

Two-Level Models for Clustered* Data

- Topics:
 - **Fixed vs. random effects for modeling clustered data**
 - **ICC and design effects in clustered data**
 - Group-Mean-Centering vs. Grand-Mean Centering
 - Model extensions under Group-MC and Grand-MC

* *Clustering = Nesting = Grouping...*

MLM for Clustered Data

- So far we've built models to account for dependency created by repeated measures (time within person)
- Now we examine two-level models for more general examples of nesting/clustering/grouping:
 - Students within schools, athletes within teams
 - Siblings within families, partners within dyads
 - Employees within businesses, patients within doctors
- Residuals of people from same group are likely to be correlated due to group differences (e.g., purposeful grouping or shared experiences create dependency)
- **Recurring theme: You still have to care about group-level variation, even if that's not the point of your study**

2 Options for Differences Across Groups

Represent Group Differences as Fixed Effects

- Include (#groups-1) contrasts for group membership in the **model for the means** (via CLASS) → so group is NOT another “level”
- Permits inference about differences between specific groups, but you cannot include between-group predictors (group is saturated)
- Snijders & Bosker (1999) ch. 4, p. 44 recommend if #groups < 10ish

Represent Group Differences as a Random Effect

- Include a **random intercept variance in the model for the variance**, such that group differences become another “level”
- Permits inference about differences across groups more generally, for which you can test effects of between-group predictors
- Better if #groups > 10ish and you want to **predict** group differences

Empty Means, Random Intercept Model

MLM for Clustered Data:

- Change in notation:
 - i = level 1, j = level 2

- Level 1:

$$y_{ij} = \beta_{0j} + e_{ij}$$

- Level 2:

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

Fixed Intercept
= grand mean
(because no
predictors yet)

Random Intercept
= group-specific
deviation from
predicted intercept

Residual = person-specific deviation
from group's predicted outcome

3 Total Parameters:

Model for the Means (1):

- Fixed Intercept γ_{00}

Model for the Variance (2):

- Level-1 Variance of $e_{ij} \rightarrow \sigma_e^2$
- Level-2 Variance of $u_{0j} \rightarrow \tau_{u_0}^2$

Composite equation:

$$y_{ij} = (\gamma_{00} + u_{0j}) + e_{ij}$$

Matrices in a Random Intercept Model

Total predicted data matrix is called **V matrix**, created from the **G [TYPE=UN]** and **R [TYPE=VC]** matrices as follows:

$$\mathbf{V} = \mathbf{Z} * \mathbf{G} * \mathbf{Z}^T + \mathbf{R} = \mathbf{V}$$

$$\mathbf{V} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} \tau_{U_0}^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} \sigma_e^2 & 0 & 0 & 0 \\ 0 & \sigma_e^2 & 0 & 0 \\ 0 & 0 & \sigma_e^2 & 0 \\ 0 & 0 & 0 & \sigma_e^2 \end{bmatrix} = \begin{bmatrix} \tau_{U_0}^2 + \sigma_e^2 & \tau_{U_0}^2 & \tau_{U_0}^2 & \tau_{U_0}^2 \\ \tau_{U_0}^2 & \tau_{U_0}^2 + \sigma_e^2 & \tau_{U_0}^2 & \tau_{U_0}^2 \\ \tau_{U_0}^2 & \tau_{U_0}^2 & \tau_{U_0}^2 + \sigma_e^2 & \tau_{U_0}^2 \\ \tau_{U_0}^2 & \tau_{U_0}^2 & \tau_{U_0}^2 & \tau_{U_0}^2 + \sigma_e^2 \end{bmatrix}$$

VCORR then provides the intraclass correlation, calculated as:

$$\text{ICC} = \tau_{U_0}^2 / (\tau_{U_0}^2 + \sigma_e^2)$$

$$\begin{bmatrix} 1 & \text{ICC} & \text{ICC} & \text{ICC} \\ \text{ICC} & 1 & \text{ICC} & \text{ICC} \\ \text{ICC} & \text{ICC} & 1 & \text{ICC} \\ \text{ICC} & \text{ICC} & \text{ICC} & 1 \end{bmatrix} \text{ assumes a constant correlation over time}$$

The **G**, **Z**, and **R** matrices still get combined to create the **V** matrix, except that they are now per group. **R** and **V** have n rows by n columns, in which $n = \#$ level-1 units, which is now people, not time. Thus, no type of **R** matrix other than VC will be used, and REPEATED is not needed.

Intraclass Correlation (ICC)

$$\begin{aligned} \text{ICC} &= \frac{\text{BG}}{\text{BG} + \text{WG}} = \frac{\text{Intercept Variance}}{\text{Intercept Variance} + \text{Residual Variance}} \\ &= \frac{\tau_{U_0}^2}{\tau_{U_0}^2 + \sigma_e^2} \end{aligned}$$

$\tau_{U_0}^2 \rightarrow$ Why don't all groups have the same mean?
 $\sigma_e^2 \rightarrow$ Why don't all people from the same group have the same outcome?

- ICC = Proportion of total variance that is between groups
- ICC = Average correlation among persons from same group
- ICC is a standardized way of expressing how much we need to worry about *dependency due to group mean differences*
(i.e., ICC is an effect size for *constant* group dependency)
 - Dependency of other kinds can still be created by differences between groups in the effects of predictors (stay tuned)

Effects of Clustering on Effective N

- **Design Effect** expresses how much effective sample size needs to be adjusted due to clustering/grouping
- **Design Effect** = ratio of the variance obtained with the given sampling design to the variance obtained for a simple random sample from the same population, given the same total sample size either way

$n = \# \text{ level-1 units}$

- Design Effect = $1 + [(n - 1) * ICC]$
- Effective sample size $\rightarrow N_{\text{effective}} = \frac{\# \text{ Total Observations}}{\text{Design Effect}}$
- As ICC goes UP and cluster size goes UP, the effective sample size goes DOWN
 - See Snijders & Bosker (1999) ch. 3, p. 22-24 for more info

Design Effects in 2-Level Nesting

- Design Effect = $1 + [(n - 1) * ICC]$
- Effective sample size $\rightarrow N_{\text{effective}} = \frac{\# \text{ Total Observations}}{\text{Design Effect}}$
- $n=5$ patients from each of 100 doctors, $ICC = .30$?
 - Patients Design Effect = $1 + (4 * .30) = 2.20$
 - **$N_{\text{effective}} = 500 / 2.20 = 227$** (not 500)
- $n=20$ students from each of 50 schools, $ICC = .05$?
 - Students Design Effect = $1 + (19 * .05) = 1.95$
 - **$N_{\text{effective}} = 1000 / 1.95 = 513$** (not 1000)

Does a non-significant ICC mean you can ignore groups and just do a regression?

- Effective sample size depends on BOTH the ICC and the number of people per group: As ICC goes UP and group size goes UP, the effective sample size goes DOWN
 - So there is NO VALUE OF ICC that is “safe” to ignore, not even 0!
 - An ICC=0 in an *empty (unconditional)* model can become ICC>0 after adding level-1 predictors, because reducing the residual variance leads to an increase in the random intercept variance (→ *conditional* ICC > 0)
- So just do a multilevel analysis anyway...
 - Even if “that’s not your question”... because people come from groups, you still have to model group dependency appropriately because of:
 - Effect of clustering on level-1 fixed effect SE’s → biased SEs
 - Potential for contextual effects of level-1 predictors

Predictors in MLM for Clustered Data

Example: Achievement in Students nested in Schools

- Level-2 predictors now refer to Group-Level Variables
 - Can only have fixed or systematically varying effects (level-2 predictors cannot have random effects in a two-level model, same as before)
 - e.g., Does mean school achievement differ b/t rural and urban schools?
- Level-1 predictors now refer to Person-Level Variables
 - Can have fixed, systematically varying, or random effects over groups
 - e.g., Does student achievement differ between boys and girls?
 - Fixed effect: Is there a gender difference in achievement, period?
 - Systematically varying effect: Does the gender effect differ b/t rural and urban schools? (but the gender effect is the same within rural and within urban schools)
 - Random effect: Does the gender effect differ *randomly* across schools?
 - We can skip all the steps for building models for “time” and head straight to predictors (given that level-1 units are exchangeable here)

Two-Level Models for Clustered* Data

- Topics:
 - Fixed vs. random effects for modeling clustered data
 - ICC and design effects in clustered data
 - **Group-Mean-Centering vs. Grand-Mean Centering**
 - Model extensions under Group-MC and Grand-MC

* *Clustering = Nesting = Grouping...*

Predictors in MLM for Clustered Data

- BUT we still need to distinguish level-2 BG effects from level-1 WG effects of level-1 predictors: NO SMUSHING ALLOWED
- Options for representing level-2 BG variance as a predictor:
 - Use **obtained** group mean of level-1 x_{ij} from your sample (labeled as **GM x_j** or \bar{X}_j), centered at a constant so that 0 is a meaningful value
 - Use **actual** group mean of level-1 x_{ij} from outside data (also centered so 0 is meaningful) → better if your sample is not the full population
- Can use either **Group-MC** or **Grand-MC** for level-1 predictors (where Group-MC is like Person-MC in longitudinal models)
 - Level-1 Group-MC → center at a VARIABLE: **WG x_{ij}** = $x_{ij} - \bar{X}_j$
 - Level-1 Grand-MC → center at a CONSTANT: **L1 x_{ij}** = $x_{ij} - C$
 - Use L1 x_{ij} when including the actual group mean instead of sample group mean

3 Kinds of Effects for Level-1 Predictors

- **Is the Between-Group (BG) effect significant?**

- Are groups with higher predictor values than other groups also higher on Y than other groups, such that the group mean of the person-level predictor GMx_j accounts for level-2 random intercept variance ($\tau_{U_0}^2$)?

- **Is the Within-Group (WG) effect significant?**

- If you have higher predictor values than others in your group, do you also have higher outcomes values than others in your group, such that the within-group deviation WGx_{ij} accounts for level-1 residual variance (σ_e^2)?

- **Are the BG and WG effects different sizes: Is there a contextual effect?**

- After controlling for the absolute value of level-1 predictor for each person, is there still an incremental contribution from having a higher group mean of the predictor (i.e., does a group's general tendency predict $\tau_{U_0}^2$ above and beyond)?
- If there is no contextual effect, then the BG and WG effects of the level-1 predictor show convergence, such that their effects are of equivalent magnitude

Clustered Data Model with Group-Mean-Centered Level-1 x_{ij}

→ WG and BG Effects directly through separate parameters

x_{ij} is group-mean-centered into WGx_{ij} , with GMx_j at L2:

$$\text{Level 1: } y_{ij} = \beta_{0j} + \beta_{1j}(WGx_{ij}) + e_{ij}$$

$WGx_{ij} = x_{ij} - \bar{X}_j \rightarrow$ it has only Level-1 WG variation

$$\text{Level 2: } \beta_{0j} = \gamma_{00} + \gamma_{01}(GMx_j) + u_{0j}$$

$$\beta_{1j} = \gamma_{10}$$

$GMx_j = \bar{X}_j - C \rightarrow$ it has only Level-2 BG variation

γ_{10} = WG main effect of having more x_{ij} than others in your group

γ_{01} = BG main effect of having more \bar{X}_j than other groups

Because WGx_{ij} and GMx_j are uncorrelated, each gets the total effect for its level (WG=L1, BG=L2)

3 Kinds of Effects for Level-1 Predictors

- **What Group-Mean-Centering tells us directly:**
- **Is the Between-Group (BG) effect significant?**
 - Are groups with higher predictor values than other groups also higher on Y than other groups, such that the group mean of the person-level predictor GMx_j accounts for level-2 random intercept variance ($\tau_{U_0}^2$)?
 - This would be indicated by a significant fixed effect of GMx_j
 - Note: this is NOT controlling for the absolute value of x_{ij} for each person
- **Is the Within-Group (WG) effect significant?**
 - If you have higher predictor values than others in your group, do you also have higher outcomes values than others in your group, such that the within-group deviation WGx_{ij} accounts for level-1 residual variance (σ_e^2)?
 - This would be indicated by a significant fixed effect of WGx_{ij}
 - Note: this is represented by the relative value of x_{ij} , NOT the absolute value of x_{ij}

3 Kinds of Effects for Level-1 Predictors

- **What Group-Mean-Centering DOES NOT tell us directly:**
- **Are the BG and WG effects different sizes: Is there a contextual effect?**
 - After controlling for the absolute value of the level-1 predictor for each person, is there still an incremental contribution from the group mean of the predictor (i.e., does a group's general tendency predict $\tau_{U_0}^2$ above and beyond just the person-specific value of the predictor)?
 - In clustered data, the contextual effect is phrased as "after controlling for the individual, what is the additional contribution of the group"?
- **To answer this question about the contextual effect for the incremental contribution of the group mean, we have two options:**
 - Ask for the contextual effect via an ESTIMATE statement in SAS (or TEST in SPSS, or NEW in Mplus, or LINCOM in STATA): **WGx -1 GMx 1**
 - Use **"grand-mean-centering"** for level-1 x_{ij} instead: **$L1x_{ij} = x_{ij} - C$**
→ **centered at a CONSTANT, NOT A LEVEL-2 VARIABLE**
 - Which constant only matters for what the reference point is; it could be the grand mean or other

Group-MC vs. Grand-MC for Level-1 Predictors

Level 2		Original	Group-MC Level 1	Grand-MC Level 1
\bar{X}_j	$GMx_j = \bar{X}_j - 5$	x_{ij}	$WGx_{ij} = x_{ij} - \bar{X}_j$	$L1x_{ij} = x_{ij} - 5$
3	-2	2	-1	-3
3	-2	4	1	-1
7	2	6	-1	1
7	2	8	1	3

Same GMx_j goes into the model using either way of centering the level-1 variable x_{ij}

Using **Group-MC**, WGx_{ij} has NO level-2 BG variation, so it is not correlated with GMx_j

Using **Grand-MC**, $L1x_{ij}$ STILL has level-2 BG variation, so it is STILL CORRELATED with GMx_j

So the effects of GMx_j and $L1x_{ij}$ when included together under Grand-MC will be different than their effects would be if they were by themselves...

Clustered Data Model with x_{ij} **represented at Level 1 Only:** → WG and BG Effects are Smushed Together

x_{ij} is grand-mean-centered into $L1x_{ij}$, WITHOUT GMx_j at L2:

Level 1: $y_{ij} = \beta_{0j} + \beta_{1j}(L1x_{ij}) + e_{ij}$

$L1x_{ij} = x_{ij} - C \rightarrow$ it still has both Level-2 BG and Level-1 WG variation

Level 2: $\beta_{0j} = Y_{00} + U_{0j}$

$\beta_{1j} = Y_{10}$

Y_{10} = *smushed*
WG and BG effects

Because $L1x_{ij}$ still contains its original 2 different kinds of variation (BG and WG), its 1 fixed effect has to do the work of 2 predictors!

A *smushed* effect is also referred to as the *convergence, conflated, or composite* effect

Convergence (Smushed) Effect of a Level-1 Predictor

$$\text{Convergence Effect: } \gamma_{\text{conv}} \approx \frac{\frac{\gamma_{\text{BG}}}{\text{SE}_{\text{BG}}^2} + \frac{\gamma_{\text{WG}}}{\text{SE}_{\text{WG}}^2}}{\frac{1}{\text{SE}_{\text{BG}}^2} + \frac{1}{\text{SE}_{\text{WG}}^2}}$$

Adapted from
Raudenbush & Bryk
(2002, p. 138)

- **The convergence effect will often be closer to the within-group effect** (due to larger level-1 sample size and thus smaller SE)
- **It is the rule, not the exception, that between and within effects differ** (Snijders & Bosker, 1999, p. 52-56, and personal experience!)
- However—when grand-mean-centering a level-1 predictor, **convergence is testable** by including a **contextual effect (carried by the group mean)** for how the **BG effect** differs from the **WG effect**...

Clustered Data Model with Grand-Mean-Centered Level-1 x_{ij}

→ Model tests difference of WG vs. BG effects (It's been fixed!)

x_{ij} is grand-mean-centered into $L1x_{ij}$, WITH GMx_j at L2:

$$\text{Level 1: } y_{ij} = \beta_{0j} + \beta_{1j}(L1x_{ij}) + e_{ij}$$

$L1x_{ij} = x_{ij} - C \rightarrow$ it still has both Level-2 BG and Level-1 WG variation

$$\begin{aligned} \text{Level 2: } \beta_{0j} &= \gamma_{00} + \gamma_{01}(GMx_j) + u_{0j} \\ \beta_{1j} &= \gamma_{10} \end{aligned}$$

$GMx_j = \bar{x}_j - C \rightarrow$ it has only Level-2 BG variation

γ_{10} becomes the **WG effect** → *unique* level-1 effect after controlling for GMx_j

γ_{01} becomes the **contextual effect** that indicates how the BG effect differs from the WG effect
→ *unique* level-2 effect after controlling for $L1x_{ij}$
→ does group matter beyond individuals?

Group-MC and Grand-MC Models are Equivalent Given a Fixed Level-1 Main Effect Only

Group-MC: $WGx_{ij} = x_{ij} - GMx_j$

Level-1: $y_{ij} = \beta_{0j} + \beta_{1j}(x_{ij} - GMx_j) + e_{ij}$

Level-2: $\beta_{0j} = Y_{00} + Y_{01}(GMx_j) + U_{0j}$

$\beta_{1j} = Y_{10}$

$\rightarrow y_{ij} = Y_{00} + Y_{01}(GMx_j) + Y_{10}(x_{ij} - GMx_j) + U_{0j} + e_{ij}$

$\rightarrow y_{ij} = Y_{00} + (Y_{01} - Y_{10})(GMx_j) + Y_{10}(x_{ij}) + U_{0j} + e_{ij}$

Composite Model:

← As Group-MC

← As Grand-MC

Grand-MC: $L1x_{ij} = x_{ij}$

Level-1: $y_{ij} = \beta_{0j} + \beta_{1j}(x_{ij}) + e_{ij}$

Level-2: $\beta_{0j} = Y_{00} + Y_{01}(GMx_j) + U_{0j}$

$\beta_{1j} = Y_{10}$

$\rightarrow y_{ij} = Y_{00} + Y_{01}(GMx_j) + Y_{10}(x_{ij}) + U_{0j} + e_{ij}$

Effect	Group-MC	Grand-MC
Intercept	Y_{00}	Y_{00}
WG Effect	Y_{10}	Y_{10}
Contextual	$Y_{01} - Y_{10}$	Y_{01}
BG Effect	Y_{01}	$Y_{01} + Y_{10}$

Contextual Effects in Clustered Data

- Group-MC is equivalent to Grand-MC if the group mean of the level-1 predictor is included and the level-1 effect is not random
- Grand-MC may be more convenient in clustered data due to its ability to directly provide contextual effects
- Example: Effect of SES for students (nested in schools) on achievement:
- **Group-MC** of level-1 student SES_{ij} , school mean \overline{SES}_j included at level 2
 - Level-1 **WG** effect: Effect of being rich kid relative to your school
(is already purely WG because of centering around \overline{SES}_j)
 - Level-2 **BG** effect: Effect of going to a rich school NOT controlling for kid SES_{ij}
- **Grand-MC** of level-1 student SES_{ij} , school mean \overline{SES}_j included at level 2
 - Level-1 **WG** effect: Effect of being rich kid relative to your school
(is purely WG after *statistically* controlling for \overline{SES}_j)
 - Level-2 **Contextual** effect: Incremental effect of going to a rich school
(after *statistically* controlling for student SES)

3 Kinds of Effects for Level-1 Predictors

- **Is the Between-Group (BG) effect significant?**
 - Are groups with higher predictor values than other groups also higher on Y than other groups, such that the group mean of the person-level predictor GMx_j accounts for level-2 random intercept variance ($\tau_{U_0}^2$)?
 - Given directly by level-2 effect of GMx_j if using Group-MC for the level-1 predictor (or can be requested via ESTIMATE if using Grand-MC for the level-1 predictor)
- **Is the Within-Group (WG) effect significant?**
 - If you have higher predictor values than others in your group, do you also have higher outcomes values than others in your group, such that the within-group deviation WGx_{ij} accounts for level-1 residual variance (σ_e^2)?
 - Given directly by the level-1 effect of WGx_{ij} if using Group-MC —OR— given directly by the level-1 effect of $L1x_{ij}$ if using Grand-MC and including GMx_j at level 2 (without GMx_j , the level-1 effect of $L1x_{ij}$ if using Grand-MC is the smushed effect)
- **Are the BG and WG effects different sizes: Is there a contextual effect?**
 - After controlling for the absolute value of the level-1 predictor for each person, is there still an incremental contribution from the group mean of the predictor (i.e., does a group's general tendency predict $\tau_{U_0}^2$ above and beyond the person-specific predictor value)?
 - Given directly by level-2 effect of GMx_j if using Grand-MC for the level-1 predictor (or can be requested via ESTIMATE if using Group-MC for the level-1 predictor)

Variance Accounted For By Level-2 Predictors

- **Fixed effects of level 2 predictors *by themselves*:**
 - Level-2 (BG) main effects reduce level-2 (BG) random intercept variance
 - Level-2 (BG) interactions also reduce level-2 (BG) random intercept variance
- **Fixed effects of *cross-level interactions* (level 1* level 2):**
 - If the interacting level-1 predictor is random, any cross-level interaction with it will reduce its corresponding level-2 BG random slope variance (that line's U)
 - If the interacting level-1 predictor not random, any cross-level interaction with it will reduce the level-1 WG residual variance instead
 - This is because the level-2 BG random slope variance would have been created by decomposing the level-1 residual variance in the first place
 - The level-1 effect would then be called "**systematically varying**" to reflect a compromise between "fixed" (all the same) and "random" (all different)—it's not that each group needs their own slope, but that the slope varies systematically across groups as a function of a known group predictor (and not otherwise)

Variance Accounted For By Level-1 Predictors

- **Fixed effects of level 1 predictors *by themselves*:**
 - Level-1 (WG) main effects reduce Level-1 (WG) residual variance
 - Level-1 (WG) interactions also reduce Level-1 (WG) residual variance
- **What happens at level 2 depends on what kind of variance the level-1 predictor has:**
 - If the level-1 predictor ALSO has level-2 variance (e.g., Grand-MC predictors), then its level-2 variance will also likely reduce level-2 random intercept variance
 - If the level-1 predictor DOES NOT have level-2 variance (e.g., Group-MC predictors), then its reduction in the level-1 residual variance will cause an INCREASE in level-2 random intercept variance
 - Same thing happens with Grand-MC level-1 predictors, but you don't generally see it
 - It's just an artifact that the estimate of true random intercept variance is:
$$\text{True } \tau_{U_0}^2 = \text{observed } \tau_{U_0}^2 - \frac{\sigma_e^2}{n} \quad \rightarrow \text{ so if only } \sigma_e^2 \text{ decreases, } \tau_{U_0}^2 \text{ increases}$$

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 - **Model extensions under Group-MC and Grand-MC**

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The Joy of Interactions Involving Level-1 Predictors

- Must consider interactions with both its BG and WG parts:
- Example: Does the effect of employee motivation (x_{ij}) on employee performance interact with type of business (for profit or non-profit; $Type_j$)?
- Group-Mean-Centering:
 - $WGx_{ij} * Type_j \rightarrow$ Does the WG motivation effect differ between business types?
 - $GMx_j * Type_j \rightarrow$ Does the BG motivation effect differ between business types?
 - Moderation of total group motivation effect (not controlling for individual motivation)
 - If forgotten, then $Type_j$ moderates the motivation effect only at level 1 (WG, not BG)
- Grand-Mean-Centering:
 - $L1x_{ij} * Type_j \rightarrow$ Does the WG motivation effect differ between business types?
 - $GMx_j * Type_j \rightarrow$ Does the *contextual* motivation effect differ b/t business types?
 - Moderation of incremental group motivation effect controlling for employee motivation (moderation of the “boost” in group performance from working with motivated people)
 - If forgotten, then although the level-1 main effect of motivation has been un-smushed via the main effect of GMx_j , the interaction of $L1x_{ij} * Type_j$ would still be smushed

Interactions with Level-1 Predictors:

Example: Employee Motivation (x_{ij}) by Business Type (Type_j)

Group-MC: $WGx_{ij} = x_{ij} - GMx_j$

Level-1: $y_{ij} = \beta_{0j} + \beta_{1j}(x_{ij} - GMx_j) + e_{ij}$

Level-2: $\beta_{0j} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{02}(\text{Type}_j) + \gamma_{03}(\text{Type}_j)(GMx_j) + U_{0j}$

$\beta_{1j} = \gamma_{10} + \gamma_{11}(\text{Sex}_j)$

Composite: $y_{ij} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{10}(x_{ij} - GMx_j) + U_{0j} + e_{ij}$
 $+ \gamma_{02}(\text{Type}_j) + \gamma_{03}(\text{Type}_j)(GMx_j) + \gamma_{11}(\text{Type}_j)(x_{ij} - GMx_j)$

Grand-MC: $L1x_{ij} = x_{ij}$

Level-1: $y_{ij} = \beta_{0j} + \beta_{1j}(x_{ij}) + e_{ij}$

Level-2: $\beta_{0j} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{02}(\text{Type}_j) + \gamma_{03}(\text{Type}_j)(GMx_j) + U_{0j}$

$\beta_{1j} = \gamma_{10} + \gamma_{11}(\text{Type}_j)$

Composite: $y_{ij} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{10}(x_{ij}) + U_{0j} + e_{ij}$
 $+ \gamma_{02}(\text{Type}_j) + \gamma_{03}(\text{Type}_j)(GMx_j) + \gamma_{11}(\text{Type}_j)(x_{ij})$

Interactions Involving Level-1 Predictors Belong at Both Levels of the Model

On the left below → Group-MC: $WGx_{ij} = x_{ij} - GMx_j$

$$y_{ij} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{10}(x_{ij} - GMx_j) + U_{0j} + e_{ij} \\ + \gamma_{02}(Type_j) + \gamma_{03}(Type_j)(GMx_j) + \gamma_{11}(Type_j)(x_{ij} - GMx_j)$$

← As Group-MC

$$y_{ij} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(GMx_j) + \gamma_{10}(x_{ij}) + U_{0j} + e_{ij} \\ + \gamma_{02}(Type_j) + (\gamma_{03} - \gamma_{11})(Type_j)(GMx_j) + \gamma_{11}(Type_j)(x_{ij})$$

← As Grand-MC

On the right below → Grand-MC: $L1x_{ij} = x_{ij}$

$$y_{ij} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{10}(x_{ij}) + U_{0j} + e_{ij} \\ + \gamma_{02}(Type_j) + \gamma_{03}(Type_j)(GMx_j) + \gamma_{11}(Type_j)(x_{ij})$$

After adding an interaction for $Type_j$ with x_{ij} at both levels, then the Group-MC and Grand-MC models are equivalent

Intercept: $\gamma_{00} = \gamma_{00}$

BG Effect: $\gamma_{01} = \gamma_{01} + \gamma_{10}$

Contextual: $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WG Effect: $\gamma_{10} = \gamma_{10}$

BG*Type Effect: $\gamma_{03} = \gamma_{03} + \gamma_{11}$

Contextual*Type: $\gamma_{03} = \gamma_{03} - \gamma_{11}$

Type Effect: $\gamma_{20} = \gamma_{20}$

BG*WG or Contextual*WG is the same: $\gamma_{11} = \gamma_{11}$

Intra-variable Interactions

- Still must consider interactions with both its BG and WG parts!
- Example: Does the effect of employee motivation (x_{ij}) on employee performance interact with business group mean motivation (GMx_j)?
- Group-Mean-Centering:
 - $WGx_{ij} * GMx_j \rightarrow$ Does the WG motivation effect differ by group motivation?
 - $GMx_j * GMx_j \rightarrow$ Does the BG motivation effect differ by group motivation?
 - Moderation of total group motivation effect (not controlling for individual motivation)
 - If forgotten, then GMx_j moderates the motivation effect only at level 1 (WG, not BG)
- Grand-Mean-Centering:
 - $L1x_{ij} * GMx_j \rightarrow$ Does the WG motivation effect differ by group motivation?
 - $GMx_j * GMx_j \rightarrow$ Does the *contextual* motivation effect differ by group motiv.?
 - Moderation of incremental group motivation effect controlling for employee motivation (moderation of the boost in group performance from working with motivated people)
 - If forgotten, then although the level-1 main effect of motivation has been un-smushed via the main effect of GMx_j , the interaction of $L1x_{ij} * GMx_j$ would still be smushed

Intra-variable Interactions:

Example: Employee Motivation (x_{ij}) by Business Mean Motivation (GMx_j)

Group-MC: $WGx_{ij} = x_{ij} - GMx_j$

Level-1: $y_{ij} = \beta_{0j} + \beta_{1j}(x_{ij} - GMx_j) + e_{ij}$

Level-2: $\beta_{0j} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{02}(GMx_j)(GMx_j) + U_{0j}$

$\beta_{1j} = \gamma_{10} + \gamma_{11}(GMx_j)$

Composite: $y_{ij} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{10}(x_{ij} - GMx_j) + U_{0j} + e_{ij}$
 $+ \gamma_{02}(GMx_j)(GMx_j) + \gamma_{11}(GMx_j)(x_{ij} - GMx_j)$

Grand-MC: $L1x_{ij} = x_{ij}$

Level-1: $y_{ij} = \beta_{0j} + \beta_{1j}(x_{ij}) + e_{ij}$

Level-2: $\beta_{0j} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{02}(GMx_j)(GMx_j) + U_{0j}$

$\beta_{1j} = \gamma_{10} + \gamma_{11}(GMx_j)$

Composite: $y_{ij} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{10}(x_{ij}) + U_{0j} + e_{ij}$
 $+ \gamma_{02}(GMx_j)(GMx_j) + \gamma_{11}(GMx_j)(x_{ij})$

Intra-variable Interactions:

Example: Employee Motivation (x_{ij}) by Business Mean Motivation (GMx_j)

On the left below → Group-MC: $WGx_{ij} = x_{ij} - GMx_j$

$$y_{ij} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{10}(x_{ij} - GMx_j) + U_{0j} + e_{ij} \\ + \gamma_{02}(GMx_j)(GMx_j) + \gamma_{11}(GMx_j)(x_{ij} - GMx_j)$$

$$y_{ij} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(GMx_j) + \gamma_{10}(x_{ij}) + U_{0j} + e_{ij} \\ + (\gamma_{02} - \gamma_{11})(GMx_j)(GMx_j) + \gamma_{11}(GMx_j)(x_{ij})$$

← As Group-MC

← As Grand-MC

On the right below → Grand-MC: $L1x_{ij} = x_{ij}$

$$y_{ij} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{10}(x_{ij}) + U_{0j} + e_{ij} \\ + \gamma_{02}(GMx_j)(GMx_j) + \gamma_{11}(GMx_j)(x_{ij})$$

After adding an interaction for $Type_j$ with x_{ij} at both levels, then the Group-MC and Grand-MC models are equivalent

Intercept: $\gamma_{00} = \gamma_{00}$

BG Effect: $\gamma_{01} = \gamma_{01} + \gamma_{10}$

Contextual: $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WG Effect: $\gamma_{10} = \gamma_{10}$

BG² Effect: $\gamma_{02} = \gamma_{02} + \gamma_{11}$

Contextual²: $\gamma_{02} = \gamma_{02} - \gamma_{11}$

BG*WG or Contextual*WG is the same: $\gamma_{11} = \gamma_{11}$

When Group-MC \neq Grand-MC: Random Effects of Level-1 Predictors

Group-MC: $WGx_{ij} = x_{ij} - GMx_j$

Level-1: $y_{ij} = \beta_{0j} + \beta_{1j}(x_{ij} - GMx_j) + e_{ij}$

Level-2: $\beta_{0j} = \gamma_{00} + \gamma_{01}(GMx_j) + U_{0j}$

$$\beta_{1j} = \gamma_{10} + U_{1j}$$

Variance due to GMx_j is removed from the random slope in Group-MC.

$$\rightarrow y_{ij} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{10}(x_{ij} - GMx_j) + U_{0j} + U_{1j}(x_{ij} - GMx_j) + e_{ij}$$

Grand-MC: $L1x_{ij} = x_{ij}$

Level-1: $y_{ij} = \beta_{0j} + \beta_{1j}(x_{ij}) + e_{ij}$

Level-2: $\beta_{0j} = \gamma_{00} + \gamma_{01}(GMx_j) + U_{0j}$

$$\beta_{1j} = \gamma_{10} + U_{1j}$$

Variance due to GMx_j is still part of the random slope in Grand-MC. So these models cannot be made equivalent.

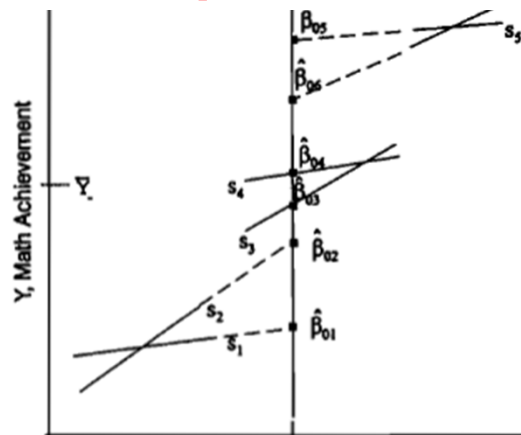
$$\rightarrow y_{ij} = \gamma_{00} + \gamma_{01}(GMx_j) + \gamma_{10}(x_{ij}) + U_{0j} + U_{1j}(x_{ij}) + e_{ij}$$

Random Effects of Level-1 Predictors

- **Random intercepts** mean different things under each model:
 - **Group-MC** → Group differences at $\mathbf{WGx}_{ij} = \mathbf{0}$ (that every group has)
 - **Grand-MC** → Group differences at $\mathbf{L1x}_{ij} = \mathbf{0}$ (that not every group will have)
- **Differential shrinkage of the random intercepts** results from differential reliability of the intercept data across models:
 - Group-MC → Won't affect shrinkage of slopes unless highly correlated
 - Grand-MC → Will affect shrinkage of slopes due to forced extrapolation
- As a result, the **random slope variance may be smaller** under Grand-MC than under Group-MC
 - Problem worsens with greater ICC of level-1 predictor (more extrapolation)
 - Anecdotal example was presented in Raudenbush & Bryk (2002; chapter 5)

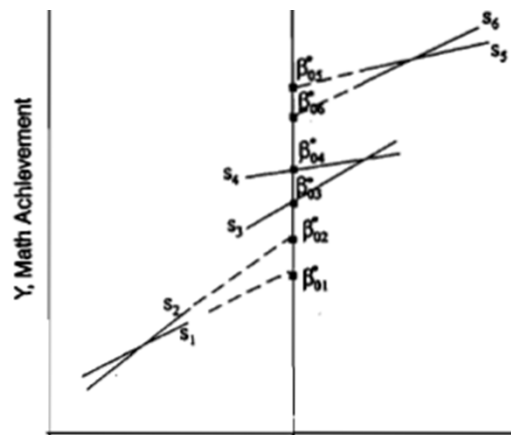
Bias in Random Slope Variance

OLS Per-Group Estimates



Level-1 X

EB Shrunk Estimates



Level-1 X

Top right: Intercepts and slopes are homogenized in Grand-MC because of intercept extrapolation

Bottom: Downwardly-biased random slope variance in Grand-MC relative to Group-MC

Unconditional Results	Conditional Results
Group-MC	
$\hat{\mathbf{T}} = \begin{bmatrix} 8.68 & 0.05 \\ 0.05 & 0.68 \end{bmatrix}$ $\hat{\sigma}^2 = 36.70$	$\hat{\mathbf{T}} = \begin{bmatrix} 2.38 & 0.19 \\ 0.19 & \boxed{0.15} \end{bmatrix}$ $\hat{\sigma}^2 = 36.70$
Grand-MC	
$\hat{\mathbf{T}} = \begin{bmatrix} 4.83 & -0.15 \\ -0.15 & 0.42 \end{bmatrix}$ $\hat{\sigma}^2 = 36.83$	$\hat{\mathbf{T}} = \begin{bmatrix} 2.41 & 0.19 \\ 0.19 & \boxed{0.06} \end{bmatrix}$ $\hat{\sigma}^2 = 36.74$

MLM for Clustered Data: Summary

- Models now come in only two kinds: “empty” and “conditional”
 - The lack of a comparable dimension to “time” simplifies things greatly!
- L2 = Between-Group, L1 = Within-Group (between-person)
 - Level-2 predictors are group variables: can have fixed or systematically varying effects (but not random effects in two-level models)
 - Level-1 predictors are person variables: can have fixed, random, or systematically varying effects
- No smushing main effects or interactions of level-1 predictors:
 - Group-MC at Level 1: Get L1=WG and L2=BG effects directly
 - Grand-MC at Level 1: Get L1=WG and L2=contextual effects directly
 - As long as some representation of the L1 effect is included in L2; otherwise, the L1 effect (and any interactions thereof) will be smushed