

Example 4: CFA of Forgiveness of Situations ($N = 1103$) in *Mplus v. 8.4*

This example comes from the Heartland Forgiveness Scale ([Yamhure Thompson et al., 2005](#)). Here we focus on the Forgiveness of Situations Subscale includes six items, three of which are reverse-coded, each on a 7-point scale:

1. When things go wrong for reasons that can't be controlled, I get stuck in negative thoughts about it. **(R)**
2. With time I can be understanding of bad circumstances in my life.
3. If I am disappointed by uncontrollable circumstances in my life, I continue to think negatively about them. **(R)**
4. I eventually make peace with bad situations in my life.
5. It's really hard for me to accept negative situations that aren't anybody's fault. **(R)**
6. Eventually I let go of negative thoughts about bad circumstances that are beyond anyone's control.

Response Anchors: 1 = Almost Always False of Me, 2 = ?, 3 = More Often False of Me, 4 = ?, 5 = More Often True of Me, 6 = ?, 7 = Almost Always True of Me

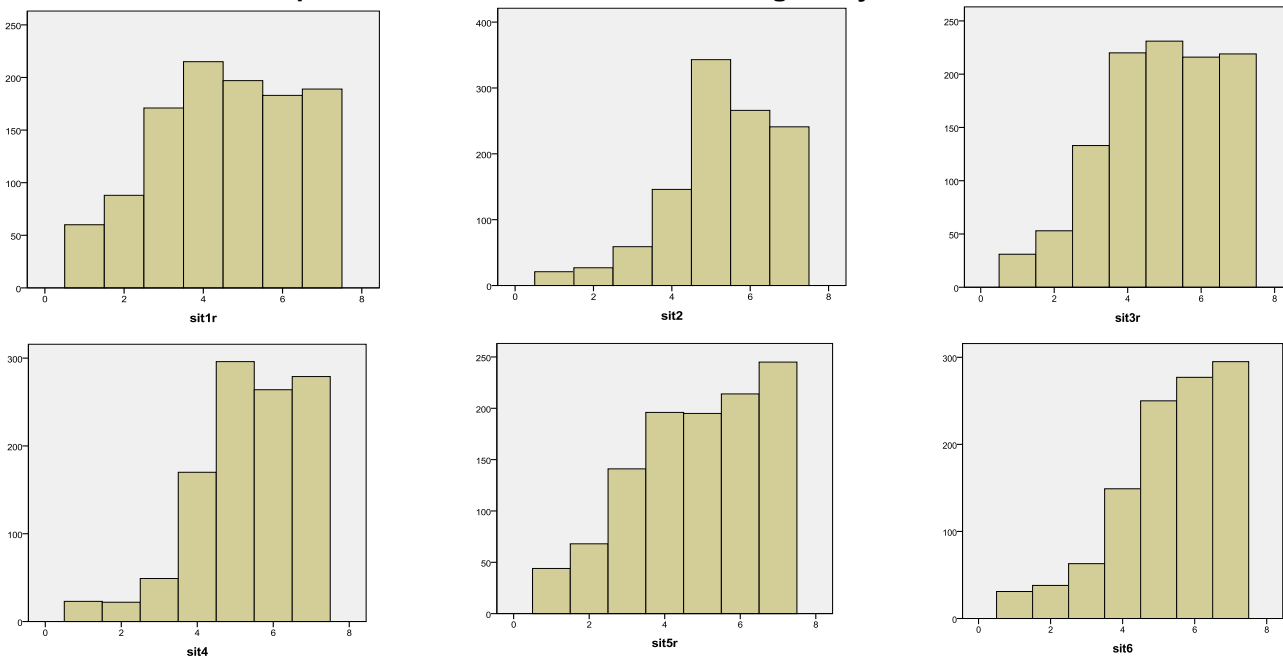
Observed Correlation Matrix	R1	2	R3	4	R5	6
R1	1.000					
2	0.240	1.000				
R3	0.647	0.317	1.000			
4	0.300	0.570	0.369	1.000		
R5	0.453	0.255	0.482	0.289	1.000	
6	0.297	0.457	0.356	0.448	0.304	1.000
Means	4.547	5.289	4.896	5.359	4.860	5.321
Variances	3.049	1.903	2.543	1.967	2.945	2.341

Observed Covariance Matrix	R1	2	R3	4	R5	6
R1	3.049					
2	0.577	1.903				
R3	1.802	0.697	2.543			
4	0.734	1.103	0.824	1.967		
R5	1.358	0.604	1.319	0.695	2.945	
6	0.795	0.965	0.868	0.962	0.798	2.341

To do CFA modeling, you only really need means, variances, and either correlations or covariances among items:

$$Cov(y_1, y_2) = Cor(y_1, y_2) * SD(y_1) * SD(y_2) \quad \text{OR} \quad Cor(y_1, y_2) = Cov(y_1, y_2) / SD(y_1) * SD(y_2)$$

Distributions of item responses – do these look “normal enough” to you?



Mplus Code to Read in Data:

```

TITLE:          CFA of Situation Factor
DATA:          FILE = Example4.csv;           ! Don't need path if in same directory
                FORMAT = free;                 ! Default
                TYPE = INDIVIDUAL;            ! Default

VARIABLE:     NAMES = PersonID Self1 Self2r Self3 Self4r Self5 Self6r
                Other1r Other2 Other3r Other4 Other5r Other6
                Sit1r Sit2 Sit3r Sit4 Sit5r Sit6
                Selfsub Othsub Sitsub HFSsum;    ! Every variable in DATASET

                USEVARIABLES = Sit1r Sit2 Sit3r Sit4 Sit5r Sit6;    ! Every variable in MODEL
                MISSING = ALL (99999);        ! Identify missing values
                IDVARIABLE = PersonID;       ! Identify person ID variable

ANALYSIS:     TYPE = GENERAL;                ! Default
                ESTIMATOR = MLR;              ! Robust ML

SAVEDATA:     SAVE = FSCORES; FILE = FactorScores.dat; ! To save factor scores (optional)

PLOT:         TYPE = PLOT1 PLOT2 PLOT3; ! To get all plots (e.g., factor score distributions)

OUTPUT:      MODINDICES (6.635) ! Voodoo to improve the model at p <.01 for df=1
                STDYX                ! Fully standardized solution
                RESIDUAL              ! Standardized and normalized residuals for local fit
                FSDETERMINACY;       ! Correlation of factor scores with "true" factor scores

MODEL:       ! (model syntax goes here, to be changed for each model as shown below)

```

**Model 1. Fully Z-Scored Factor Model Identification
(Factor Variance = 1, Factor Mean = 0, All Loadings and Intercepts Estimated)**

The following code refers to EVERY model parameter for completeness:

```

! Model 1 - Fully Z-Scored Factor Identification Approach

! Item factor loadings --> @=fixed, *=free → * REQUIRED for first item if free
Sit BY Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;

! Item intercepts --> [ ] indicates means or intercepts, @=fixed, *=free
[Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*];

! Item error variances --> just list item by itself, @=fixed, *=free
Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;

! Factor variance --> just list factor by itself, @=fixed, *=free
Sit@1;

! Factor mean --> [ ] indicates means or intercepts, @=fixed, *=free
[Sit@0];

```

In reality, all you'd need to write to define this model is:

```

! Item factor loadings --> @=fixed, *=free → * REQUIRED for first item if free
Sit BY Sit1r* Sit2 Sit3r Sit4 Sit5r Sit6;

! Factor variance --> just list factor by itself, @=fixed, *=free
Sit@1;

```

By default, all intercepts are estimated separately and the factor mean is fixed at 0.
By default, all residual variances for the items are estimated separately, too.
By default, factor variances and covariances are estimated freely.

Model 1. Fully Z-Scored Identification Approach for a Single Factor Model (Factor Variance = 1, Factor Mean = 0, All Loadings and Intercepts Estimated)

UNSTANDARDIZED MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS (regression slopes of item response on factor)					
SIT	BY				
	SIT1R	1.234	0.069	17.906	0.000
	SIT2	0.702	0.074	9.441	0.000
	SIT3R	1.241	0.063	19.846	0.000
	SIT4	0.784	0.069	11.334	0.000
	SIT5R	1.023	0.053	19.179	0.000
	SIT6	0.819	0.069	11.942	0.000

Means (of Factor)

999 = "cannot be computed" - here, because the parameter is fixed to 0 already

SIT	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SIT	0.000	0.000	999.000	999.000

Intercepts (of Items) - HERE, ARE ACTUAL ITEM MEANS BECAUSE FACTOR MEAN IS ZERO

SIT	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SIT1R	4.547	0.053	86.474	0.000
SIT2	5.289	0.042	127.347	0.000
SIT3R	4.896	0.048	101.959	0.000
SIT4	5.359	0.042	126.895	0.000
SIT5R	4.860	0.052	94.060	0.000
SIT6	5.321	0.046	115.493	0.000

Variances (of Factor)

999 = "cannot be computed" - here, because the parameter is fixed to 1 already

SIT	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SIT	1.000	0.000	999.000	999.000

Residual Variances (variance of e's)

SIT	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SIT1R	1.526	0.149	10.217	0.000
SIT2	1.409	0.128	11.014	0.000
SIT3R	1.004	0.135	7.456	0.000
SIT4	1.352	0.127	10.672	0.000
SIT5R	1.899	0.118	16.025	0.000
SIT6	1.671	0.159	10.517	0.000

Making use of the unstandardized model estimates:

Writing out the model—individual predicted values:

$$y_{1s} = \mu_1 + \lambda_1 F_s + e_{1s}$$

$$y_{1s} = 4.547 + 1.234F_s + e_{1s}$$

Writing out the model—predicted item variances and covariances:

$$Var(y_1) = (\lambda_1^2) Var(F) + Var(e_1)$$

$$Var(y_1) = (1.234^2)(1) + 1.526 = 3.049 \text{ (= original item variance)}$$

$$Cov(y_1, y_2) = \lambda_1 * Var(F) * \lambda_2$$

$$Cov(y_1, y_2) = 1.234 * 1 * 0.702 = 0.866$$

(actual covariance = 0.577, so the model over-predicted how related items 1 and 2 should be)

STDYX STANDARDIZED MODEL RESULTS (FULLY STANDARDIZED WITH RESPECT TO X & Y)

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
FACTOR LOADINGS (correlations of item response with factor)					
Square these to get reliability (proportion "true variance") per item					
SIT	BY				
SIT1R		0.707	0.035	19.983	0.000
SIT2		0.509	0.053	9.545	0.000
SIT3R		0.778	0.034	22.655	0.000
SIT4		0.559	0.048	11.641	0.000
SIT5R		0.596	0.029	20.528	0.000
SIT6		0.535	0.047	11.392	0.000
Means (of Factor)					
SIT		0.000	0.000	999.000	999.000
Intercepts (of Items) → is intercept / SD(Y) → not usually reported					
SIT1R		2.604	0.057	45.888	0.000
SIT2		3.834	0.111	34.394	0.000
SIT3R		3.070	0.072	42.921	0.000
SIT4		3.821	0.111	34.441	0.000
SIT5R		2.832	0.066	43.095	0.000
SIT6		3.477	0.101	34.573	0.000
Variances (of Factor) → will always be 1 in a standardized solution					
SIT		1.000	0.000	999.000	999.000
Residual Variances (standardized variance of e's)					
SIT1R		0.500	0.050	10.009	0.000
SIT2		0.741	0.054	13.628	0.000
SIT3R		0.395	0.053	7.388	0.000
SIT4		0.687	0.054	12.786	0.000
SIT5R		0.645	0.035	18.619	0.000
SIT6		0.714	0.050	14.187	0.000
R-SQUARE (equals 1-residual variance OR standardized loading squared)					
SIT1R		0.500	0.050	9.991	0.000
SIT2		0.259	0.054	4.772	0.000
SIT3R		0.605	0.053	11.327	0.000
SIT4		0.313	0.054	5.821	0.000
SIT5R		0.355	0.035	10.264	0.000
SIT6		0.286	0.050	5.696	0.000

The standardized solution will look identical across methods of model identification with respect to the factor loadings, error variances, and R-square values for the items. The standardized intercepts will change because they depend on the unstandardized intercepts (but nobody reports them anyway).

Making use of the standardized model estimates:

Writing out the model – predicted item correlations:

$$Cor(y_1, y_2) = \lambda_1 * Var(F) * \lambda_2$$

$$Cor(y_1, y_2) = .707 * 1 * .509 = .360$$

(actual correlation = .240, so the model over-predicted how related 1 and 2 should be)

Next up: two equivalent ways of getting the same model, but with different scaling (i.e., illustrating the results of different methods of identification...)

**Now let's see the model parameters when using the marker item for model identification instead...
Model 2. Marker Item Loading = 1, Factor Mean = 0 (Factor Variance, All Intercepts Estimated)**

```
! Model 2 -- Marker Item Loading with Factor Mean = 0 - MOST COMMON APPROACH AND DEFAULT IN MPLUS
  Sit BY Sit1r@1 Sit2* Sit3r* Sit4* Sit5r* Sit6*;      ! Loadings (#1 fixed=1)
  [Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*];          ! Intercepts (all free)
  Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;           ! Residual variances (all free)
  Sit*;                                               ! Factor variance (free)
  [Sit@0];                                           ! Factor mean (fixed=0)
```

UNSTANDARDIZED MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS (regression slopes of item response on factor)				
Here, loading for SIT1R is not tested because it is fixed=1				
SIT	BY			
SIT1R		1.000	0.000	999.000
SIT2		0.569	0.083	6.830
SIT3R		1.005	0.035	28.555
SIT4		0.636	0.082	7.741
SIT5R		0.829	0.053	15.698
SIT6		0.664	0.081	8.143
Means (of Factor)				
SIT		0.000	0.000	999.000
Intercepts (of Items) - EXPECTED Y WHEN FACTOR = 0, or for mean of factor in sample				
SIT1R		4.547	0.053	86.474
SIT2		5.289	0.042	127.347
SIT3R		4.896	0.048	101.960
SIT4		5.359	0.042	126.896
SIT5R		4.860	0.052	94.060
SIT6		5.321	0.046	115.492
Variances (of Factor)				
SIT		1.523	0.170	8.954
Residual Variances (variances of e's)				
SIT1R		1.526	0.149	10.217
SIT2		1.409	0.128	11.014
SIT3R		1.004	0.135	7.456
SIT4		1.352	0.127	10.673
SIT5R		1.899	0.118	16.026
SIT6		1.671	0.159	10.517

Yet another equivalent alternative method for scaling the factor...

Model 3. Marker Item Loading = 1 and Intercept = 0 (Factor Variance and Mean Estimated)

```
! Model 3 -- Marker Item Loading and Intercept
  Sit BY Sit1r@1 Sit2* Sit3r* Sit4* Sit5r* Sit6*;      ! Loadings (1 fixed=1)
  [Sit1r@0 Sit2* Sit3r* Sit4* Sit5r* Sit6*];          ! Intercepts (1 fixed=0)
  Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;           ! Residual variances (all free)
  Sit*;                                               ! Factor variance (free)
  [Sit*];                                           ! Factor mean (free)
```

Means (of Factor) → Note is mean of marker item 1

SIT	4.547	0.053	86.474	0.000
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Intercepts (of Items) - EXPECTED Y WHEN FACTOR = 0
HERE, WHICH IS WHEN ITEM 1 = 0 → beyond scale of item, so values are very low

SIT1R	0.000	0.000	999.000	999.000
SIT2	2.701	0.383	7.046	0.000
SIT3R	0.325	0.171	1.899	0.058
SIT4	2.469	0.380	6.504	0.000
SIT5R	1.092	0.246	4.431	0.000
SIT6	2.304	0.369	6.250	0.000

Calculating model degrees of freedom:

$$\text{Total DF} = \frac{v(v+1)}{2} + v = \frac{6(6+1)}{2} + 6 = 27$$

$$\text{Spent: Model DF} = 18$$

$$\text{Leftover: DF} = 9$$

Model fit information for a single-factor model (same regardless of factor scaling method):

Number of Free Parameters **18** → is # of estimated parameters ("free" to be not 0)

Loglikelihood - use for testing differences in model fit across nested models

H0 Value	-11536.404	→ this is for your specified model
H0 Scaling Correction Factor for MLR	1.4158	→ indicates how far off from normal=1
H1 Value	-11322.435	→ this is for a saturated (perfect) model
H1 Scaling Correction Factor for MLR	1.4073	→ indicates how far off from normal=1

Information Criteria → "smaller is better" - use for nested or non-nested model comparisons

Akaike (AIC)	23108.808	→ $AIC = (-2*LL_{H0}) + (2*\text{estimated parameters})$
Bayesian (BIC)	23198.912	→ $BIC = (-2*LL_{H0}) + (\text{LN } N*\text{estimated parameters})$
Sample-Size Adjusted BIC ($n^* = (n + 2) / 24$)	23141.739	→ BIC replacing N with $(N + 2) / 24$

Chi-Square Test of Model Fit (Significance is bad here) → for your specified model

Value	307.799	
Degrees of Freedom	9	→ leftover after estimating our one-factor model
P-Value	0.0000	
Scaling Correction Factor for MLR	1.3903	→ indicates how far off from normal=1 > 1 = leptokurtic distribution (too-fat tails) < 1 = platykurtotic distribution (too-thin tails)

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

Where does this χ^2 value for "model fit" come from? A rescaled -2LL model comparison of this one-factor model (H0) against the saturated model (H1) that perfectly reproduces the data covariances:

Step 1: Original $-2\Delta LL = -2*(LL_{\text{fewer}} - LL_{\text{more}}) = -2(-11,536.404 + 11,322.435) = 427.938$

Step 2: Scaling correction = $[(\#parms_{\text{fewer}} * scale_{\text{fewer}}) - (\#parms_{\text{more}} * scale_{\text{more}})] / (\#parms_{\text{fewer}} - \#parms_{\text{more}})$
 $= [(18 * 1.4158) - (27 * 1.4073)] / (18 - 27) = -12.501 / -9 = 1.3903$

Step 3: Rescaled $-2\Delta LL = -2\Delta LL / \text{scaling correction} = 427.938 / 1.903 = \mathbf{307.803}$ → ~matches model χ^2

Step 4: Difference in df = $\#parms_{\text{more}} - \#parms_{\text{fewer}} = 27 - 18 = 9$

FYI for demonstration purposes, here is how to fit the saturated (Unstructured) Baseline Model: Item means, variances, and covariances in original data**! Saturated Model**

```
! Item means --> [ ] indicates means or intercepts, @=fixed, *=free
[Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*];
! Item variances --> just list item by itself, @=fixed, *=free
Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;
! Item covariances --> just list all by all, @=fixed, *=free
Sit1r Sit2 Sit3r Sit4 Sit5r Sit6 WITH
Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;
```

Model fit information for the saturated model: illustrating what the χ^2 test of global model fit means

Number of Free Parameters	27	→ all possible means, variances, covariances
Loglikelihood		
H0 Value	-11322.435	
H0 Scaling Correction Factor for MLR	1.4073	
H1 Value	-11322.435	
H1 Scaling Correction Factor for MLR	1.4073	
Information Criteria		
Akaike (AIC)	22698.870	
Bayesian (BIC)	22834.027	
Sample-Size Adjusted BIC ($n^* = (n + 2) / 24$)	22748.268	
Chi-Square Test of Model Fit		
Value	0.000*	
Degrees of Freedom	0	
P-Value	0.0000	
Scaling Correction Factor for MLR	1.0000	

Note that H0 and H1 are now the same!
Our H0 model IS the H1 saturated model.

Now back to the rest of the one-factor model fit statistics:**RMSEA (Root Mean Square Error Of Approximation) (want close to 0 = saturated model)**

Estimate	0.173
90 Percent C.I.	0.157 0.190
Probability RMSEA ≤ .05	0.000 → so RMSEA does NOT overlap .05 (is signif > .05)

CFI/TLI (want close to 1 = saturated model)

CFI	0.732
TLI	0.553

SRMR (Standardized Root Mean Square Residual) (want close to 0 = saturated model)

Value	0.086
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Chi-Square Test of Model Fit for the Baseline Model → for the "no covariances" model

Value	1128.693
Degrees of Freedom	15
P-Value	0.0000

Where does this χ^2 value for "fit of the baseline model" come from? A rescaled -2LL model comparison of the independence model with NO covariances to the saturated model:

Step 1: Original $-2\Delta LL = -2*(LL_{\text{fewer}} - LL_{\text{more}}) = -2(-12,312.952 + 11,322.435) = 1,981.034$

Step 2: Scaling correction = $[(\#parms_{\text{fewer}} * scale_{\text{fewer}}) - (\#parms_{\text{more}} * scale_{\text{more}})] / (\#parms_{\text{fewer}} - \#parms_{\text{more}})$
 $= [(12 * 0.9725) - (27 * 1.4073)] / (12 - 27) = -26.372 / -15 = 1.7551$

Step 3: Rescaled $-2\Delta LL = -2\Delta LL / \text{scaling correction} = 1,981.034 / 1.7551 = \mathbf{1,128.704}$ → ~matches baseline χ^2

Step 4: Difference in df = $\#parms_{\text{more}} - \#parms_{\text{fewer}} = 27 - 12 = 15$

What's the point? This baseline model fit test tells us whether there are any covariances at all (i.e., whether it even makes sense to try to fit latent factors to predict them).

As another FYI for demonstration purposes, here is how to fit the Independence (Null) Baseline Model: Item means and variances, but NO covariances

```
! Independence Model
! Item means --> [ ] indicates means or intercepts, @=fixed, *=free
[Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*];
! Item variances --> just list item by itself, @=fixed, *=free
Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;
! NO Item covariances --> just list all by all, @=fixed to 0
Sit1r Sit2 Sit3r Sit4 Sit5r Sit6 WITH
Sit1r@0 Sit2@0 Sit3r@0 Sit4@0 Sit5r@0 Sit6@0;
```

Model fit for the independence “null” model: illustrating what RMSEA, CFI, and TLI mean

Number of Free Parameters 12

Loglikelihood
 H0 Value -12312.952
 H0 Scaling Correction Factor 0.9725
 for MLR
 H1 Value -11322.435
 H1 Scaling Correction Factor 1.4073
 for MLR

Information Criteria
 Akaike (AIC) 24649.904
 Bayesian (BIC) 24709.974
 Sample-Size Adjusted BIC 24671.859
 (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
 Value 1128.692*
 Degrees of Freedom 15
 P-Value 0.0000
 Scaling Correction Factor 1.7552
 for MLR

Note that the model fit is the same as the “baseline” model fit given before.

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)
 Estimate 0.259
 90 Percent C.I. 0.247 0.272
 Probability RMSEA <= .05 0.000

CFI/TLI
 CFI 0.000
 TLI 0.000

Chi-Square Test of Model Fit for the Baseline Model
 Value 1128.693
 Degrees of Freedom 15
 P-Value 0.0000

SRMR (Standardized Root Mean Square Residual)
 Value 0.300

Although not 0, this is the worst possible RMSEA while still allowing separate means and variances per item in these data. RMSEA is a parsimony-corrected absolute fit index (so, its fit is relative to the saturated model).

CFI and TLI are 0 because they are “incremental fit” indices relative to the independence model (which this is).

SRMR is also an absolute fit index (relative to saturated model), so this is the worst it gets for these data, too.

**So global fit for the one-factor model is not so good... (RMSEA = .173, CFI = .732)
What do the voo-doo modification indices suggest we do to fix it?**

MODEL MODIFICATION INDICES

Minimum M.I. value for printing the modification index 6.635
EPC = EXPECTED PARAMETER CHANGE

		M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
WITH Statements (SUGGESTED ERROR COVARIANCES for unknown multidimensionality)					
SIT2	WITH SIT1R	49.618	-0.464	-0.464	-0.316
SIT3R	WITH SIT1R	143.624	1.023	1.023	0.827
SIT3R	WITH SIT2	34.877	-0.357	-0.357	-0.300
SIT4	WITH SIT1R	36.280	-0.403	-0.403	-0.280
SIT4	WITH SIT2	161.318	0.702	0.702	0.509
SIT4	WITH SIT3R	29.202	-0.336	-0.336	-0.288
SIT6	WITH SIT1R	24.079	-0.358	-0.358	-0.224
SIT6	WITH SIT2	63.893	0.486	0.486	0.317
SIT6	WITH SIT3R	22.386	-0.319	-0.319	-0.246
SIT6	WITH SIT4	46.541	0.415	0.415	0.276

Another approach—how about we examine local fit and see where the problems seem to be?

The means and variances of the items will be perfectly reproduced, so that's not an issue...
misfit results from the difference between the observed and model-predicted covariances.

Mplus gives us the "residual" (defined as observed – predicted) or "leftover" matrix of indicator covariances, as well as the correlation version (NOT the same as "residual correlations", which are error correlations):

Residuals for Correlations (Observed - Predicted Correlations)

	SIT1R	SIT2	SIT3R	SIT4	SIT5R	SIT6
SIT1R	0.000					
SIT2	-0.120	0.000				
SIT3R	0.097	-0.079	0.000			
SIT4	-0.096	0.285	-0.067	0.000		
SIT5R	0.032	-0.048	0.018	-0.045	0.000	
SIT6	-0.081	0.185	-0.061	0.149	-0.015	0.000

Mplus also gives us "normalized" residuals, which can be thought of as z-scores for how large the residual leftover covariance is in absolute terms. Because the denominator decreases with sample size, however, these values may be inflated in large samples, so look for *relatively* large values.

"Normalized" Residuals for Inter-Item Covariances = (observed – predicted) / SE(observed)

Normalized Residuals for Covariances

	SIT1R	SIT2	SIT3R	SIT4	SIT5R	SIT6
SIT1R	0.000					
SIT2	-3.503	0.000				
SIT3R	2.977	-2.253	0.000			
SIT4	-2.928	6.560	-1.959	0.000		
SIT5R	0.960	-1.434	0.548	-1.372	0.000	
SIT6	-2.345	4.721	-1.756	3.925	-0.444	0.000

NEGATIVE NORMALIZED RESIDUAL → Less related than you predicted (don't want to be together)
POSITIVE NORMALIZED RESIDUAL → More related than you predicted (want to be more together)

Why might the normalized residuals (leftover correlations) for the positive-worded items be larger than for the negatively-worded items?

These results suggest that wording valence is playing a larger role in the pattern of covariance across items than what the one-factor model predicts. Rather than adding voo-doo covariances among the residuals for specific items, how about a two-factor model based on wording instead?

Model 4. Model with Two Fully Z-Scored Factors

! Model 4 -- Fully Z-Scored 2-Factor Model

```
SitP BY Sit2* Sit4* Sit6*;           ! SitP loadings (all free)
SitN BY Sit1r* Sit3r* Sit5r*;       ! SitN loadings (all free)
[Sit2* Sit4* Sit6*];               ! SitP intercepts (all free)
[Sit1r* Sit3r* Sit5r*];           ! SitN intercepts (all free)
Sit2* Sit4* Sit6*;                 ! SitP residual variances (all free)
Sit1r* Sit3r* Sit5r*;             ! SitN residual variances (all free)
SitP@1; SitN@1;                     ! Factor variances (fixed=1)
SitP WITH SitN*;                   ! Factor covariance (free)
[SitP@0 SitN@0];                   ! Factor means (fixed=0)
```

MODEL FIT INFORMATION

Number of Free Parameters	19
Loglikelihood	
H0 Value	-11340.140
H0 Scaling Correction Factor	1.4017
for MLR	
H1 Value	-11322.435
H1 Scaling Correction Factor	1.4073
for MLR	
Information Criteria	
Akaike (AIC)	22718.281
Bayesian (BIC)	22813.391
Sample-Size Adjusted BIC	22753.042
(n* = (n + 2) / 24)	
Chi-Square Test of Model Fit	
Value	24.924*
Degrees of Freedom	8
P-Value	0.0016
Scaling Correction Factor	1.4207
for MLR	

Is the 2-factor model better than the 1-factor model? How do we know?

Rescaled likelihood ratio test
(-2LL rescaled difference test):

1. **-2ΔLL = -2* difference in LL:**
 $-2*(-11,536.404 + 11,340.140) = 392.528$

2. **difference scaling correction:**
 $(\text{parms}_1 * \text{scale}_1) - (\text{parms}_2 * \text{scale}_2) / (\text{parms}_1 - \text{parms}_2)$
 $(18 * 1.4158) - (19 * 1.4017) / (18 - 19) = 1.1479$

3. **rescaled difference = -2ΔLL / scaling correction:**
 $392.528 / 1.1479 = 341.953$

4. **compare rescaled difference to χ^2 with DF = ΔDF:**
critical χ^2 for DF = 1 is 3.84, so because 341.953 is > 3.84, the model fit significantly improved

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.044
90 Percent C.I.	0.025 0.064
Probability RMSEA <= .05	0.667

CFI/TLI

CFI	0.985
TLI	0.972

Chi-Square Test of Model Fit for the Baseline Model

Value	1128.693
Degrees of Freedom	15
P-Value	0.0000

SRMR (Standardized Root Mean Square Residual)

Value	0.029
-------	-------

UNSTANDARDIZED RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SITP BY				
SIT2	1.007	0.052	19.487	0.000
SIT4	1.064	0.050	21.195	0.000
SIT6	0.956	0.053	18.203	0.000
SITN BY				
SIT1R	1.325	0.048	27.698	0.000
SIT3R	1.349	0.044	30.514	0.000
SIT5R	1.009	0.055	18.358	0.000
SITP WITH SITN = factor covariance (= correlation if variances=1)				
	0.564	0.041	13.776	0.000
Means				
SITP	0.000	0.000	999.000	999.000
SITN	0.000	0.000	999.000	999.000
Intercepts				
SIT1R	4.547	0.053	86.474	0.000
SIT2	5.289	0.042	127.347	0.000
SIT3R	4.896	0.048	101.959	0.000
SIT4	5.359	0.042	126.896	0.000
SIT5R	4.860	0.052	94.060	0.000
SIT6	5.321	0.046	115.492	0.000
Variations				
SITP	1.000	0.000	999.000	999.000
SITN	1.000	0.000	999.000	999.000
Residual Variations				
SIT1R	1.294	0.103	12.547	0.000
SIT2	0.888	0.097	9.173	0.000
SIT3R	0.724	0.092	7.857	0.000
SIT4	0.835	0.093	9.003	0.000
SIT5R	1.926	0.119	16.128	0.000
SIT6	1.428	0.134	10.684	0.000

Omega =

$$\frac{\text{Var}(\text{Factor}) * (\text{Sum of loadings})^2 / \text{Var}(\text{Factor}) * (\text{Sum of loadings})^2 + \text{Sum of error variances} + 2 * \text{Sum of error covariances}}{\text{Omega for Positive Factor} = .744}$$

$$\frac{1.0*(1.007+1.064+0.956)^2 / 1.0*(1.007+1.064+0.956)^2 + (0.888+0.835+1.428) + 2*0}{\text{Omega for Negative Factor} = .775}$$

$$\frac{1.0*(1.325+1.349+1.009)^2 / 1.0*(1.325+1.349+1.009)^2 + (1.294+0.724+1.926) + 2*0}{\text{Omega for Negative Factor} = .775}$$
(alpha was .746, btw)
(alpha was .780)

STDYX STANDARDIZED RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SITP BY				
SIT2	0.730	0.032	22.794	0.000
SIT4	0.759	0.029	25.995	0.000
SIT6	0.625	0.035	17.949	0.000
SITN BY				
SIT1R	0.759	0.022	34.072	0.000
SIT3R	0.846	0.021	39.657	0.000
SIT5R	0.588	0.030	19.651	0.000
SITP WITH SITN				
	0.564	0.041	13.776	0.000
Residual Variations				
SIT1R	0.425	0.034	12.567	0.000
SIT2	0.467	0.047	9.976	0.000
SIT3R	0.285	0.036	7.895	0.000
SIT4	0.425	0.044	9.589	0.000
SIT5R	0.654	0.035	18.576	0.000
SIT6	0.610	0.043	14.029	0.000
R-SQUARE				
SIT1R	0.575	0.034	17.036	0.000
SIT2	0.533	0.047	11.397	0.000
SIT3R	0.715	0.036	19.829	0.000
SIT4	0.575	0.044	12.998	0.000
SIT5R	0.346	0.035	9.826	0.000
SIT6	0.390	0.043	8.974	0.000

Wouldn't it be nice if Mplus would compute Omegas for you? It can, if you (a) label the parameters it needs to do the math, and (b) create new terms for the Omega estimates via MODEL CONSTRAINT:

Model 4. Fully Z-Scored, 2-Factor Model again, now with parameter labels

```
! Model 4 -- Fully Z-Scored 2-Factor Model with all parameters labeled for reference
SitP BY Sit2* Sit4* Sit6* (L1-L3); ! SitP loadings (all free)
SitN BY Sit1r* Sit3r* Sit5r* (L4-L6); ! SitN loadings (all free)
[Sit2* Sit4* Sit6*] (I1-I3); ! SitP intercepts (all free)
[Sit1r* Sit3r* Sit5r*] (I4-I6); ! SitN intercepts (all free)
Sit2* Sit4* Sit6* (E1-E3); ! SitP residual variances (all free)
Sit1r* Sit3r* Sit5r* (E4-E6); ! SitN residual variances (all free)
SitP@1 (VarP); SitN@1 (VarN); ! Factor variances (fixed=1)
SitP WITH SitN* (FactCov); ! Factor covariance (free)
[SitP@0 SitN@0] (MeanP MeanN); ! Factor means (fixed=0)
```

```
MODEL CONSTRAINT: ! Calculate omega model-based reliability per factor
NEW(OmegaP OmegaN); ! Using 1 as placeholder for factor variances
OmegaP = (1*(L1+L2+L3)**2) / ((1*(L1+L2+L3)**2) + (E1+E2+E3));
OmegaN = (1*(L4+L5+L6)**2) / ((1*(L4+L5+L6)**2) + (E4+E5+E6));
```

Output now provided in unstandardized solution:

New/Additional Parameters				
OMEGAP	0.744	0.020	37.956	0.000
OMEGAN	0.775	0.014	56.803	0.000

Any more local fit problems? Let's see...

Residuals of correlation matrix (how far off each predicted correlation is from data):

	SIT1R	SIT2	SIT3R	SIT4	SIT5R	SIT6
SIT1R	0.000					
SIT2	-0.073	0.000				
SIT3R	0.006	-0.031	0.000			
SIT4	-0.025	0.016	0.007	0.000		
SIT5R	0.007	0.013	-0.015	0.037	0.000	
SIT6	0.030	0.001	0.057	-0.026	0.097	0.000

"Normalized" residuals (z-like statistic for how far off each covariance is):

	SIT1R	SIT2	SIT3R	SIT4	SIT5R	SIT6
SIT1R	0.000					
SIT2	-2.125	0.000				
SIT3R	0.172	-0.896	0.000			
SIT4	-0.768	0.370	0.192	0.000		
SIT5R	0.212	0.382	-0.464	1.128	0.000	
SIT6	0.869	0.031	1.658	-0.676	2.847	0.000

Any suggested voo-doo? (only available when not using MODEL CONSTRAINT, though)

MODEL MODIFICATION INDICES

Minimum M.I. value for printing the modification index 6.635

M.I. E.P.C. Std E.P.C. StdYX E.P.C.

BY Statements - these are cross-loadings

SITN	BY SIT2	9.775	-0.224	-0.224	-0.162
SITN	BY SIT6	10.828	0.245	0.245	0.160

WITH Statements - these are error covariances

SIT4	WITH SIT2	10.830	0.332	0.332	0.386
SIT6	WITH SIT4	9.773	-0.273	-0.273	-0.250

Because we have no real theoretical or defensible reason to fit any of these suggested parameters, we will not add any new parameters. This will be about as good as it gets.

Let's examine the estimated distribution of the factor scores for each factor:

SUMMARY OF FACTOR SCORES

FACTOR SCORE INFORMATION (COMPLETE-DATA PATTERN)	
FACTOR DETERMINACIES	
SITP	0.882
SITN	0.908

The factor determinacy, the correlation between the estimated and true factor scores, is .882 for the positive factor and .908 for the negative factor.

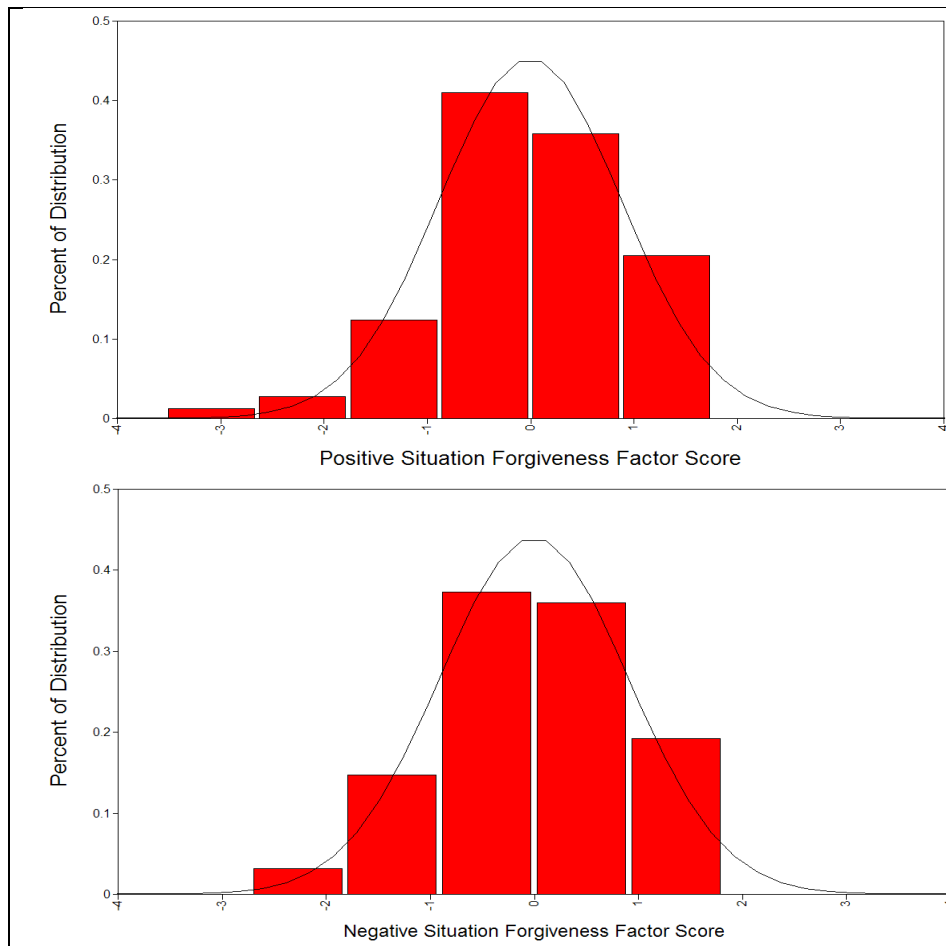
Positive factor score SE = 0.472
 Negative factor score SE = 0.418

SAMPLE STATISTICS FOR ESTIMATED FACTOR SCORES

SAMPLE STATISTICS				
Means				
	SITP	SITP_SE	SITN	SITN_SE
1	0.000	0.472	0.000	0.418
Covariances				
	SITP	SITP_SE	SITN	SITN_SE
SITP	0.777			
SITP_SE	0.000	0.000		
SITN	0.533	0.000	0.825	
SITN_SE	0.000	0.000	0.000	0.000
Correlations				
	SITP	SITP_SE	SITN	SITN_SE
SITP	1.000			
SITP_SE	999.000	1.000		
SITN	0.665	999.000	1.000	
SITN_SE	999.000	999.000	999.000	1.000

Although the variance of each factor was supposed to be 1.0, the variance of the factor scores is < 1.0 because of shrinkage.

Likewise, the correlation between the factors was .56, but the correlation between the estimated factor scores is .67 instead (given the shrinkage).



The positive factor scores have an estimated mean of 0 with a variance of 0.78 instead of 1.00.

The SE for each person's factor score is 0.472. Treating factor scores as observed variables is like saying SE = 0.

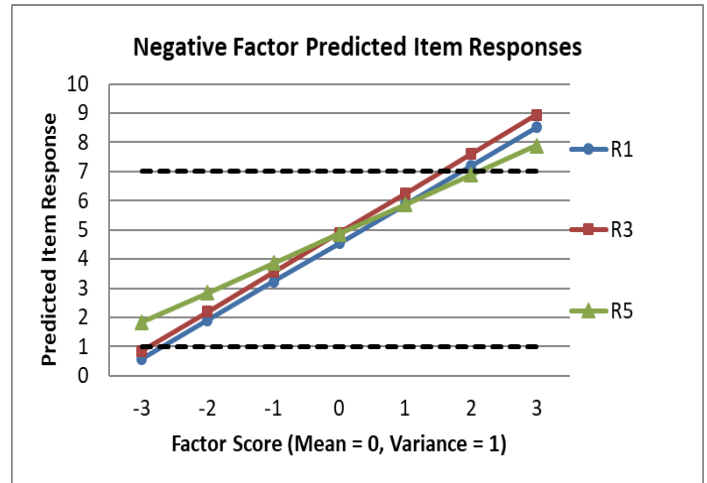
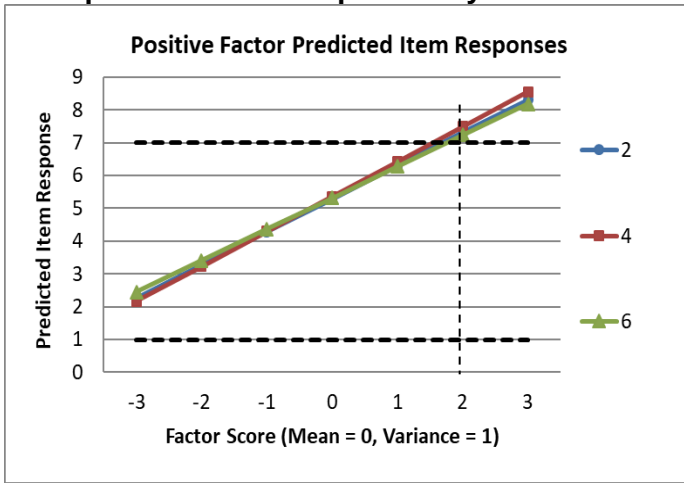
Positive factor score =
 Score ± 2*0.472 = Score ± 0.944!

The negative factor scores have an estimated mean of 0 with a variance of 0.825 instead of 1.00.

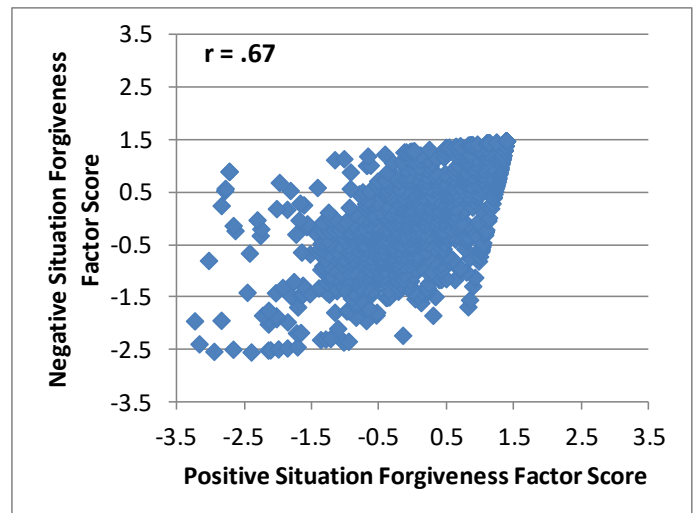
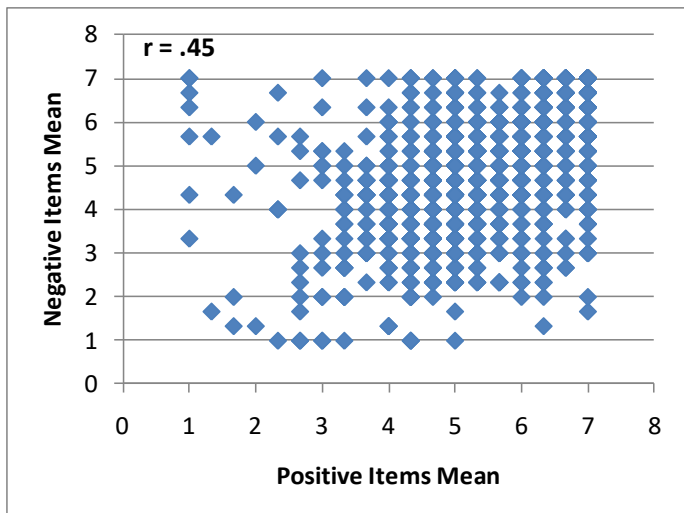
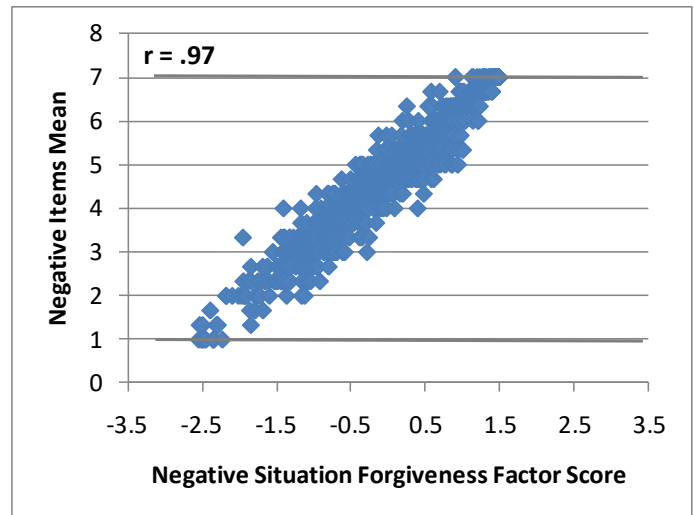
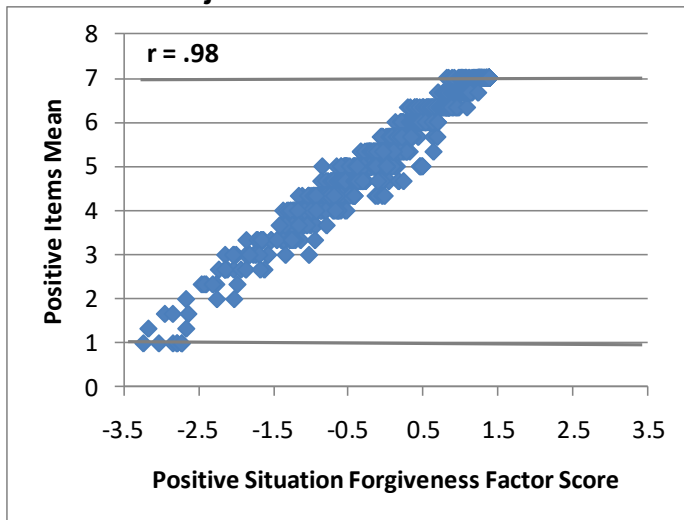
The SE for each person's factor score is 0.418, so ± 0.836!

The negative factor scores retain more variance (and have a smaller SE) because there is more information in them, due to higher factor loadings (greater reliability) of their items.

Model-predicted item responses by factor scores:



What if we had just taken the mean of the three items for each subscale?



There are problems with either of these observed variable approaches: The **mean of the items** appears to have less variability (i.e., fewer possible scores) and assumes that all items should be weighted equally and have no error. The **estimated factor scores** do not have the same properties as estimated for the factor in the model (i.e., less variance for each factor, higher correlation among the factors).

What to do instead of either of these? Stay tuned for options involving plausible values...

Another example: Formal Tests of CTT Assumptions

We will test the CTT assumption of tau-equivalence (equal factor loadings), one factor at a time. If those hold, we can then test the assumption of parallel items (equal error variances, too).

First, in Model 5 we will test “tau-equivalence” (equal loadings) of the negative factor only:

```
! Model 5 -- Tau-Equivalent Negative Items Only 2-Factor Model
  SitP BY Sit2* Sit4* Sit6*;           ! SitP loadings (all free)
  SitN BY Sit1r* Sit3r* Sit5r* (NegLoad); ! SitN loadings (all held equal)
  [Sit2* Sit4* Sit6*];                 ! SitP intercepts (all free)
  [Sit1r* Sit3r* Sit5r*];             ! SitN intercepts (all free)
  Sit2* Sit4* Sit6*;                   ! SitP residual variances (all free)
  Sit1r* Sit3r* Sit5r*;               ! SitN residual variances (all free)
  SitP@1; SitN@1;                       ! Factor variances (fixed=1)
  SitP WITH SitN*;                      ! Factor covariance (free)
  [SitP@0 SitN@0];                     ! Factor means (fixed=0)
```

UNSTANDARDIZED MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SITP	BY				
	SIT2	1.007	0.052	19.491	0.000
	SIT4	1.063	0.050	21.202	0.000
	SIT6	0.957	0.052	18.257	0.000
SITN	BY				
	SIT1R	1.254	0.032	38.957	0.000
	SIT3R	1.254	0.032	38.957	0.000
	SIT5R	1.254	0.032	38.957	0.000
SITP	WITH				
	SITN	0.575	0.041	13.855	0.000
Residual Variances					
	SIT1R	1.335	0.083	16.150	0.000
	SIT2	0.889	0.096	9.217	0.000
	SIT3R	0.857	0.069	12.337	0.000
	SIT4	0.837	0.092	9.045	0.000
	SIT5R	1.806	0.115	15.716	0.000
	SIT6	1.425	0.134	10.630	0.000

STANDARDIZED STYDX MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SITP	BY				
	SIT2	0.730	0.032	22.840	0.000
	SIT4	0.758	0.029	26.037	0.000
	SIT6	0.626	0.035	17.958	0.000
SITN	BY				
	SIT1R	0.735	0.016	46.189	0.000
	SIT3R	0.805	0.016	50.774	0.000
	SIT5R	0.682	0.015	45.076	0.000
Residual Variances					
	SIT1R	0.459	0.023	19.604	0.000
	SIT2	0.467	0.047	10.017	0.000
	SIT3R	0.353	0.025	13.835	0.000
	SIT4	0.425	0.044	9.633	0.000
	SIT5R	0.535	0.021	25.887	0.000
	SIT6	0.609	0.044	13.969	0.000

Why are the standardized factor loadings for the negative factor not held equal like the unstandardized loadings are?

Fit of previous 2-factor model as baseline:		Fit of tau-equivalent negative items 2-factor model:	
Number of Free Parameters	19	Number of Free Parameters	17
Loglikelihood		Loglikelihood	
H0 Value	-11340.140	H0 Value	-11357.612
H0 Scaling Correction Factor for MLR	1.4017	H0 Scaling Correction Factor for MLR	1.4474
H1 Value	-11322.435	H1 Value	-11322.435
H1 Scaling Correction Factor for MLR	1.4073	H1 Scaling Correction Factor for MLR	1.4073
RMSEA (Root Mean Square Error Of Approximation)		RMSEA (Root Mean Square Error Of Approximation)	
Estimate	0.044	Estimate	0.062
90 Percent C.I.	0.025 0.064	90 Percent C.I.	0.046 0.079
Probability RMSEA <= .05	0.667	Probability RMSEA <= .05	0.102
CFI/TLI		CFI/TLI	
CFI	0.985	CFI	0.962
TLI	0.972	TLI	0.943

Does the assumption of tau-equivalence hold for the negative items? How do we know?

Second, Model 6 tests tau-equivalence of the factor loadings for the positive factor only:

```
! Model 6 -- Tau-Equivalent Positive Items Only 2-Factor Model
  SitP BY Sit2* Sit4* Sit6* (PosLoad);      ! SitP loadings (all held equal)
  SitN BY Sit1r* Sit3r* Sit5r*;           ! SitN loadings (all free)
  [Sit2* Sit4* Sit6*];                   ! SitP intercepts (all free)
  [Sit1r* Sit3r* Sit5r*];               ! SitN intercepts (all free)
  Sit2* Sit4* Sit6*                      (E1-E3); ! SitP residual variances (all free)
  Sit1r* Sit3r* Sit5r*;                 ! SitN residual variances (all free)
  SitP@1; SitN@1;                       ! Factor variances (fixed=1)
  SitP WITH SitN*;                      ! Factor covariance (free)
  [SitP@0 SitN@0];                      ! Factor means (fixed=0)
```

MODEL CONSTRAINT:

```
NEW(AlphaP);      ! This is now equivalent to alpha
AlphaP = (1*(Posload*3)**2) / ((1*(Posload*3)**2) + (E1+E2+E3));
```

Number of Free Parameters	17
Loglikelihood	
H0 Value	-11341.773
H0 Scaling Correction Factor for MLR	1.4187
H1 Value	-11322.435
H1 Scaling Correction Factor for MLR	1.4073
RMSEA (Root Mean Square Error Of Approximation)	
Estimate	0.040
90 Percent C.I.	0.023 0.058
Probability RMSEA <= .05	0.797
CFI/TLI	
CFI	0.984
TLI	0.976

Does the assumption of tau-equivalence hold for the positive items? How do we know?

UNSTANDARDIZED MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SITP	BY				
	SIT2	1.014	0.036	28.389	0.000
	SIT4	1.014	0.036	28.389	0.000
	SIT6	1.014	0.036	28.389	0.000
SITN	BY				
	SIT1R	1.325	0.048	27.727	0.000
	SIT3R	1.349	0.044	30.531	0.000
	SIT5R	1.010	0.055	18.370	0.000
SITP	WITH				
	SITN	0.567	0.040	14.131	0.000
Residual Variances					
	SIT1R	1.295	0.103	12.580	0.000
	SIT2	0.881	0.083	10.587	0.000
	SIT3R	0.725	0.092	7.873	0.000
	SIT4	0.886	0.075	11.767	0.000
	SIT5R	1.925	0.119	16.117	0.000
	SIT6	1.384	0.118	11.737	0.000
New/Additional Parameters					
	ALPHAP	0.746	0.020	38.200	0.000

STANDARDIZED STDYX MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SITP	BY				
	SIT2	0.734	0.023	32.593	0.000
	SIT4	0.733	0.021	35.611	0.000
	SIT6	0.653	0.022	29.743	0.000
SITN	BY				
	SIT1R	0.759	0.022	34.139	0.000
	SIT3R	0.846	0.021	39.706	0.000
	SIT5R	0.588	0.030	19.663	0.000
SITP	WITH				
	SITN	0.567	0.040	14.131	0.000
Residual Variances					
	SIT1R	0.425	0.034	12.598	0.000
	SIT2	0.461	0.033	13.965	0.000
	SIT3R	0.285	0.036	7.910	0.000
	SIT4	0.463	0.030	15.350	0.000
	SIT5R	0.654	0.035	18.562	0.000
	SIT6	0.574	0.029	20.019	0.000

Given that tau-equivalence held for the positive factor, in Model 7 we can also test the assumption of parallel items as equal residual variances (in addition to equal factor loadings):

! Model 7 -- Parallel Items on Positive Only 2-Factor Model

```

SitP BY Sit2* Sit4* Sit6* (PosLoad);      ! SitP loadings (all held equal)
SitN BY Sit1r* Sit3r* Sit5r*;            ! SitN loadings (all free)
[Sit2* Sit4* Sit6*];                      ! SitP intercepts (all free)
[Sit1r* Sit3r* Sit5r*];                  ! SitN intercepts (all free)
Sit2* Sit4* Sit6* (PosError);            ! SitP residual variances (all held equal)
Sit1r* Sit3r* Sit5r*;                    ! SitN residual variances (all free)
SitP@1; SitN@1;                           ! Factor variances (fixed=1)
SitP WITH SitN*;                          ! Factor covariance (free)
[SitP@0 SitN@0];                          ! Factor means (fixed=0)

```

MODEL CONSTRAINT:

```

NEW(SpearP);      ! This is now Spearman-Brown reliability
SpearP = (1*(Posload*3)**2) / ((1*(Posload*3)**2) + (PosError*3));

```

Number of Free Parameters 15

Loglikelihood
 H0 Value -11361.960
 H0 Scaling Correction Factor 1.3443
 for MLR
 H1 Value -11322.435
 H1 Scaling Correction Factor 1.4073
 for MLR

RMSEA (Root Mean Square Error Of Approximation)
 Estimate 0.056
 90 Percent C.I. 0.041 0.072
 Probability RMSEA <= .05 0.244

CFI/TLI
 CFI 0.963
 TLI 0.954

Does the assumption of parallel items hold for the positive items? How do we know?

UNSTANDARDIZED MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SITP BY				
SIT2	1.005	0.035	28.455	0.000
SIT4	1.005	0.035	28.455	0.000
SIT6	1.005	0.035	28.455	0.000
SITN BY				
SIT1R	1.325	0.048	27.816	0.000
SIT3R	1.347	0.044	30.623	0.000
SIT5R	1.011	0.055	18.408	0.000
SITP WITH				
SITN	0.581	0.040	14.581	0.000
Residual Variances				
SIT1R	1.294	0.102	12.645	0.000
SIT2	1.060	0.061	17.452	0.000
SIT3R	0.728	0.091	7.992	0.000
SIT4	1.060	0.061	17.452	0.000
SIT5R	1.922	0.119	16.095	0.000
SIT6	1.060	0.061	17.452	0.000

STANDARDIZED STDYX MODEL RESULTS

SITP BY				
SIT2	0.698	0.019	37.365	0.000
SIT4	0.698	0.019	37.365	0.000
SIT6	0.698	0.019	37.365	0.000
SITN BY				
SIT1R	0.759	0.022	34.339	0.000
SIT3R	0.845	0.021	40.011	0.000
SIT5R	0.589	0.030	19.713	0.000
SITP WITH				
SITN	0.581	0.040	14.581	0.000
Residual Variances				
SIT1R	0.424	0.034	12.652	0.000
SIT2	0.512	0.026	19.616	0.000
SIT3R	0.286	0.036	8.024	0.000
SIT4	0.512	0.026	19.616	0.000
SIT5R	0.653	0.035	18.520	0.000
SIT6	0.512	0.026	19.616	0.000

Example results section describing these analyses...

(Note: You may borrow the phrasing contained in this example to describe various aspects of your analyses, but your own results sections will not mimic this example exactly—they should be customized to describe the how and the why of what you did, specifically).

(Descriptive information for the sample and items would have already been given in the method section...)

The reliability and dimensionality of six items each assessing forgiveness of situations was assessed in a sample of 1,103 persons with a confirmatory factor analysis using robust maximum likelihood estimation (MLR) in *Mplus* v. 8.4 (Muthén & Muthén, 1998–2017). All models were identified by setting any latent factor means to 0 and latent factor variances to 1, such that all item intercepts, item factor loadings, and item residual variances were then estimated. The six items utilized a seven-point response scale, and three items were reverse-coded prior to analysis such that higher values then indicated greater levels of situation forgiveness for all items. As reported in Table 1, model fit statistics include the obtained model χ^2 , its scaling factor (in which values different than 1.000 indicate deviations from multivariate normality), its degrees of freedom, and its *p*-value (in which non-significance is desirable for good fit), CFI, or Comparative Fit Index (in which values higher than .95 are desirable for good fit), and the RMSEA, or Root Mean Square Error of Approximation, point estimate and 90% confidence interval (in which values lower than .06 are desirable for good fit). As reported in Table 2, nested model comparisons were conducted using the rescaled $-2\Delta LL$ with degrees of freedom equal to the rescaled difference in the number of parameters between models (i.e., a rescaled likelihood ratio test). The specific models examined are described in detail below.

Although a one-factor model was initially posited to account for the pattern of covariance across these six items, it resulted in poor fit, as shown in Table 1. Although each item had a significant factor loading (with standardized loadings ranging from .509 to .778), a single latent factor did not adequately describe the pattern of relationship across these six items as initially hypothesized. Sources of local misfit were identified using the normalized residual covariance matrix, available via the RESIDUAL output option in *Mplus*, in which individual values were calculated as: (observed covariance – expected covariance) / SE of observed covariance. Relatively large positive residual covariances were observed among items 2, 4, and 6 (the positively-worded items), indicating that these items were more related than was predicted by the single-factor model. Modification indices, available via the MODINDICES output option in *Mplus*, corroborated this pattern, further suggesting additional remaining relationships among the negatively-worded items as well.

The necessity of separate latent factors for the positively-worded and negatively-worded items was tested by specifying a two-factor model in which the positively-worded items 2, 4, and 6 indicated a *forgiveness* factor, and in which negatively-worded items 1, 3, and 5 indicated a *not unforgiveness* factor, and in which the two factors were allowed to correlate. The two-factor model fit was acceptable by every criterion except the significant χ^2 , likely due to the large sample. In addition, the two-factor model fit significantly better than the one-factor model, as reported in Table 2, indicating that the estimated correlation between the two factors of .564 was significantly less than 1.000. Thus, the six items appeared to measure two separate but related constructs. Further examination of local fit via normalized residual covariances and modification indices yielded no interpretable remaining relationships, and thus this two-factor model was retained.

Table 3 provides the estimates and their standard errors for the item factor loadings, intercepts, and residual (error) variances from both the unstandardized and standardized solutions. All factor loadings and the factor covariance were statistically significant. As shown in Table 3, standardized loadings for the forgiveness factor items ranged from .625 to .759 (with R^2 values for the amount of item variance accounted for by the factor ranging from .390 to .575), and standardized loadings for the not unforgiveness factor ranged from .588 to .846 (with R^2 values of .346 to .715), suggesting the factor loadings were practically significant as well. Omega model-based reliability was calculated for the sum scores of each factor as described in Brown (2015) as the squared sum of the factor loadings divided by the squared sum of the factor loadings plus the sum of the error variances plus twice the sum of the error covariances (although no error covariances were included here). Omega was .744 for the forgiveness factor and .775 for the not unforgiveness factor, suggesting marginal reliability for each of the three-item scales.

The resulting distribution of the factors was examined by requesting empirical Bayes estimates of the individual scores for each factor, as shown in Figure 1. Factor determinacy estimates, available via the FSDETERMINACY output option in *Mplus*, were .882 and .908, respectively, for the forgiveness and not unforgiveness factors (with standard errors for the factor scores of .472 and .418), indicating that the estimated factor scores were strongly related to their model-based counterparts. In addition, Figure 2 shows the predicted response for each item as a linear function of the latent factor based on the estimated model parameters. As shown, the predicted item response goes above the highest response option just before a latent factor score of +2 (i.e., 2 SDs above the mean), resulting in a ceiling effect for both sets of factor scores, as also shown in Figure 1. In addition, for the not unforgiveness factor, the predicted item response goes below the lowest response option just before a latent factor score of -3 (i.e., 3 SDs below the mean), resulting in a floor effect for the not unforgiveness factor, as also shown in Figure 1.

The extent to which the items within each factor could be seen as exchangeable was then examined via an additional set of nested model comparisons, as reported in Table 1 (for fit) and Table 2 (for comparisons of fit). First, the assumption of tau-equivalence (i.e., true-score equivalence, equal discrimination across items) was examined by constraining the factor loadings to be equal within a factor. For the not unforgiveness factor, the tau-equivalent model fit was acceptable but was significantly worse than the original two-factor model fit (i.e., in which all loadings were estimated freely). For the forgiveness factor, however, the tau-equivalent model fit was acceptable and was not significantly worse than the original two-factor model fit. Thus, the assumption of tau-equivalence held for the forgiveness factor items only. Finally, the assumption of parallel items (i.e., equal factor loadings and equal residual variances, or equal reliability across items) was examined for the forgiveness factor items only, and the resulting model fit was acceptable but was significantly worse than the tau-equivalent forgiveness factor model fit. Thus, the assumption of parallel items did not hold for the forgiveness factor items. In summary, while the not unforgiveness factor items were not exchangeable, the forgiveness factor items were exchangeable with respect to their factor loadings only (i.e., equal discrimination, but not equal item residual variances or item reliability).

Tables would be built as seen in the excel workbook:

Table 1 → “Model Fit Table 1” worksheet
 Table 2 → “MLR Comparisons Table 2” worksheet
 Table 3 → “Model Estimates Table 3” worksheet

Figures would be built as seen in this example:

Figure 1 → Can be built in *Mplus*
 Figure 2 → Can be built using “Factor Model Predictions” worksheet

References:

Brown, T. A. (2015). *Confirmatory factor analysis for applied research* (2nd ed.). New York, NY: Guilford.

Muthén, L. K., & Muthén, B.O. (1998–2017). *Mplus user's guide* (8th ed.). Los Angeles, CA: Muthén & Muthén.

Example 4 Continued: CFA of Forgiveness of Situations (N = 1103) using SAS MIXED

SAS Code to Read in Mplus Data:

```
* Import data from Mplus, becomes var1-var23 without names at top;
PROC IMPORT OUT=work.Situation DATAFILE= "&example.\Study2.csv" DBMS=CSV REPLACE;
  GETNAMES=NO; DATAROW=1; RUN;

* Rename variables, remove missing values;
DATA Situation; SET Situation;
  ARRAY old(23) var1-var23;
  ARRAY new(23) PersonID Self1 Self2r Self3 Self4r Self5 Self6r
    Other1r Other2 Other3r Other4 Other5r Other6
    Sit1r Sit2 Sit3r Sit4 Sit5r Sit6
    Selfsub Othsub Sitsub HFSsum;
  DO i=1 TO 23; new(i)=old(i); IF new(i)=99999 THEN new(i)=.; END;
  DROP i var1-var23; RUN;

* Stack situation items;
DATA SituationStacked; SET Situation;
  ARRAY aitem(6) Sit1r Sit2 Sit3r Sit4 Sit5r Sit6;
  DO i=1 TO 6; itemnum=i; response=aitem(i); OUTPUT; END; DROP i; RUN;
```

Independence (Null) Baseline Model: Item means and variances, but NO covariances

```
TITLE "Independence (Null) CFA Model in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
  CLASS PersonID itemnum;
  MODEL response = itemnum / SOLUTION NOINT NOTEST;
  REPEATED itemnum / TYPE=TOEPH(1) SUBJECT=PersonID R; RUN;
```

TYPE=TOEPH(1) predicts a diagonal matrix (would be the same as TYPE=UN(1).

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	3.0493					
2		1.9028				
3			2.5431			
4				1.9672		
5					2.9451	
6						2.3412

The R matrix shows the unconditional variances per item—repeated in the next piece of output as Var(item). Note that this independence “null” model predicts NO covariances between items.

Covariance Parameter Estimates

Cov	Subject	Estimate	Standard Error	Z	Pr > Z
Var(1)	PersonID	3.0493	0.1298	23.48	<.0001
Var(2)	PersonID	1.9028	0.08102	23.48	<.0001
Var(3)	PersonID	2.5431	0.1083	23.48	<.0001
Var(4)	PersonID	1.9672	0.08377	23.48	<.0001
Var(5)	PersonID	2.9451	0.1254	23.48	<.0001
Var(6)	PersonID	2.3412	0.09969	23.48	<.0001

Model fit is given as -2LL rather than LL (but otherwise is the same as given from Mplus).

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
24625.9	12	24649.9	24650.0	24672.6	24710.0	24722.0

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > t
itemnum	1	4.5467	0.05258	5509	86.47	<.0001
itemnum	2	5.2892	0.04153	5509	127.35	<.0001
itemnum	3	4.8957	0.04802	5509	101.96	<.0001
itemnum	4	5.3590	0.04223	5509	126.90	<.0001
itemnum	5	4.8604	0.05167	5509	94.06	<.0001
itemnum	6	5.3209	0.04607	5509	115.49	<.0001

The fixed effects show the unconditional means per item.

Saturated (Unstructured) Baseline Model: Item means, variances, and covariances in original data

```
TITLE "Saturated (Unstructured) CFA Model in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
CLASS PersonID itemnum;
MODEL response = itemnum / SOLUTION NOINT NOTEST;
REPEATED itemnum / TYPE=UN(6) SUBJECT=PersonID R RCORR; RUN;
```

TYPE=UN(6) predicts a fully-estimated matrix with no constraints whatsoever.

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	3.0493	0.5772	1.8022	0.7339	1.3583	0.7946
2	0.5772	1.9028	0.6974	1.1029	0.6043	0.9652
3	1.8022	0.6974	2.5431	0.8244	1.3191	0.8676
4	0.7339	1.1029	0.8244	1.9672	0.6947	0.9618
5	1.3583	0.6043	1.3191	0.6947	2.9451	0.7982
6	0.7946	0.9652	0.8676	0.9618	0.7982	2.3412

The R matrix shows the unconditional variances and covariances for the items.

RCORR is the unconditional correlation matrix.

Note THIS IS THE DATA—the only discrepancies you'd see relative to descriptive statistics would be from missing data, as these are ML estimates (that assume MAR rather than MCAR as in listwise deletion).

Estimated R Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.2396	0.6472	0.2997	0.4533	0.2974
2	0.2396	1.0000	0.3170	0.5700	0.2553	0.4573
3	0.6472	0.3170	1.0000	0.3686	0.4820	0.3555
4	0.2997	0.5700	0.3686	1.0000	0.2886	0.4482
5	0.4533	0.2553	0.4820	0.2886	1.0000	0.3040
6	0.2974	0.4573	0.3555	0.4482	0.3040	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
UN(1,1)	PersonID	3.0493	0.1298	23.48	<.0001
UN(2,1)	PersonID	0.5772	0.07458	7.74	<.0001
UN(2,2)	PersonID	1.9028	0.08102	23.48	<.0001
UN(3,1)	PersonID	1.8022	0.09988	18.04	<.0001
UN(3,2)	PersonID	0.6974	0.06948	10.04	<.0001
UN(3,3)	PersonID	2.5431	0.1083	23.48	<.0001
UN(4,1)	PersonID	0.7339	0.07699	9.53	<.0001
UN(4,2)	PersonID	1.1029	0.06705	16.45	<.0001
UN(4,3)	PersonID	0.8244	0.07178	11.49	<.0001
UN(4,4)	PersonID	1.9672	0.08377	23.48	<.0001
UN(5,1)	PersonID	1.3583	0.09907	13.71	<.0001
UN(5,2)	PersonID	0.6043	0.07356	8.21	<.0001
UN(5,3)	PersonID	1.3191	0.09148	14.42	<.0001
UN(5,4)	PersonID	0.6947	0.07543	9.21	<.0001
UN(5,5)	PersonID	2.9451	0.1254	23.48	<.0001
UN(6,1)	PersonID	0.7946	0.08393	9.47	<.0001
UN(6,2)	PersonID	0.9652	0.06988	13.81	<.0001
UN(6,3)	PersonID	0.8676	0.07798	11.13	<.0001
UN(6,4)	PersonID	0.9618	0.07081	13.58	<.0001
UN(6,5)	PersonID	0.7982	0.08264	9.66	<.0001
UN(6,6)	PersonID	2.3412	0.09969	23.48	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
22644.9	27	22698.9	22699.1	22750.0	22834.0	22861.0

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > t
itemnum	1	4.5467	0.05258	5509	86.47	<.0001
itemnum	2	5.2892	0.04153	5509	127.35	<.0001
itemnum	3	4.8957	0.04802	5509	101.96	<.0001
itemnum	4	5.3590	0.04223	5509	126.90	<.0001
itemnum	5	4.8604	0.05167	5509	94.06	<.0001
itemnum	6	5.3209	0.04607	5509	115.49	<.0001

The fixed effects again show the unconditional means per item.

Model 1. Single Factor with Fully Z-Scored Factor Model Identification (Factor Variance = 1, Factor Mean = 0, All Loadings and Intercepts Estimated)

```
TITLE "Single-Factor CFA Model (Factor Variance=1, Factor Mean=0) in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
  CLASS PersonID itemnum;
  MODEL response = itemnum / SOLUTION NOINT NOTEST;
  REPEATED itemnum / TYPE=FA(1) SUBJECT=PersonID R RCORR;
RUN;
```

TYPE=FA(1) creates the covariance matrix that would be predicted by a single-factor model.

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	3.0493	0.8670	1.5313	0.9682	1.2626	1.0108
2	0.8670	1.9028	0.8716	0.5511	0.7187	0.5753
3	1.5313	0.8716	2.5431	0.9733	1.2692	1.0161
4	0.9682	0.5511	0.9733	1.9672	0.8025	0.6424
5	1.2626	0.7187	1.2692	0.8025	2.9451	0.8378
6	1.0108	0.5753	1.0161	0.6424	0.8378	2.3412

The R matrix shows the predicted variances and covariances for the items.

RCORR is the single-factor predicted correlation matrix.

THIS IS NO LONGER THE DATA. So the objective is to see how close this predicted covariance matrix is from the one given by the saturated model (which was the data).

Estimated R Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.3600	0.5499	0.3953	0.4213	0.3783
2	0.3600	1.0000	0.3962	0.2848	0.3036	0.2726
3	0.5499	0.3962	1.0000	0.4351	0.4638	0.4164
4	0.3953	0.2848	0.4351	1.0000	0.3334	0.2994
5	0.4213	0.3036	0.4638	0.3334	1.0000	0.3191
6	0.3783	0.2726	0.4164	0.2994	0.3191	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
FA(1)	PersonID	1.5259	0.09440	16.16	<.0001
FA(2)	PersonID	1.4093	0.07096	19.86	<.0001
FA(3)	PersonID	1.0038	0.07755	12.94	<.0001
FA(4)	PersonID	1.3518	0.07071	19.12	<.0001
FA(5)	PersonID	1.8986	0.09312	20.39	<.0001
FA(6)	PersonID	1.6706	0.08330	20.05	<.0001
FA(1,1)	PersonID	1.2342	0.05332	23.15	<.0001
FA(2,1)	PersonID	0.7025	0.04720	14.88	<.0001
FA(3,1)	PersonID	1.2407	0.04783	25.94	<.0001
FA(4,1)	PersonID	0.7845	0.04679	16.76	<.0001
FA(5,1)	PersonID	1.0230	0.05202	19.67	<.0001
FA(6,1)	PersonID	0.8190	0.05019	16.32	<.0001

The FA(item) terms are the item residual variances. The FA(item, factor) terms are the item factor loadings.

So the total variance per item is given by: $\text{loading}^2(1) + \text{error variance}$, as shown in the R matrix above.

Item 1 = $1.2342^2 + 1.5259 = 3.0493$

The covariance between items is given by their loadings multiplied together.

Item 1 and 2 cov = $1.2342 * 0.7025 = 0.8670$

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
23072.8	18	23108.8	23108.9	23142.9	23198.9	23216.9

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > t
itemnum	1	4.5467	0.05258	5509	86.47	<.0001
itemnum	2	5.2892	0.04153	5509	127.35	<.0001
itemnum	3	4.8957	0.04802	5509	101.96	<.0001
itemnum	4	5.3590	0.04223	5509	126.90	<.0001
itemnum	5	4.8604	0.05167	5509	94.06	<.0001
itemnum	6	5.3209	0.04607	5509	115.49	<.0001

The fixed effects now show the intercepts per item conditional on factor = 0 (which then are equal to the original item means).

Tau-Equivalent Items Single-Factor Model with Marker Item Factor Model Identification (Factor Variance = ?, Factor Mean = 0, All Loadings Equal at 1)

```
TITLE "Tau-Equivalent Items Single-Factor CFA Model (Factor Variance=1, Factor Mean=0) in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
CLASS PersonID itemnum;
MODEL response = itemnum / SOLUTION NOINT NOTEST;
RANDOM INTERCEPT / TYPE=UN SUBJECT=PersonID G V VCORR;
REPEATED itemnum / TYPE=TOEPH(1) SUBJECT=PersonID R; RUN;
```

A random intercept creates a constant source of covariance across all items.

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	2.0017					
2		1.1357				
3			1.4550			
4				1.0866		
5					2.0552	
6						1.4565

The R matrix shows the item residual variances.

The G matrix shows the variance due to the factor for all items.

V is the predicted covariance matrix from putting G and R back together, and VCORR is the predicted correlation matrix.

Estimated G Matrix

Row	Effect	Person ID	Col1
1	Intercept	1	0.9127

Estimated V Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	2.9143	0.9127	0.9127	0.9127	0.9127	0.9127
2	0.9127	2.0483	0.9127	0.9127	0.9127	0.9127
3	0.9127	0.9127	2.3677	0.9127	0.9127	0.9127
4	0.9127	0.9127	0.9127	1.9993	0.9127	0.9127
5	0.9127	0.9127	0.9127	0.9127	2.9679	0.9127
6	0.9127	0.9127	0.9127	0.9127	0.9127	2.3691

Estimated V Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.3735	0.3474	0.3781	0.3103	0.3473
2	0.3735	1.0000	0.4144	0.4510	0.3702	0.4143
3	0.3474	0.4144	1.0000	0.4195	0.3443	0.3853
4	0.3781	0.4510	0.4195	1.0000	0.3747	0.4194
5	0.3103	0.3702	0.3443	0.3747	1.0000	0.3442
6	0.3473	0.4143	0.3853	0.4194	0.3442	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z	Pr > Z
UN(1,1)	PersonID	0.9127	0.04938	18.48	<.0001
Var(1)	PersonID	2.0017	0.09613	20.82	<.0001
Var(2)	PersonID	1.1357	0.05929	19.15	<.0001
Var(3)	PersonID	1.4550	0.07304	19.92	<.0001
Var(4)	PersonID	1.0866	0.05703	19.05	<.0001
Var(5)	PersonID	2.0552	0.09729	21.13	<.0001
Var(6)	PersonID	1.4565	0.07161	20.34	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
23131.1	13	23157.1	23157.1	23181.7	23222.2	23235.2

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > t
itemnum	1	4.5467	0.05140	5510	88.45	<.0001
itemnum	2	5.2892	0.04309	5510	122.74	<.0001
itemnum	3	4.8957	0.04633	5510	105.67	<.0001
itemnum	4	5.3590	0.04257	5510	125.87	<.0001
itemnum	5	4.8604	0.05187	5510	93.70	<.0001
itemnum	6	5.3209	0.04635	5510	114.81	<.0001

The fixed effects still show the intercepts per item conditional on factor = 0 (which then are equal to the original item means).

Parallel Items Single-Factor Model with Marker Item Factor Model Identification (Factor Variance = ?, Factor Mean = 0, All Loadings = 1 and All Error Variances Equal)

```
TITLE "Parallel Items Single-Factor CFA Model (Factor Variance=1, Factor Mean=0) in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
CLASS PersonID itemnum;
MODEL response = itemnum / SOLUTION NOINT NOTEST;
RANDOM INTERCEPT / TYPE=UN SUBJECT=PersonID G V VCORR;
REPEATED itemnum / TYPE=VC SUBJECT=PersonID R; RUN;
```

A random intercept creates a constant source of covariance across all items. A Type=VC R matrix means equal residual variance across items.

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.5180					
2		1.5180				
3			1.5180			
4				1.5180		
5					1.5180	
6						1.5180

Estimated G Matrix

Row	Effect	Person ID	Col1
1	Intercept	1	0.9401

Estimated V Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	2.4581	0.9401	0.9401	0.9401	0.9401	0.9401
2	0.9401	2.4581	0.9401	0.9401	0.9401	0.9401
3	0.9401	0.9401	2.4581	0.9401	0.9401	0.9401
4	0.9401	0.9401	0.9401	2.4581	0.9401	0.9401
5	0.9401	0.9401	0.9401	0.9401	2.4581	0.9401
6	0.9401	0.9401	0.9401	0.9401	0.9401	2.4581

The **R** matrix shows the item residual variances.

The **G** matrix shows the variance due to the factor for all items.

V is the predicted covariance matrix from putting **G** and **R** back together, and **VCORR** is the predicted correlation matrix.

This type of predicted covariance matrix has a special name: **compound symmetry**.

Estimated V Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.3825	0.3825	0.3825	0.3825	0.3825
2	0.3825	1.0000	0.3825	0.3825	0.3825	0.3825
3	0.3825	0.3825	1.0000	0.3825	0.3825	0.3825
4	0.3825	0.3825	0.3825	1.0000	0.3825	0.3825
5	0.3825	0.3825	0.3825	0.3825	1.0000	0.3825
6	0.3825	0.3825	0.3825	0.3825	0.3825	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z	Pr > Z
UN(1,1)	PersonID	0.9401	0.05103	18.42	<.0001
itemnum	PersonID	1.5180	0.02891	52.51	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
23254.0	8	23270.0	23270.1	23285.2	23310.1	23318.1

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > t
itemnum	1	4.5467	0.04721	5510	96.31	<.0001
itemnum	2	5.2892	0.04721	5510	112.04	<.0001
itemnum	3	4.8957	0.04721	5510	103.71	<.0001
itemnum	4	5.3590	0.04721	5510	113.52	<.0001
itemnum	5	4.8604	0.04721	5510	102.96	<.0001
itemnum	6	5.3209	0.04721	5510	112.71	<.0001

The fixed effects still show the intercepts per item conditional on factor = 0 (which then are equal to the original item means).

Unfortunately, multiple factor models in MIXED appear to be EFA models instead of CFA models, so no examples of two-factor models are given here. PROC CALIS can be used for CFA in SAS.