

Example 6: Clustered Longitudinal Data for Time within Person within Twin Pair

The data for this example come from the Octogenarian Twin Study of Aging, a longitudinal study (with 5 occasions spanning 8 years) of same-sex twin pairs initially age 79-100. We will be examining change over time in a measure of crystallized intelligence (information test), as well as prediction of that change from a measure of physical functioning (grip strength measured in pounds). These data are already stacked such that one row contains the data for one occasion for one person. The ID variables PairID and TwinID index which twin pair and which person, respectively, and Case is a unique identifier for each person. Time is unbalanced across persons, so the REPEATED statement will not be used (because we have to assume a VC R matrix anyway).

Model 1a: Empty Means, 2-Level Model for Information Test Outcome

$$\begin{aligned} \text{Level 1: } \text{Info}_{ti} &= \beta_{0i} + e_{ti} \\ \text{Level 2: } \beta_{0i} &= \gamma_{00} + U_{0i} \end{aligned}$$

This model has 2 variance components: residual at level 1 and random intercept at level 2. It assumes that all people are independent (does not account for twin pair membership).

```
TITLE "SAS Model 1a: Empty Means, 2-Level Model for Information Test Outcome";
PROC MIXED DATA=work.octodata NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
  CLASS PairID TwinID;
  MODEL info = / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT / TYPE=UN SUBJECT=PairID*TwinID; RUN;
```

```
TITLE "SPSS Model 1a: Empty Means, 2-Level Model for Information Test Outcome".
MIXED info BY PairID TwinID
  /METHOD = REML
  /PRINT = SOLUTION TESTCOV
  /FIXED =
  /RANDOM = INTERCEPT | SUBJECT(PairID*TwinID) COVTYPE(UN).
```

```
* STATA Model 1a: Empty Means, 2-Level Model for Information Test Outcome
xtmixed info , || Case: , variance reml covariance(unstructured)
  estat ic, n(594)
  estimates store TwoLevel
```

STATA output:

```
Mixed-effects REML regression
Group variable: Case
Number of obs      =      1734
Number of groups   =       594
Obs per group: min =         1
                  avg =        2.9
                  max =         5
Wald chi2(0)       =         .
Prob > chi2        =         .

Log restricted-likelihood = -6073.7202

-----+-----
      info |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
      _cons |   25.46294   .4909978    51.86   0.000     24.5006     26.42527

-----+-----
Random-effects Parameters |   Estimate  Std. Err.   [95% Conf. Interval]
-----+-----
Case: Identity
      var(_cons) |   130.5222   8.38369    115.0827    148.0331
      var(Residual) |   26.66816   1.120232    24.5605    28.95669

LR test vs. linear regression: chibar2(01) = 1411.30 Prob >= chibar2 = 0.0000
      estat ic, n(594)

-----+-----
      Model |   Obs     ll(null)   ll(model)    df         AIC         BIC
-----+-----
      . |   594         .   -6073.72     3     12153.44     12166.6

-----+-----
```

Note: N=594 used in calculating BIC

Calculate the ICC for the proportion of between-person variation in Info:

$$\text{ICC} = \frac{130.52}{130.52 + 26.67} = .83$$

This LR test tells us that the random intercept variance is significantly greater than 0,

This model now has 3 variance components: residual at level-1, random intercept at level 2, and random intercept at level 3. It now allows a correlation between people from the same twin pair.

```
lrtest ThreeLevel TwoLevel
```

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```
. estimates store ThreeLevel
. lrtest ThreeLevel TwoLevel
```

Is the 3-level model a better fit than the 2-level model?
 Yes, $-2\Delta LL(1) = 101.5$, $p < .001$

```
Likelihood-ratio test                LR chi2(1)  =    101.50
(Assumption: TwoLevel nested in ThreeLevel)  Prob > chi2 =    0.0000
```

Note: The reported degrees of freedom assumes the null hypothesis is not on the boundary of the parameter space. If this is not true, then the reported test is conservative. Note: LR tests based on REML are valid only when the fixed-effects specification is identical for both models.

Proportion variance at each level:

```
Level 1 (time) = 26.75 / 157.83 = .17
Level 2 (person) = 47.34 / 157.83 = .30
Level 3 (pair) = 83.73 / 157.83 = .53
```

ICC for time within person & pair =

```
83.73 + 47.34 / (83.73 + 47.34 + 26.75) = .83
```

ICC for person within pair = 83.72 / (83.72 + 47.33) = .64

This ICC = .64 is significantly greater than 0 via $-2\Delta LL$ for 3- vs. 2-level.

Now let's do the same thing for our two time-varying predictors: age and grip strength.

Age Model: Empty Means, 3-Level Model for Age Predictor

```
TITLE "SAS Age Model: Empty Means, 3-Level Model for Age Predictor";
PROC MIXED DATA=work.octodata NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
  CLASS PairID TwinID;
  MODEL age = / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT / TYPE=UN SUBJECT=PairID;          * Level 3;
  RANDOM INTERCEPT / TYPE=UN SUBJECT=PairID*TwinID;    * Level 2; RUN;
```

```
TITLE "SPSS Age Model: Empty Means, 3-Level Model for Age Predictor".
```

```
MIXED age BY PairID TwinID
  /METHOD = REML
  /PRINT = SOLUTION TESTCOV
  /FIXED =
  /RANDOM = INTERCEPT | SUBJECT(PairID) COVTYPE(UN)
  /RANDOM = INTERCEPT | SUBJECT(PairID*TwinID) COVTYPE(UN).
```

```
* STATA Age Model: Empty Means, 3-Level Model for Age Predictor
```

```
xtmixed age , || PairID: , covariance(unstructured) ///
  || Case: , variance reml covariance(unstructured)
```

STATA output:

```
-----+-----
      age |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
      _cons |   85.96476   .1585134   542.32   0.000    85.65408    86.27544
-----+-----

Random-effects Parameters   |   Estimate   Std. Err.     [95% Conf. Interval]
-----+-----
PairID: Identity  var(_cons) |   6.553374   .6752503     5.354986    8.019948  level-3 between-pair = 47%
-----+-----
Case: Identity    var(_cons) |   2.84e-23   5.82e-23     5.09e-25    1.58e-21  level-2 within-pair = 0%
-----+-----
                        var(Residual) |   7.466046   .2842018     6.929293    8.044377  level-1 within-person = 53%
-----+-----

LR test vs. linear regression:      chi2(2) =    459.38   Prob > chi2 = 0.0000
```

Because there is no age variance at level 2, age will be a predictor at levels 1 and 3 only.

Grip Strength Model: Empty Means, 3-Level Model for Grip Strength Predictor

```

TITLE "SAS Grip Model: Empty Means, 3-Level Model for Grip Strength Predictor";
PROC MIXED DATA=work.octodata NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
  CLASS PairID TwinID;
  MODEL gripp = / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT / TYPE=UN SUBJECT=PairID;          * Level 3;
  RANDOM INTERCEPT / TYPE=UN SUBJECT=PairID*TwinID;    * Level 2; RUN;

TITLE "SPSS Grip Model: Empty Means, 3-Level Model for Grip Strength Predictor".
MIXED gripp BY PairID TwinID
  /METHOD = REML
  /PRINT = SOLUTION TESTCOV
  /FIXED =
  /RANDOM = INTERCEPT | SUBJECT(PairID) COVTYPE(UN)
  /RANDOM = INTERCEPT | SUBJECT(PairID*TwinID) COVTYPE(UN).

* STATA Grip Model: Empty Means, 3-Level Model for Grip Strength Predictor
xtmixed gripp , || PairID: , covariance(unstructured) ///
  || Case: , variance reml covariance(unstructured)

```

STATA output:

```

-----
      gripp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
      _cons |   8.06599   .1268694    63.58   0.000     7.817331    8.31465
-----+-----

Random-effects Parameters   |   Estimate   Std. Err.     [95% Conf. Interval]
-----+-----
PairID: Identity  var(_cons) |   3.085847   .4673646     2.293276    4.152336 level-3 between-pair = 36%
-----+-----
Case: Identity    var(_cons) |   2.552534   .3436612     1.960513    3.323329 level-2 within-pair = 29%
-----+-----
                        var(Residual) |   3.049563   .1271551     2.810255    3.309249 level-1 within-person = 35%
-----+-----
LR test vs. linear regression:      chi2(2) =    795.50   Prob > chi2 = 0.0000

```

Because there is grip strength variance at all levels, grip strength will be a predictor at all levels.

We now need to create our predictor variables, including a mean of grip strength at the pair and person levels. We then code time as “time-in-study” and use baseline age as between-pair age. This gives us a convenient demarcation of age at baseline as the cross-sectional effect of age, and time-in-study as the longitudinal effect of age.

SAS Data Manipulation:

```

* Importing data into work library and creating person mean gripp for level-2;
DATA work.octodata; SET octo.octodata;
  PMgripp = MEAN(OF gripp1-gripp5);
  LABEL PMgripp= "PMgripp: Person Mean Gripp"; RUN;

* Getting twin pair means for grip strength to use at level-3;
PROC SORT DATA=work.octodata; BY PairID TwinID Wave; run;
PROC MEANS NOPRINT DATA=work.octodata; BY PairID; VAR PMgripp;
  OUTPUT OUT=PairMeans MEAN(PMgripp) = FMgripp; RUN;

* Merging PairMeans with datafile and centering predictors;
DATA work.octodata; MERGE work.octodata work.PairMeans; BY PairID;
  LABEL FMgripp= "FMgripp: Family Mean Gripp";

*** Age Variables ***;
  * Centering age at time 1 at 85 to use at level-3;
  BFage85 = agew1 - 85; LABEL BFage85= "BFage85: Age at Time1 (0=85)";
  * Within-person centering age at level-1 (like PERSON MEAN CENTERING);
  time = age - agew1; LABEL time= "time: Time Since Entry (0= Age Wave 1)";

```

```

*** Grip Strength Variables ***;
* Centering family mean gripp at 9 to use at level-3;
  BFgripp9 = FMgripp - 9;
* Centering person mean gripp at 9 to use at level-2;
  BPgripp9 = PMgripp - 9;          * GRAND MEAN CENTERING;
  WFgripp  = PMgripp - FMgripp;    * PERSON MEAN CENTERING;
* Centering time-varying gripp to use at level-1;
  TVgripp9 = gripp - 9;          * GRAND MEAN CENTERING;
  WPgripp  = gripp - PMgripp;    * PERSON MEAN CENTERING;
LABEL BFgripp9= "BFgripp9: Between-Family Mean Grip Strength in Pounds (0=9)"
      BPgripp9= "BPgripp9: Between-Person Mean Grip Strength in Pounds (0=9)"
      WFgripp=  "WFgripp: Within-Family Deviation from Mean Grip Strength in Pounds"
      TVgripp9= "TVgripp9: Time-Varying Grip Strength in Pounds (0=9)"
      WPgripp=  "WPgripp: Within-Person Deviation from Mean Grip Strength in Pounds";

* Selecting only cases with complete data;
  IF NMISS(agew1, age, FMgripp, PMgripp, gripp, info)>0 THEN DELETE; RUN;

```

SPSS Data Manipulation:

```

SORT CASES BY PairID TwinID Wave.
* Getting person gripp means to use as level-2 predictor.
COMPUTE PMgripp = MEAN(gripp1 TO gripp5).
EXECUTE.
* Getting pair gripp means to use as level-3 predictor.
AGGREGATE /OUTFILE=* MODE=ADDVARIABLES /PRESORTED /BREAK = PairID /FMgripp = MEAN(PMgripp).
VARIABLE LABELS FMgripp "FMgripp: Family Mean Gripp" PMgripp "PMgripp: Person Mean Gripp".

*** Age Variables ***.
* Centering age at time 1 at 85 to use at level-3.
  COMPUTE BFace85 = agew1 - 85.
* Within-person centering age at level-1 (like PERSON MEAN CENTERING).
  COMPUTE time = age - agew1.
  VARIABLE LABELS BFace85 "BFace85: Age at Time1 (0=85)"
                 time    "time: Time Since Entry (0= Age Wave 1)".

*** Grip Strength Variables ***.
* Centering family mean gripp at 9 to use at level-3.
  COMPUTE BFgripp9 = FMgripp - 9.
* Centering person mean gripp at 9 to use at level-2.
  COMPUTE BPgripp9 = PMgripp - 9.
  COMPUTE WFgripp  = PMgripp - FMgripp.
* Centering time-varying gripp to use at level-1.
  COMPUTE TVgripp9 = gripp - 9.
  COMPUTE WPgripp  = gripp - PMgripp.
VARIABLE LABELS
  BFgripp9 "BFgripp9: Between-Family Mean Grip Strength in Pounds (0=9)"
  BPgripp9 "BPgripp9: Between-Person Mean Grip Strength in Pounds (0=9)"
  WFgripp  "WFgripp: Within-Family Deviation from Mean Grip Strength in Pounds"
  TVgripp9 "TVgripp9: Time-Varying Grip Strength in Pounds (0=9)"
  WPgripp  "WPgripp: Within-Person Deviation from Mean Grip Strength in Pounds".

* Selecting only complete cases.
  SELECT IF (NMISS(agew1, age, FMgripp, PMgripp, gripp, info)=0).
  EXECUTE.

```

STATA Data Manipulation:

```

* Creating person mean gripp for level-2
egen PMgripp = rmean(GRIPP1-GRIPP5)
label variable PMgripp "PMgripp: Person Mean Gripp"
* Creating family mean gripp for level-3
egen FMgripp = mean(PMgripp), by(PairID)
label variable FMgripp "FMgripp: Family Mean Gripp"

```

```

* Age variables
* centering age at time 1 at 85 to use at level-3
gen BFage85 = agew1 - 85
label variable BFage85 "BFage85: Age at Time1 (0=85)"
* within person centering age at level-1 (like PERSON MEAN CENTERING)
gen time = age - agew1
label variable time "time: Time since entry (0= Age Wave 1)"

* Grip Strength Variables
* centering family mean gripp at 9 use at level-3
gen BFgripp9 = FMgripp - 9
* centering person mean gripp at 9 to use at level-2
gen BPgripp9 = PMgripp - 9          // GRAND MEAN CENTERING
gen WFgripp9 = PMgripp - FMgripp    // PERSON MEAN CENTERING
* centering time-varying gripp to use at level-1
gen TVgripp9 = gripp - 9           // GRAND MEAN CENTERING
gen WPgripp9 = gripp - PMgripp      // PERSON MEAN CENTERING

label variable BFgripp9 "BFgripp9: Between-Family Mean Grip Strength in Pounds (0=9)"
label variable BPgripp9 "BPgripp9: Between-Person mean gripp strength in pounds (0=9)"
label variable WFgripp9 "WFgripp9: Within-Family deviation from mean grip strength in Pounds"
label variable TVgripp9 "TVgripp9: Time-Varying Grip Strength in Pounds (0=9)"
label variable WPgripp9 "WPgripp9: Within-Person Deviation from Mean Grip Strength in Pounds"

* Selecting only cases with complete data
egen nummiss = rowmiss(agew1 age FMgripp PMgripp gripp info)
drop if nummiss>0

```

Model 2a: Fixed Quadratic, Random Intercepts at Levels 2 and 3

Level 1: $\text{Info}_{ij} = \beta_{0ij} + \beta_{1ij}(\text{Age}_{ij} - \text{PairAge1}_j) + \beta_{2ij}(\text{Age}_{ij} - \text{PairAge1}_j)^2 + e_{ij}$

Level 2:

Intercept: $\beta_{0ij} = \delta_{00j} + U_{0ij}$

Linear Time: $\beta_{1ij} = \delta_{10j}$

Quadratic Time: $\beta_{2ij} = \delta_{20j}$

Level 3:

Intercept: $\delta_{00j} = \gamma_{000} + \gamma_{001}(\text{PairAge1}_j - 85) + V_{00j}$

Linear Time: $\delta_{10j} = \gamma_{100}$

Quadratic Time: $\delta_{20j} = \gamma_{200}$

```

TITLE "SAS Model 2a: Fixed Quadratic, Random Intercept for Pair and Twin";
PROC MIXED DATA=work.octodata NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
  CLASS PairID TwinID;
  MODEL info = BFage85 time time*time / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT / TYPE=UN SUBJECT=PairID;          * Level 3;
  RANDOM INTERCEPT / TYPE=UN SUBJECT=PairID*TwinID;    * Level 2; RUN;

```

```

TITLE "SPSS Model 2a: Fixed Quadratic, Random Intercept for Pair and Twin".
MIXED info BY PairID TwinID WITH BFage85 time

```

```

/METHOD = REML
/PRINT = SOLUTION TESTCOV
/FIXED = BFage85 time time*time
/RANDOM = INTERCEPT | SUBJECT(PairID) COVTYPE(UN)
/RANDOM = INTERCEPT | SUBJECT(PairID*TwinID) COVTYPE(UN).

```

```

* STATA Model 2a: Fixed Quadratic, Random Intercepts at Levels 2 and 3
xtmixed info c.BFage85 c.time c.time#c.time , || PairID: , covariance(unstructured) ///
  || Case: , variance reml covariance(unstructured)
estat ic, n(337)
estimates store FixQuad

```

STATA output:

```

Wald chi2(3)      = 195.45
Log restricted-likelihood = -5939.0225      Prob > chi2      = 0.0000
-----+-----
      info |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
      BFace85 | -.8073689   .1942406    -4.16   0.000   -1.188074   -.4266643
        time | -.2350914   .1456677    -1.61   0.107   -.5205948    .050412
c.time#c.time | -.0555854   .0187153    -2.97   0.003   -.0922667   -.018904
        _cons | 25.10103   .6834791    36.73   0.000    23.76144   26.44062
-----+-----

Random-effects Parameters |   Estimate    Std. Err.     [95% Conf. Interval]
-----+-----
PairID: Identity  var(_cons) |   79.53662    9.694711     62.6346    100.9997
-----+-----
Case: Identity   var(_cons) |   52.4136     5.67978     42.38419    64.81628
-----+-----
                var(Residual) |   22.77218    .9601037     20.96606    24.73389
-----+-----

LR test vs. linear regression:      chi2(2) = 1636.90   Prob > chi2 = 0.0000
-----+-----
      Model |   Obs    ll(null)   ll(model)    df         AIC         BIC
-----+-----
          . |   337         .   -5939.023      7    11892.05    11918.79

```

This model has 3 variance components: residual at level-1, random intercept at level-2, and random intercept at level-3. It now also has 3 new fixed effects: BFace85, time, and time².

We do not compare REML deviances because these models differ in fixed effects. Instead, we use their p-values. This is our new unconditional growth model baseline, as obtained from testing sequential models not shown here.

Model 2b: Fixed Quadratic, Random Linear Slope at Level 2

Level 1: $\text{Info}_{tij} = \beta_{0ij} + \beta_{1ij}(\text{Age}_{tij} - \text{PairAge1}_j) + \beta_{2ij}(\text{Age}_{tij} - \text{PairAge1}_j)^2 + e_{tij}$

Level 2:

Intercept: $\beta_{0ij} = \delta_{00j} + U_{0ij}$

Linear Time: $\beta_{1ij} = \delta_{10j} + U_{1ij}$ 

Quadratic Time: $\beta_{2ij} = \delta_{20j}$

Level 3:

Intercept: $\delta_{00j} = \gamma_{000} + \gamma_{001}(\text{PairAge1}_j - 85) + V_{00j}$

Linear Time: $\delta_{10j} = \gamma_{100}$

Quadratic Time: $\delta_{20j} = \gamma_{200}$

```

TITLE "SAS Model 2b: Add Random Linear Slope for Twin";
PROC MIXED DATA=work.octodata NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
  CLASS PairID TwinID;
  MODEL info = BFace85 time time*time / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT / TYPE=UN SUBJECT=PairID; * Level 3;
  RANDOM INTERCEPT time / TYPE=UN SUBJECT=PairID*TwinID; * Level 2; RUN;

```

```

TITLE "SPSS Model 2b: Add Random Linear Slope for Twin".
MIXED info BY PairID TwinID WITH BFace85 time
  /METHOD = REML
  /PRINT = SOLUTION TESTCOV
  /FIXED = BFace85 time time*time
  /RANDOM = INTERCEPT | SUBJECT(PairID) COVTYPE(UN)
  /RANDOM = INTERCEPT time | SUBJECT(PairID*TwinID) COVTYPE(UN).

```

```

* STATA Model 2b: Add Random Linear Slope for Twin
xtmixed info c.BFace85 c.time c.time#c.time , || PairID: , covariance(unstructured) ///
  || Case: time , variance reml covariance(unstructured)
estat ic, n(337)
estimates store RandLin2
lrtest RandLin2 FixQuad

```


STATA output:

Log restricted-likelihood = -5872.9993 Wald chi2(3) = 188.20
 Prob > chi2 = 0.0000

info	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
BFace85	-.7307761	.1909202	-3.83	0.000	-1.104973	-.3565793
time	-.1454705	.132939	-1.09	0.274	-.4060262	.1150853
c.time#c.time	-.1021417	.0165422	-6.17	0.000	-.1345639	-.0697195
_cons	25.27722	.6626819	38.14	0.000	23.97839	26.57605

Random-effects Parameters		Estimate	Std. Err.	[95% Conf. Interval]	
PairID: Identity	var(_cons)	80.10376	9.410732	63.62858	100.8448 → level-3 intercept var
Case: Unstructured					
	var(time)	1.178443	.1805631	.8727425	1.591224 → level-2 linear var
	var(_cons)	44.31214	5.257737	35.11767	55.91389 → level-2 intercept var
	cov(time,_cons)	1.622178	.7900245	.0737584	3.170598 → level-2 int-linear cov
	var(Residual)	15.12274	.8324702	13.57607	16.84563 → level-1 residual var

LR test vs. linear regression: chi2(4) = 1768.94 Prob > chi2 = 0.0000

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	337	.	-5872.999	9	11764	11798.38

```
. estimates store RandLin2
. lrtest RandLin2 FixQuad
```

This model has 2 new variance components at level 2:
 random linear slope and intercept-slope covariance.
Do we need the random linear slope for twin?
 Yes, $-2\Delta LL(2) = 132, p < .001$

Likelihood-ratio test LR chi2(2) = 132.05
 (Assumption: FixQuad nested in RandLin2) Prob > chi2 = 0.0000

Note: The reported degrees of freedom assumes the null hypothesis is not on the boundary of the parameter space. If this is not true, then the reported test is conservative. Note: LR tests based on REML are valid only when the fixed-effects specification is identical for both models.

Model 2c: Fixed Quadratic, Random Linear Slope at Levels 2 and 3

Level 1: $\text{Info}_{tij} = \beta_{0ij} + \beta_{1ij}(\text{Age}_{tij} - \text{PairAge1}_j) + \beta_{2ij}(\text{Age}_{tij} - \text{PairAge1}_j)^2 + e_{tij}$

Level 2:

Intercept: $\beta_{0ij} = \delta_{00j} + U_{0ij}$

Linear Time: $\beta_{1ij} = \delta_{10j} + U_{1ij}$

Quadratic Time: $\beta_{2ij} = \delta_{20j}$

Level 3:

Intercept: $\delta_{00j} = \gamma_{000} + \gamma_{001}(\text{PairAge1}_j - 85) + V_{00j}$

Linear Time: $\delta_{10j} = \gamma_{100} + V_{10j}$ ←

Quadratic Time: $\delta_{20j} = \gamma_{200}$


```

TITLE "SAS Model 2c: Add Random Linear Slope for Pair";
PROC MIXED DATA=work.octodata NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
  CLASS PairID TwinID;
  MODEL info = BFace85 time time*time / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT time / TYPE=UN SUBJECT=PairID;          * Level 3;
  RANDOM INTERCEPT time / TYPE=UN SUBJECT=PairID*TwinID;    * Level 2; RUN;

TITLE "SPSS Model 2c: Add Random Linear Slope for Pair".
MIXED info BY PairID TwinID WITH BFace85 time
  /METHOD = REML
  /PRINT = SOLUTION TESTCOV
  /FIXED = BFace85 time time*time
  /RANDOM = INTERCEPT time | SUBJECT(PairID) COVTYPE(UN)
  /RANDOM = INTERCEPT time | SUBJECT(PairID*TwinID) COVTYPE(UN).

* STATA Model 2c: Add Random Linear Slope for Pair
xtmixed info c.BFace85 c.time c.time#c.time , || PairID: time, covariance(unstructured) ///
  || Case: time , variance reml covariance(unstructured)
estat ic, n(337)
estimates store RandLin23
lrtest RandLin23 RandLin2

```

STATA output:

```

Log restricted-likelihood = -5872.6076      Wald chi2(3)      =      182.94
                                           Prob > chi2      =      0.0000
-----+-----
      info |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
      BFace85 |  -.7438709   .190867   -3.90   0.000   -1.117963   -.3697784
         time |  -.1429383   .133292   -1.07   0.284   -.4041859   .1183093
c.time#c.time |  -.1016908   .0165408   -6.15   0.000   -.1341103   -.0692713
         _cons |   25.25502   .6639108   38.04   0.000    23.95378   26.55626
-----+-----

Random-effects Parameters   |   Estimate   Std. Err.   [95% Conf. Interval]
-----+-----
PairID: Unstructured        |
      var(time) |   .0640187   .1696797     .000355    11.5449 → level-3 linear var
      var(_cons) |   80.86105   9.503706    64.22388   101.8081 → level-3 intercept var
      cov(time,_cons) |  -.7329904   .9257944    -2.547514    1.081533 → level-3 int-linear cov
-----+-----
Case: Unstructured          |
      var(time) |   1.116498   .2415957     .7305816    1.706266 → level-2 linear var
      var(_cons) |   44.00753   5.22105     34.87711    55.52819 → level-2 intercept var
      cov(time,_cons) |   1.957119   .8826687     .2271198    3.687117 → level-2 int-linear cov
-----+-----
      var(Residual) |   15.11455   .8311075     13.57031    16.8345 → level-1 residual var
-----+-----
LR test vs. linear regression:      chi2(6) = 1769.73   Prob > chi2 = 0.0000
-----+-----
      Model |   Obs    ll(null)   ll(model)    df        AIC        BIC
-----+-----
      . |   337      .   -5872.608    11    11767.22    11809.24
.      estimates store RandLin23
.      lrtest RandLin23 RandLin2

```

This model has 2 new variance components at level 3: random linear slope and intercept-slope covariance. **Do we need the random linear slope for pair, too?** No, $-2\Delta LL(2) = 0.8$, $p = .67$

```

Likelihood-ratio test
(Assumption: RandLin2 nested in RandLin23)      LR chi2(2) =      0.78
                                                  Prob > chi2 =      0.6759

```

ICC of person within pair:

For Intercepts = $80.86 / (80.86 + 44.01) = .65$

For Slopes = $0.06 / (0.06 + 1.12) = .05$ (≈ 0)

Because the ICC for the slope at the pair level is not significantly different from 0, we will remove it.

TWO EQUIVALENT MODELS: PERSON-MEAN-CENTERING VS. GRAND-MEAN-CENTERING

Model 3a: Separate Effects of Grip Strength at Each Level via Person-Mean-Centering

```

TITLE "SAS Model 3a: Grip Strength at each level via PERSON MEAN CENTERING";
PROC MIXED DATA=work.octodata NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
  CLASS PairID TwinID;
  MODEL info = BFace85 time time*time WPgripp WFgripp BFgripp9
    / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT / TYPE=UN SUBJECT=PairID; * Level 3;
  RANDOM INTERCEPT time / TYPE=UN SUBJECT=PairID*TwinID; * Level 2;
  ESTIMATE "Level-2 Contextual Effect" WFgripp 1 WPgripp -1;
  ESTIMATE "Level-3 Contextual Effect" BFgripp9 1 WFgripp -1; RUN;

TITLE "SPSS Model 3a: Grip Strength at each level via PERSON MEAN CENTERING".
MIXED info BY PairID TwinID WITH BFace85 time WPgripp WFgripp BFgripp9
  /METHOD = REML
  /PRINT = SOLUTION TESTCOV
  /FIXED = BFace85 time time*time WPgripp WFgripp BFgripp9
  /RANDOM = INTERCEPT | SUBJECT(PairID) COVTYPE(UN)
  /RANDOM = INTERCEPT time | SUBJECT(PairID*TwinID) COVTYPE(UN)
  /TEST = "Level-2 Contextual Effect" WFgripp 1 WPgripp -1
  /TEST = "Level-3 Contextual Effect" BFgripp9 1 WFgripp -1.

* STATA Model 3a: Grip Strength at each level via PERSON MEAN CENTERING
xtmixed info c.BFace85 c.time c.time#c.time c.WPgripp c.WFgripp c.BFgripp9 , ///
  || PairID: , covariance(unstructured) || Case: time, variance reml covariance(unstructured)
  estat ic, n(337)
  lincom 1*c.WFgripp - 1*c.WPgripp // Level-2 Contextual Effect
  lincom 1*c.BFgripp9 - 1*c.WFgripp // Level-3 Contextual Effect

```

Model 3b: Testing 3-Level Convergence of Grip Strength Effects via Grand-Mean-Centering

```

TITLE "SAS Model 3b: Grip Strength Convergence across levels via GRAND MEAN CENTERING";
PROC MIXED DATA=work.octodata NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
  CLASS PairID TwinID;
  MODEL info = BFace85 time time*time TVgripp9 BPgripp9 BFgripp9
    / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT / TYPE=UN SUBJECT=PairID; * Level 3;
  RANDOM INTERCEPT time / TYPE=UN SUBJECT=PairID*TwinID; * Level 2;
  ESTIMATE "Level-2 Within-Family Effect" TVgripp9 1 BPgripp9 1;
  ESTIMATE "Level-3 Between-Pair Effect" TVgripp9 1 BPgripp9 1 BFgripp9 1; RUN;

TITLE "SPSS Model 3b: Grip Strength Convergence across levels via GRAND MEAN CENTERING".
MIXED info BY PairID TwinID WITH BFace85 time TVgripp9 BPgripp9 BFgripp9
  /METHOD = REML
  /PRINT = SOLUTION TESTCOV
  /FIXED = BFace85 time time*time TVgripp9 BPgripp9 BFgripp9
  /RANDOM = INTERCEPT | SUBJECT(PairID) COVTYPE(UN)
  /RANDOM = INTERCEPT time | SUBJECT(PairID*TwinID) COVTYPE(UN)
  /TEST = "Level-2 Within-Family Effect" TVgripp9 1 BPgripp9 1
  /TEST = "Level-3 Between-Pair Effect" TVgripp9 1 BPgripp9 1 BFgripp9 1.

* STATA Model 3b: Grip Strength Convergence across levels via GRAND MEAN CENTERING
xtmixed info c.BFace85 c.time c.time#c.time c.TVgripp9 c.BPgripp9 c.BFgripp9 , ///
  || PairID: , covariance(unstructured) || Case: time, variance reml covariance(unstructured)
  estat ic, n(337)
  lincom 1*c.TVgripp9 + 1*c.BPgripp9 // Level-2 Within-Family Effect
  lincom 1*c.TVgripp9 + 1*c.BPgripp9 + 1*c.BFgripp9 // Level-3 Between-Pair Effect

```

STATA output:

```

                                Wald chi2(6)      =      270.77
Log restricted-likelihood = -5838.9589      Prob > chi2      =      0.0000
-----

```

Random-effects Parameters		Estimate	Std. Err.	[95% Conf. Interval]		
PairID: Identity	var(_cons)	71.39084	8.596088	56.38327	90.39297	
Case: Unstructured						
	var(time)	.9945399	.1647409	.7188274	1.376004	
	var(_cons)	41.90059	5.043539	33.09495	53.04917	
	cov(time,_cons)	1.224168	.7247081	-.1962339	2.64457	
	var(Residual)	15.31228	.8413071	13.74903	17.05327	
LR test vs. linear regression:		chi2(4) =	1700.54	Prob > chi2 = 0.0000		
Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	337	.	-5838.959	12	11701.92	11747.76

Because the models we will examine for grip strength are equivalent, the variance components and fit statistics are the same for both.

Model 3a: Separate Effects of Grip Strength at Each Level via Person-Mean-Centering

Level 1: $\text{Info}_{ij} = \beta_{0ij} + \beta_{1ij}(\text{Age}_{ij} - \text{PairAge1}_j) + \beta_{2ij}(\text{Age}_{ij} - \text{PairAge1}_j)^2 + \beta_{3ij}(\text{Grip}_{ij} - \overline{\text{Grip}}_{ij}) + e_{ij}$

Level 2:

Intercept: $\beta_{0ij} = \delta_{00j} + \delta_{01j}(\overline{\text{Grip}}_{ij} - \overline{\text{Grip}}_j) + U_{0ij}$ **Within-person grip (WPgripp)**

Linear Time: $\beta_{1ij} = \delta_{10j} + U_{1ij}$ **Within-family grip (WFgripp)**

Quadratic Time: $\beta_{2ij} = \delta_{20j}$

Within-Person Grip: $\beta_{3ij} = \delta_{30j}$

Level 3:

Intercept: $\delta_{00j} = \gamma_{000} + \gamma_{001}(\text{PairAge1}_j - 85) + \gamma_{002}(\overline{\text{Grip}}_j - 9) + V_{00j}$ **Between-family grip (BFgripp9)**

Linear Time: $\delta_{10j} = \gamma_{100}$

Quadratic Time: $\delta_{20j} = \gamma_{200}$

Within-Person Grip: $\delta_{30j} = \gamma_{300}$

Within-Family Grip: $\delta_{01j} = \gamma_{010}$

info	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
BFAge85	-.3463256	.1921102	-1.80	0.071	-.7228547	.0302034
time	.0884511	.1386187	0.64	0.523	-.1832365	.3601387
c.time#c.time	-.1010033	.0165328	-6.11	0.000	-.133407	-.0685995
WPgripp	.5031221	.0979629	5.14	0.000	.3111184	.6951259
WFgripp	.9143609	.2251347	4.06	0.000	.4731049	1.355617
BFgripp9	1.511445	.2463971	6.13	0.000	1.028515	1.994374
_cons	27.04317	.7528849	35.92	0.000	25.56754	28.5188
. lincom 1*c.WFgripp - 1*c.WPgripp // Level-2 Contextual Effect						
info	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
(1)	.4112388	.241575	1.70	0.089	-.0622395	.8847171
. lincom 1*c.BFgripp9 - 1*c.WFgripp // Level-3 Contextual Effect						
info	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
(1)	.5970839	.3275152	1.82	0.068	-.044834	1.239002

level-1, total within-person
level-2, total within-family
level-3, total between-family

Model 3b: Testing 3-Level Convergence of Grip Strength Effects via Grand-Mean-Centering

Level 1: $\text{Info}_{tij} = \beta_{0ij} + \beta_{1ij}(\text{Age}_{tij} - \text{PairAge1}_j) + \beta_{2ij}(\text{Age}_{tij} - \text{PairAge1}_j)^2 + \beta_{3ij}(\text{Grip}_{tij} - 9) + e_{tij}$

Level 2:

Intercept: $\beta_{0ij} = \delta_{00j} + \delta_{01j}(\overline{\text{Grip}}_j - 9) + U_{0ij}$ **Within-person grip (TVgripp9)**

Linear Time: $\beta_{1ij} = \delta_{10j} + U_{1ij}$ **Contextual between-person grip (BPgripp9)**

Quadratic Time: $\beta_{2ij} = \delta_{20j}$

Within-Person Grip: $\beta_{3ij} = \delta_{30j}$

Level 3:

Intercept: $\delta_{00j} = \gamma_{000} + \gamma_{001}(\text{PairAge1}_j - 85) + \gamma_{002}(\overline{\text{Grip}}_j - 9) + V_{00j}$ **Contextual between-family grip (BFgripp9)**

Linear Time: $\delta_{10j} = \gamma_{100}$

Quadratic Time: $\delta_{20j} = \gamma_{200}$

Within-Person Grip: $\delta_{30j} = \gamma_{300}$

Within-Family Grip: $\delta_{01j} = \gamma_{010}$

	info	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
	+						
	BFace85	-.3463256	.1921102	-1.80	0.071	-.7228547	.0302034
	time	.0884511	.1386187	0.64	0.523	-.1832365	.3601387
c.time#c.time		-.1010033	.0165328	-6.11	0.000	-.133407	-.0685995
TVgripp9		.5031221	.0979629	5.14	0.000	.3111184	.6951259 level-1, total within-person
BPgripp9		.4112388	.241575	1.70	0.089	-.0622395	.884717 level-1 = level-2 effect?
BFgripp9		.5970839	.3275152	1.82	0.068	-.044834	1.239002 level-2 = level-3 effect?
_cons		27.04317	.7528849	35.92	0.000	25.56754	28.5188

. lincom 1*c.TVgripp9 + 1*c.BPgripp9				// Level-2 Within-Family Level 2 Effect			

	info	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
	+						
	(1)	.9143609	.2251347	4.06	0.000	.4731049	1.355617

. lincom 1*c.TVgripp9 + 1*c.BPgripp9 + 1*c.BFgripp9				// Level-3 Between-Pair, Level 3 Effect			

	info	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
	+						
	(1)	1.511445	.2463971	6.13	0.000	1.028515	1.994374

It appears that although there is a significant positive effect of grip strength at each level, those effects may not be significantly different in magnitude. Accordingly, let's simplify the model by removing the contextual effect at level 3, such that the level-2 and level-3 effects of grip strength are assumed to be the same.

Model 3c: Separate Effects of Grip Strength at Level 1 and Level-2&3 via Person-Mean-Centering

```
TITLE "SAS Model 3c: Grip Strength at Level 1 and Level 2&3 via PERSON MEAN CENTERING";
PROC MIXED DATA=work.octodata NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
  CLASS PairID TwinID;
  MODEL info = BFace85 time time*time WPgripp BPgripp9 / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT / TYPE=UN SUBJECT=PairID;                                * Level 3;
  RANDOM INTERCEPT time / TYPE=UN SUBJECT=PairID*TwinID;                      * Level 2;
  ESTIMATE "Level-2&3 Contextual Effect" BFgripp9 1 WPgripp -1; RUN;
```

TITLE "SPSS Model 3c: Grip Strength at Level 1 and Level 2&3 via PERSON MEAN CENTERING".

MIXED info **BY** PairID TwinID **WITH** BFace85 time WPgripp BPgripp9

/METHOD = REML

/PRINT = SOLUTION TESTCOV

/FIXED = BFace85 time time*time WPgripp BPgripp9

/RANDOM = INTERCEPT | SUBJECT(PairID) COVTYPE(UN)

/RANDOM = INTERCEPT time | SUBJECT(PairID*TwinID) COVTYPE(UN)

/TEST = "Level-2&3 Contextual Effect" BPgripp9 1 WPgripp -1.

*** STATA Model 3c: Grip Strength at Level 1 and Level 2&3 via PERSON MEAN CENTERING**

xtmixed info c.BFace85 c.time c.time#c.time c.WPgripp c.BPgripp9 , ///

|| PairID: , covariance(unstructured) || Case: time, variance reml covariance(unstructured)
estat ic, n(337)

lincom 1*c.BPgripp9 - 1*c.WPgripp // Level-2&3 Contextual Effect

Level 1: $\text{Info}_{ij} = \beta_{0ij} + \beta_{1ij}(\text{Age}_{ij} - \text{PairAge1}_j) + \beta_{2ij}(\text{Age}_{ij} - \text{PairAge1}_j)^2 + \beta_{3ij}(\text{Grip}_{ij} - \overline{\text{Grip}}_{ij}) + e_{ij}$

Level 2:

Intercept: $\beta_{0ij} = \delta_{00j} + \delta_{01j}(\overline{\text{Grip}}_{ij} - 9) + U_{0ij}$ **Within-person grip (WPgripp)**

Linear Time: $\beta_{1ij} = \delta_{10j} + U_{1ij}$

Quadratic Time: $\beta_{2ij} = \delta_{20j}$ **Between-person grip (BPgripp9)**

Within-Person Grip: $\beta_{3ij} = \delta_{30j}$

Level 3:

Intercept: $\delta_{00j} = \gamma_{000} + \gamma_{001}(\text{PairAge1}_j - 85) + V_{00j}$

Linear Time: $\delta_{10j} = \gamma_{100}$

Quadratic Time: $\delta_{20j} = \gamma_{200}$

Within-Person Grip: $\delta_{30j} = \gamma_{300}$

Within-Family Grip: $\delta_{01j} = \gamma_{010}$

STATA output:

Log restricted-likelihood = -5840.4202				Wald chi2(5)	=	267.14
				Prob > chi2	=	0.0000
<hr/>						
	info	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
<hr/>						
	BFace85	-.4275427	.1874185	-2.28	0.023	-.7948762 -.0602091
	time	.0904993	.1386117	0.65	0.514	-.1811746 .3621732
c.time#c.time		-.1011004	.0165312	-6.12	0.000	-.133501 -.0686997
	WPgripp	.5071004	.097934	5.18	0.000	.3151532 .6990476
	BPgripp9	1.184309	.1695658	6.98	0.000	.8519664 1.516652
	_cons	26.47672	.687956	38.49	0.000	25.12835 27.82508
<hr/>						
Random-effects Parameters			Estimate	Std. Err.	[95% Conf. Interval]	
<hr/>						
PairID: Identity	var(_cons)	71.96326	8.654375	56.85183	91.09136	
<hr/>						
Case: Unstructured						
	var(time)	.9952698	.1647394	.7195284	1.376682	
	var(_cons)	41.97828	5.046668	33.16595	53.13209	
	cov(time,_cons)	1.234527	.7220033	-.180573	2.649628	
<hr/>						
	var(Residual)	15.3081	.8408553	13.74567	17.04813	
<hr/>						
LR test vs. linear regression:			chi2(4) =	1705.54	Prob > chi2 = 0.0000	

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	337	.	-5840.42	11	11702.84	11744.86
. lincom 1*c.BPgripp9 - 1*c.WPgripp // Level-2&3 Contextual Effect						
info	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
(1)	.6772089	.1925793	3.52	0.000	.2997605	1.054657

One could then test interactions, keeping in mind the need to differentiate effects across all three levels as needed...

Sample Results Section (note this combines across models somewhat)

The extent of individual change in crystallized intelligence (as measured by the information test) and the relationship between intelligence, age, and grip strength was examined in a sample of 337 same-sex twins measured every two years for up to five occasions. Multilevel models were estimated using restricted maximum likelihood. The significance of fixed effects was evaluated with individual Wald tests (i.e., of estimate / SE), whereas random effects were evaluated via likelihood ratio tests (i.e., $-2\Delta LL$ with degrees of freedom equal to the number of new random effects variances and covariances).

A two-level empty means, random intercept model of time nested within person was initially specified and indicated that 83% of the information test outcome variance was between persons. The addition of a random intercept for twin pair resulted in a significant improvement in model fit, $-2\Delta LL(1) = 101.5$, $p < .001$, and revealed that 64% of that between-person variance was due to twin pair (i.e., shared variance between twins from the same pair). Thus, a three-level model was necessary, given that 17% of the variance was at level 1 (within persons over time), 30% was at level 2 (within pairs), and 53% was at level 3 (between pairs). A three-level empty means, random intercept model to decompose the variance in time-varying age revealed that 47% was between pairs (given that the twins initially varied in age from 80 to 100), whereas the remaining 53% was within persons over time—there was no level-2 age variance. Thus, the level-3 cross-sectional and level-1 longitudinal effects of age were modeled separately using baseline age (centered at 85) and time in study, respectively. Preliminary analyses revealed that a linear effect of age at baseline and a quadratic effect of time in study resulted in the best-fitting model to describe mean change. Although a random linear time slope for twin significantly improved model fit, $-2\Delta LL(2) = 132.0$, $p < .001$, the subsequent addition of a random linear time slope for twin pair did not significantly improve model fit, $-2\Delta LL(2) = 0.8$, $p = .67$, indicating that the 5% of the random linear time slope variance that was due to twin pair was not distinguishable from 0. As a result, the random linear time slope was retained at the twin level only (i.e., level 2 but not level 3).

The prediction of the information test outcome from time-varying grip strength was then examined. A three-level empty means, random intercept model to decompose the variance in grip strength revealed that 36% was between pairs, 29% was within pairs, and 35% was within persons over time. Predictors for grip strength were included via person-mean-centering, in which the within-person effect was represented by the deviation of each occasion's grip strength around each person's mean, the within-pair effect was represented by the deviation of each twin's mean grip strength around each pair's mean, and the between-pair effect was represented by the family mean grip strength (centered at 9 pounds). There was a significant main effect of grip strength at each level. Within persons, for every additional pound of grip strength more than one's own mean, information test at that occasion was expected to be higher by 0.50. Within pairs, for every additional pound of person mean grip strength more than one's family mean, information test for that twin was expected to be higher by 0.91. Between pairs, for every additional pound of family mean grip strength more than other families, information test for the twin pair was expected to be higher by 1.51.

Contextual effects for the differences in effect size across levels were requested using separate statements (i.e., as would be provided directly using grand-mean-centering but including the person and pair means). The pair-level contextual effect was not significant, indicating that the within-pair and between-pair effects were equivalent. Consequently, the model was re-specified to include within-person grip strength, as described previously, along with between-person grip strength to represent the combination of the twin and pair levels, calculated as each person's mean grip strength centered at 9. The between-person effect of grip strength was significant, such that for every additional pound of mean grip strength more than other people, information test for that twin was expected to be higher by 1.18. This effect was significantly larger than the within-person effect of grip strength of 0.51 (i.e., a significant person contextual effect), and thus both the within-person and between-person effects of grip strength were retained.