theta)</u>  <b>Threshold = expected logit of (y=0) for someone with Theta=0</b>  <b>When *-1, threshold becomes intercept: expected logit for (y=1) instead</b></p> <p><b>Loading = regression of item logit on Theta</b>  <b>= change in logit(y) for a one-unit change in Theta</b></p> <p><u>IFA Models:</u>  <b>Logit (DIA1=1) = 1.629 + 4.328(Theta) → if Theta=0, prob(y=1)= .836</b>  <b>Logit (DIA7=1) = 5.962 + 3.283(Theta) → if Theta=0, prob(y=1)= .997</b></p> <p><b>USING RESULTS FROM IRT MODEL (RIGHT PANEL):</b></p> <p><u>IRT model: Logit(y=1) = a(theta - difficulty)</u>  <b>a = discrimination (rescaled slope) = loading/1.7</b>  <b>b = difficulty (location on latent metric) = threshold/loading</b></p> <p><u>IRT Models:</u>  <b>Logit (DIA1=1) = 4.328*(Theta - -0.376) → if Theta=0, prob(y=1)= .836</b>  <b>Logit (DIA7=1) = 3.283*(Theta - -1.816) → if Theta=0, prob(y=1)= .997</b></p>					DIA1		4.328	0.560	7.725	0.000	DIA2		4.978	0.808	6.159	0.000	DIA3		4.323	0.570	7.579	0.000	DIA4		7.511	1.696	4.429	0.000	DIA5		4.248	0.527	8.062	0.000	DIA6		3.451	0.401	8.600	0.000	DIA7		3.283	0.601	5.467	0.000	DIA1\$1		-0.376	0.052	-7.298	0.000	DIA2\$1		-1.045	0.065	-15.978	0.000	DIA3\$1		-0.801	0.059	-13.562	0.000	DIA4\$1		-0.415	0.047	-8.849	0.000	DIA5\$1		-0.432	0.052	-8.296	0.000	DIA6\$1		-0.708	0.060	-11.889	0.000	DIA7\$1		-1.816	0.126	-14.454	0.000
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DIA7		0.875	0.037	23.380	0.000																																																																																																																																																																																																																																																																																																										
DIA1\$1		-0.347	0.048	-7.303	0.000																																																																																																																																																																																																																																																																																																										
DIA2\$1		-0.982	0.056	-17.409	0.000																																																																																																																																																																																																																																																																																																										
DIA3\$1		-0.739	0.051	-14.373	0.000																																																																																																																																																																																																																																																																																																										
DIA4\$1		-0.404	0.045	-8.928	0.000																																																																																																																																																																																																																																																																																																										
DIA5\$1		-0.397	0.048	-8.348	0.000																																																																																																																																																																																																																																																																																																										
DIA6\$1		-0.626	0.050	-12.558	0.000																																																																																																																																																																																																																																																																																																										
DIA7\$1		-1.590	0.080	-19.949	0.000																																																																																																																																																																																																																																																																																																										
DIA1		0.851	0.033	25.856	0.000																																																																																																																																																																																																																																																																																																										
DIA2		0.883	0.034	26.278	0.000																																																																																																																																																																																																																																																																																																										
DIA3		0.850	0.034	25.311	0.000																																																																																																																																																																																																																																																																																																										
DIA4		0.945	0.024	40.190	0.000																																																																																																																																																																																																																																																																																																										
DIA5		0.846	0.032	26.145	0.000																																																																																																																																																																																																																																																																																																										
DIA6		0.784	0.039	19.865	0.000																																																																																																																																																																																																																																																																																																										
DIA7		0.766	0.066	11.690	0.000																																																																																																																																																																																																																																																																																																										
DIA1		4.328	0.560	7.725	0.000																																																																																																																																																																																																																																																																																																										
DIA2		4.978	0.808	6.159	0.000																																																																																																																																																																																																																																																																																																										
DIA3		4.323	0.570	7.579	0.000																																																																																																																																																																																																																																																																																																										
DIA4		7.511	1.696	4.429	0.000																																																																																																																																																																																																																																																																																																										
DIA5		4.248	0.527	8.062	0.000																																																																																																																																																																																																																																																																																																										
DIA6		3.451	0.401	8.600	0.000																																																																																																																																																																																																																																																																																																										
DIA7		3.283	0.601	5.467	0.000																																																																																																																																																																																																																																																																																																										
DIA1\$1		-0.376	0.052	-7.298	0.000																																																																																																																																																																																																																																																																																																										
DIA2\$1		-1.045	0.065	-15.978	0.000																																																																																																																																																																																																																																																																																																										
DIA3\$1		-0.801	0.059	-13.562	0.000																																																																																																																																																																																																																																																																																																										
DIA4\$1		-0.415	0.047	-8.849	0.000																																																																																																																																																																																																																																																																																																										
DIA5\$1		-0.432	0.052	-8.296	0.000																																																																																																																																																																																																																																																																																																										
DIA6\$1		-0.708	0.060	-11.889	0.000																																																																																																																																																																																																																																																																																																										
DIA7\$1		-1.816	0.126	-14.454	0.000																																																																																																																																																																																																																																																																																																										



**Distribution of Theta under 2 PL (made in Mplus):** Although reliability is > .80 from -1.5 to 0.3 or so, we see a huge ceiling effect: most respondents can do all the tasks.



The estimated theta scores are supposed to have a mean of 0 and a variance of 1, but this table shows that they have a variance of only .741 instead. Such shrinkage is why it can be problematic to use these estimated theta scores as observed variables in other analyses.



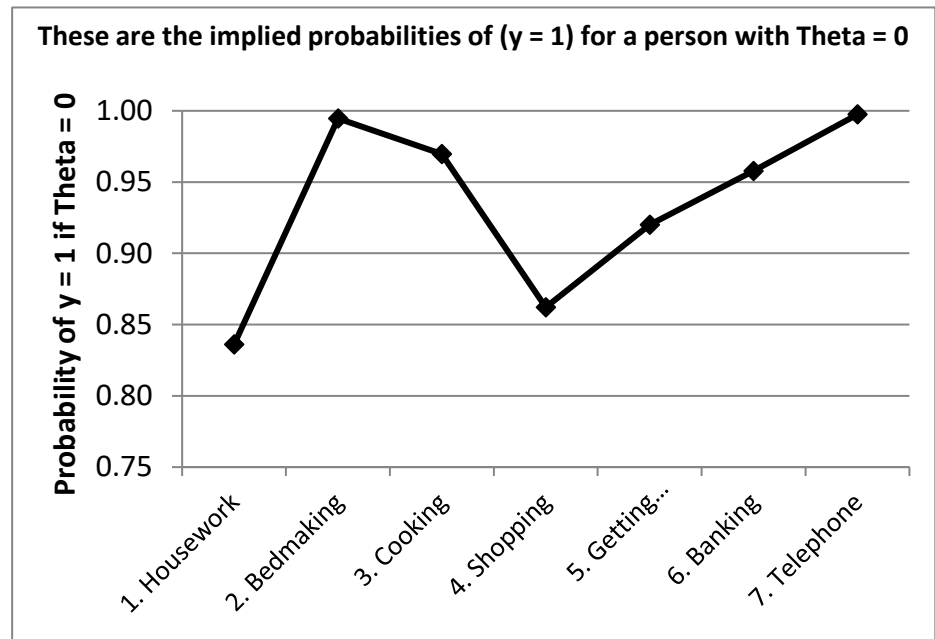
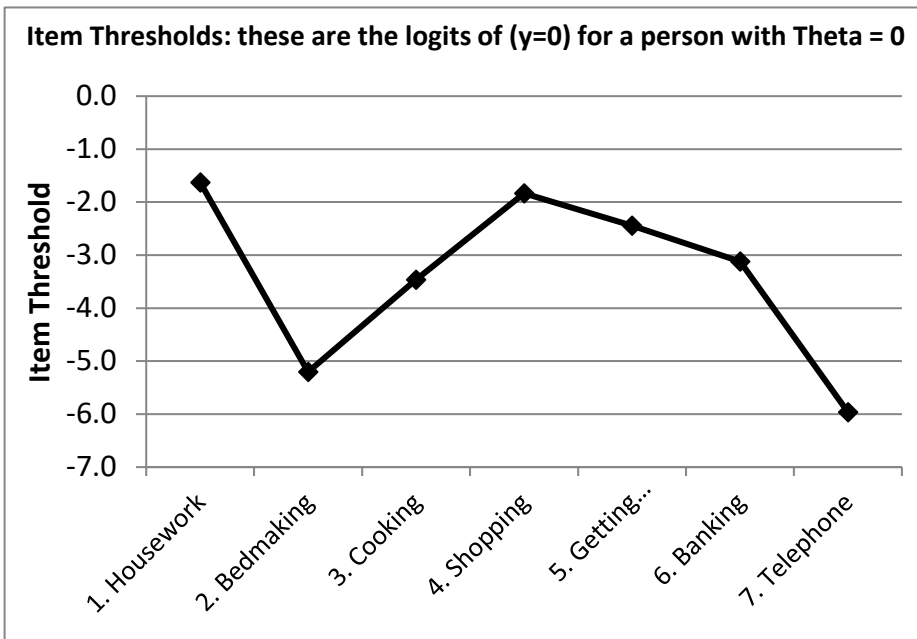
**Plots of item parameters and predicted probabilities of item responses (made in excel):**

**Top Left:** Note that no items are available to measure above-average abilities well! The item difficulty for most items covers values of Theta between  $-1.0$  to  $-0.5$ .

**Bottom Left:** These are the thresholds for each item, or the logit of  $(y=0)$  if  $\text{Theta}=0$ . These are hard to interpret as is....

**Bottom Right:** These are the probability of  $y=1$  if  $\text{Theta}=0$ , as given by  $1 - [\exp(\text{threshold}) / (1 + \exp(\text{threshold}))]$

See excel workbook for calculations and plots



### Here is another estimation approach: a 2PL vs. a 1PL for Binary Responses using WLSMV Probit model

<pre> <b>TITLE:</b> 2PL Binary Model under limited-info WLSMV <b>DATA:</b> FILE = Example5.csv; ! Don't need path if in same folder <b>VARIABLE:</b> NAMES = case dial-dia7 cial-cia7;               USEVARIABLES = dial-dia7; ! All vars in data               CATEGORICAL = dial-dia7; ! All vars in model               MISSING = ALL (99999); ! Missing value code               IDVARIABLE = case; ! Person ID variable  <b>ANALYSIS:</b> ESTIMATOR = WLSMV; ! Limited-info in probits               PARAMETERIZATION = THETA; ! Error vars=1 scaling               CONVERGENCE = 0.0000001; ! For OS comparability  <b>MODEL:</b> ! Factor loadings all estimated in 2PL IADL BY dial-dia7*; ! Item thresholds all estimated [dial\$1-dia7\$1*]; ! Factor mean=0 and variance=1 for identification [IADL@0]; IADL@1;  <b>OUTPUT:</b> STDYX RESIDUAL; ! Standardized solution, local fit <b>SAVEDATA:</b> DIFFTEST=2PL.dat; ! Save info from bigger model               SAVE = FSCORES; ! Save factor scores (thetas)               FILE = IADL_2PLThetas.dat; ! File thetas saved to               MISSFLAG = 99999; ! Missing data value in file  <b>PLOT:</b> TYPE IS PLOT1 PLOT2 PLOT3; ! Get all IRT plots  MODEL FIT INFORMATION Number of Free Parameters 14 Chi-Square Test of Model Fit Value 54.820* Degrees of Freedom 14 P-Value 0.0000  * The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.  RMSEA (Root Mean Square Error Of Approximation) Estimate 0.068 90 Percent C.I. 0.049 0.087 Probability RMSEA &lt;= .05 0.055  CFI/TLI CFI 0.997 TLI 0.995 Chi-Square Test of Model Fit for the Baseline Model Value 12351.798 Degrees of Freedom 21 P-Value 0.0000 SRMR (Standardized Root Mean Square Residual) Value 0.037 </pre>	<pre> <b>TITLE:</b> 1PL Binary Model under limited-info WLSMV <b>DATA:</b> FILE = Example5.csv; ! Don't need path if in same folder <b>VARIABLE:</b> NAMES = case dial-dia7 cial-cia7;               USEVARIABLES = dial-dia7; ! All vars in data               CATEGORICAL = dial-dia7; ! All vars in model               MISSING = ALL (99999); ! Missing value code               IDVARIABLE = case; ! Person ID variable  <b>ANALYSIS:</b> ESTIMATOR = WLSMV; ! Limited-info in probits               PARAMETERIZATION = THETA; ! Error vars=1 scaling               CONVERGENCE = 0.0000001; ! For OS comparability               DIFFTEST=2PL.dat; ! Use saved info from bigger model  <b>MODEL:</b> ! Factor loadings all equal in 1PL IADL BY dial-dia7* (loading); ! Item thresholds all estimated [dial\$1-dia7\$1*]; ! Factor mean=0 and variance=1 for identification [IADL@0]; IADL@1;  <b>OUTPUT:</b> STDYX RESIDUAL; ! Standardized solution, local fit <b>SAVEDATA:</b> SAVE = FSCORES; ! Save factor scores (thetas)               FILE = IADL_1PLThetas.dat; ! File thetas saved to               MISSFLAG = 99999; ! Missing data value in file  <b>PLOT:</b> TYPE IS PLOT1 PLOT2 PLOT3; ! Get all IRT plots  MODEL FIT INFORMATION Number of Free Parameters 8 Chi-Square Test of Model Fit Value 64.889* Degrees of Freedom 20 P-Value 0.0000  <b>Chi-Square Test for Difference Testing</b> Value 17.874 Degrees of Freedom 6 P-Value 0.0066  RMSEA (Root Mean Square Error Of Approximation) Estimate 0.059 90 Percent C.I. 0.044 0.076 Probability RMSEA &lt;= .05 0.154  CFI/TLI CFI 0.996 TLI 0.996 SRMR (Standardized Root Mean Square Residual) Value 0.056  <b>The Chi-Square for Difference Testing tells us directly that the 2PL version of the binary model fits significantly better (now under WLSMV, same as it did under ML).</b> </pre>
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**Here are the parameter estimates under WLSMV Theta Parameterization (Probit) for the 2PL version of binary items**

**UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)**

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
<b>FACTOR LOADINGS = CHANGE IN PROBIT(Y=1) PER UNIT CHANGE IN THETA</b>				
IADL BY				
DIA1	2.686	0.317	8.461	0.000
DIA2	2.941	0.493	5.966	0.000
DIA3	2.803	0.384	7.290	0.000
DIA4	3.654	0.575	6.356	0.000
DIA5	2.486	0.294	8.449	0.000
DIA6	1.991	0.223	8.940	0.000
DIA7	1.571	0.299	5.246	0.000
<b>THRESHOLDS = EXPECTED PROBIT(Y=0) WHEN THETA IS 0</b>				
DIA1\$1	-1.004	0.179	-5.607	0.000
DIA2\$1	-3.097	0.481	-6.444	0.000
DIA3\$1	-2.221	0.307	-7.240	0.000
DIA4\$1	-1.581	0.298	-5.312	0.000
DIA5\$1	-1.057	0.174	-6.071	0.000
DIA6\$1	-1.391	0.166	-8.359	0.000
DIA7\$1	-2.946	0.398	-7.401	0.000
<b>STDYX MODEL RESULTS (STANDARDIZED IFA MODEL SOLUTION)</b>				
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
<b>FACTOR LOADINGS IN STANDARDIZED METRIC = loading*SD(Theta)/SD(Y)</b>				
IADL BY				
DIA1	0.937	0.013	69.487	0.000
DIA2	0.947	0.016	57.551	0.000
DIA3	0.942	0.015	64.551	0.000
DIA4	0.965	0.011	91.196	0.000
DIA5	0.928	0.015	60.671	0.000
DIA6	0.894	0.020	44.371	0.000
DIA7	0.844	0.046	18.195	0.000
<b>Thresholds IN STANDARDIZED METRIC = threshold/SD(Y)</b>				
DIA1\$1	-0.350	0.052	-6.790	0.000
DIA2\$1	-0.997	0.061	-16.474	0.000
DIA3\$1	-0.746	0.056	-13.326	0.000
DIA4\$1	-0.417	0.052	-8.041	0.000
DIA5\$1	-0.395	0.051	-7.676	0.000
DIA6\$1	-0.624	0.054	-11.648	0.000
DIA7\$1	-1.582	0.081	-19.628	0.000
<b>R-SQUARE = standardized loading<sup>2</sup></b>				
DIA1	0.878	0.025	34.744	0.000
DIA2	0.896	0.031	28.775	0.000
DIA3	0.887	0.027	32.276	0.000
DIA4	0.930	0.020	45.598	0.000
DIA5	0.861	0.028	30.336	0.000
DIA6	0.799	0.036	22.185	0.000
DIA7	0.712	0.078	9.097	0.000

(output from same model continued)

**IRT PARAMETERIZATION IN TWO-PARAMETER PROBIT METRIC WHERE THE PROBIT IS DISCRIMINATION\*(THETA - DIFFICULTY)**

**Item Discriminations**

IADL BY					
DIA1	2.686	0.317	8.461	0.000	
DIA2	2.941	0.493	5.966	0.000	
DIA3	2.803	0.384	7.290	0.000	
DIA4	3.654	0.575	6.356	0.000	
DIA5	2.486	0.294	8.449	0.000	
DIA6	1.991	0.223	8.940	0.000	
DIA7	1.571	0.299	5.246	0.000	

**Item Difficulties**

IADL BY					
DIA1\$1	-0.374	0.055	-6.743	0.000	
DIA2\$1	-1.053	0.069	-15.360	0.000	
DIA3\$1	-0.792	0.062	-12.863	0.000	
DIA4\$1	-0.433	0.054	-7.982	0.000	
DIA5\$1	-0.425	0.056	-7.607	0.000	
DIA6\$1	-0.699	0.063	-11.084	0.000	
DIA7\$1	-1.875	0.154	-12.191	0.000	

Logit = 1.7\*probit, or Probit = Logit/1.7

**IFA model:  $PROBIT(y) = -threshold + loading(Theta)$**   
 Threshold = expected probit of (y=0) for someone with Theta=0  
 When \*-1, threshold → intercept: expected probit for (y=1) instead  
 Loading = regression of item probit on Theta

**IRT model:  $Probit(y=1) = a(theta - difficulty)$**   
 a = discrimination (rescaled slope) = loading/1  
 b = difficulty (location on latent metric) = threshold/loading

**LOCAL FIT VIA STANDARDIZED RESIDUAL CORRELATIONS**

**LEFTOVER TETRACHORIC CORRELATION (HOW FAR OFF FROM DATA)**

**Residuals for Covariances/Correlations/Residual Correlations**

	DIA1	DIA2	DIA3	DIA4	DIA5	DIA6
DIA1						
DIA2	0.028					
DIA3	0.038	0.029				
DIA4	-0.022	-0.040	-0.046			
DIA5	-0.032	-0.034	<b>-0.103</b>	0.029		
DIA6	-0.052	-0.056	-0.046	0.026	0.032	
DIA7	<b>-0.112</b>	-0.003	0.010	0.031	-0.027	0.064

**Extensive Results Section (in which model fit via WLSMV is reported first, followed by full-information MML as “better” version of model parameters). Note this is \*way\* more text than one would typically write, but I provide it here for completeness:**

Psychometric assessment for the extent to which a single latent trait could predict that associations among the 7 binary items measuring physical capacity was conducted using Item Factor Analysis (IFA) in *Mplus* v 8.4 (Muthén and Muthén, 1998–2017). These models use a link function (i.e., logit or probit) and a conditional Bernoulli response distribution to predict the conditional probability of a response = 1 (instead of 0) from a linear model as  $Link(y_{is} = 1) = -\tau_i + \lambda_i F_s$ . In this item model,  $-\tau_i$  is the item-specific threshold, which when multiplied by  $-1$  becomes an intercept that gives the link-transformed probability of response  $y_{is} = 1$  (for item  $i$  and subject  $s$ ) at a latent trait score  $F$  for subject  $s$  of 0, and  $\lambda_i$  is an item-specific factor loading for the expected change in the link-transformed response for a one-unit change in  $F_s$ . No separate item-specific residual variances can be estimated given these items' binary response options.

The current gold standard of estimation for IFA models is marginal maximum likelihood (MML), in which the term *marginal* refers to the full-information process of allowing all possible trait values for each person in the analysis using adaptive Gaussian quadrature with 15 points per latent trait. Accordingly, measures of model fit when using MML involve the contingency table of all possible responses to all items. In our 7 items, the full contingency table generates up to  $2^7 = 128$  possible cells. Consequently, no measures of absolute fit would be valid for the current sample of 635 respondents (which would need a minimum expected count of 5 respondents within each possible cell). Instead, we conducted assessment of model fit via a limited-information diagonally weighted least squares estimator using a mean- and variance-corrected  $\chi^2$  (i.e., WLSMV in *Mplus* with the THETA parameterization and a probit link function). In the WLSMV estimator, the item responses are first summarized into an estimated tetrachoric correlation matrix using the cross-tabulation of responses for each possible pair of items. The IFA models are then fitted to the estimated tetrachoric correlation matrix, such that traditional measures of global and local absolute fit (i.e., traditional in confirmatory factor analyses of continuous responses) can be computed by comparing the model-predicted and data-estimated tetrachoric correlation matrices. In addition to  $\chi^2$  tests of absolute fit, it also provides the Comparative Fit Index (CFI), the Standardized Root Mean Square Residual (SRMR), and the Root Mean Square Error of Approximation (RMSEA). The CFI indexes the fit of the specified model relative to a null model (of no tetrachoric correlations across items), in which CFI values  $\geq .95$  traditionally indicate excellent fit. Conversely, the SRMR and RMSEA index the fit of the specified model relative to a saturated model (i.e., the data-estimated tetrachoric correlations), in which SRMR and RMSEA values  $\leq .06$  traditionally indicate excellent fit. RMSEA also offers a 90% confidence interval and a significance test of “close fit” with a null hypothesis of .05. Local misfit can be diagnosed by examining the specific sources of discrepancy between the model-predicted and data-estimated tetrachoric correlations (i.e., as available using the RESIDUAL option in *Mplus*). Finally, the fit of nested models can be compared using the DIFFTEST procedure in *Mplus*.

A single-trait model was first fit for the 7 binary items using WLSMV, in which the latent trait mean and variance were fixed for identification to 0 and 1, respectively, and separate thresholds and factor loadings were estimated for each item. This model exhibited acceptable fit by every measure except the  $\chi^2$  test of absolute fit,  $\chi^2(14) = 54.820$ ,  $p < .001$ , CFI = .997, SRMR = .037, RMSEA = .068 [CI = .049–.087,  $p = .055$ ]. Examination of local misfit revealed all discrepancies between the model-predicted and data-estimated tetrachoric correlations were less than .113 in absolute value, indicating no practically significant bivariate item misfit. A reduced model in which all loadings were constrained equal across items fit significantly worse, DIFFTEST(6) = 17.874,  $p = .007$ , indicating differences in item discrimination (i.e., the extent to which each item was related to the latent trait). Thus, the original model was retained for further examination using full-information marginal maximum likelihood (MML) estimation instead (given the presence of missing item-level responses).

Model parameters obtained using MML and a logit link are shown in Table 1, which includes the IFA item parameters (thresholds and loadings), as well as their Item Response Theory (IRT) analogous parameter of item difficulty, computed as  $b_i = \tau_i / \lambda_i$ ; IRT discrimination  $a_i$  is the same as the loading  $\lambda_i$  in this case. The net result of these item parameters can be described more succinctly by examining the overall reliability with which the latent trait has been measured. In IFA or IRT models—as in any kind of psychometric model with a nonlinear relationship between the item response and the latent trait—reliability is trait-specific, most often characterized by a quantity known as *test information*. For ease of interpretation, the test information function created by the items was converted to a traditional measure of reliability that ranges from 0 to 1 as  $reliability = information / (information + 1)$ . Figure 1 shows that test reliability is  $\geq .80$  only from  $\sim 1.8$  SD below the mean to 0.20 SD above the mean, after which point reliability drops off precipitously due to a lack of items with difficulty levels above 0.

(See Example 5 spreadsheet for Table 1 and Figure 1)

Reference: Muthén, L. K., & Muthén, B.O. (1998–2017). *Mplus user's guide* (8th ed.). Los Angeles, CA: Muthén & Muthén.