Example 8a Models using Slides Example: L1 students nested in L2 schools

1. Empty means, random intercept predicting language
   (~Model 1b in Lecture 2 slides 10–11)

2. Add L2 observed school predictors; add latent-centered student verbal at L1 and latent school mean verbal at L2
   (~Model 2a in Lecture 3 slides 16–18)

3. Add random slope of L1 within verbal across L2 schools
   (~Model 2b in Lecture 4 slides 8–9)

4. Add moderation of L1 within and L2 between verbal slopes by mixed grade *(~Model 4e in Lecture 4 slides 29–32)*

5. Add moderation of L1 within and L2 between verbal slopes by L2 verbal intercept (L2 quadratic interaction)
   (*~Model 5b in Lecture 4 slides 34, 36–37*)

6. New: add a random scale factor (to become location–scale model)

Empty Means, Random Intercept Model: (1b) Syntax by Univariate MLM Program

SAS:

```
PROC MIXED DATA=work.Example COVTEST NOCLPRINT IC METHOD=REML;
     CLASS schoolID;
     MODEL langpost = / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT / VCORR TYPE=UN SUBJECT=schoolID; * VCORR gives ICC;
RUN;
```

R lmer from lme4 package—*using lmerTest package to get Satterthwaite denominator DF, and using performance package to get ICC from lmer:*

```
name = lmer(data=Example, REML=TRUE, formula=langpost~1+(1+|schoolID))
summary(name, ddf="Satterthwaite")
icc(name); ranova(name) # ICC and LRT for random intercept
```

STATA:

```
mixed langpost , || schoolID: , ///
      reml dfmethod(satterthwaite) dftable(pvalue) nolog
estat icc // Get ICC
```

SPSS:

```
MIXED langpost BY schoolID
      /METHOD    = REML
      /CRITERIA = DFMETHOD(SATTERTHWAITE)
      /PRINT     = SOLUTION TESTCOV
      /FIXED     =
      /RANDOM = INTERCEPT | COVTYPE(UN) SUBJECT(schoolID).
```

Electronic materials for this example from my 2023 APA training sessions are **here**

Model 1b: Level-1 Students in Level-2 Schools

Example from Snijders & Bosker (2012): Predict language outcomes (M = 41.46, VAR = 77.69) for 3,566 students (p) from 191 schools (c)

Level-1: $Lang_{pc} = \beta_{0c} + e_{pc}$

Level-2: $\beta_{0c} = \gamma_{00} + U_{0c}$

$$\text{ICC} = \frac{\tau^2_{U_0}}{\tau^2_{U_0} + \sigma^2_e} = \frac{17.809}{17.809 + 62.230} = .223$$

22.3% of total language variance is due to school mean differences (WC r = .22)

Results from SAS MIXED:

Without random intercept U_{0c}:

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	41.4635	0.1476	3565	280.91	<.0001

Covariance Parameter Estimates

Cov Parm	Estimate	Standard Error	V…	Pr	Z
Residual	77.6905	1.8402	42…	…001	

With random intercept U_{0c}:

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	41.0791	0.3371	175	121.87	<.0001

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	…	…	Z
UN(1,1)	schoolID	17.8085	2.3063	7.72	0	1
Residual		62.2296	1.5179	4…	…001	

Model 2

- Add L2 observed school predictors; add latent-centered student verbal at L1 and latent school mean verbal at L2
  (~Model 2a in Lecture 3 slides 16–18)

Adding L2 Between and L1 Within Predictors: (2a) Syntax by Univariate MLM Program

SAS:

```
PROC MIXED DATA=work.Example COVTEST NOCLPRINT IC METHOD=REML;
     CLASS schoolID;
     MODEL langpost = hw2 mixgrd CMverb10 WCverb / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT / TYPE=UN SUBJECT=schoolID;
     ESTIMATE "L2 Contextual Effect of Verbal"  CMverb10 1 WCverb -1;
RUN;
```

R lmer from lme4 package—using lmerTest package to get Satterthwaite denominator DF and contest1D:

```
name = lmer(data=Example, REML=TRUE,
            formula=langpost~1+hw2+mixgrd+CMverb10+WCverb+(1+|schoolID))
summary(name, ddf="Satterthwaite")
contest1D(name, ddf="Satterthwaite", L=c(0,0,0,1,-1)) # L2 Contextual effect of verbal
```

STATA:

```
mixed langpost c.hw2 c.mixgrd c.CMverb10 c.WCverb, || schoolID:, ///
      reml dfmethod(satterthwaite) dftable(pvalue) nolog
lincom c.CMverb10*1 + c.WCverb*-1, small // L2 Contextual effect of verbal
```

SPSS:

```
MIXED langpost BY schoolID WITH hw2 mixgrd CMverb10 WCverb
      /METHOD    = REML
      /CRITERIA = DFMETHOD(SATTERTHWAITE)
      /PRINT     = SOLUTION TESTCOV
      /FIXED     = hw2 mixgrd CMverb10 WCverb
      /RANDOM    = INTERCEPT | COVTYPE(UN) SUBJECT(schoolID)
      /TEST      = "L2 Contextual effect of verbal" CMverb10 1 WCverb -1.
```

Electronic materials for this example from my 2023 APA training sessions are **here**

Model 2a: Cluster-MC Level-1 Predictor

Example from Snijders & Bosker (2012) ch. 9: Predicting language outcomes for 3,566 students (p) from 191 schools (c) → **adding student verbal ability**

Level-1: $Lang_{pc} = \beta_{0c} + \beta_{1c}(Verbal_{pc} - \overline{Verbal_c}) + e_{pc}$

Level-2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(HW_c - 2) + \gamma_{02}(MixGrd_c) + \gamma_{03}(\overline{Verbal_c} - 10) + U_{0c}$

$\beta_{1c} = \gamma_{10}$

Results from SAS MIXED:

L1 WCverb = $Verbal_{pc} - \overline{Verbal_c}$

L2 CMverb10 = $\overline{Verbal_c} - 10$

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	41.5794	0.3624	172	114.73	<.0001
hw2	-0.05255	0.4585	179	-0.11	0.9089
mixgrd	-1.1209	0.5157	197	-2.17	0.0309
CMverb10	3.6599	0.2709	207	13.51	<.0001
WCverb	2.4227	0.05718	3373	42.37	<.0001

Btw, L2 Contextual = 1.237, SE = 0.277, $p < .0001$

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	
UN(1,1)	schoolID	8.3939	1.1326	
Residual		40.5508	0.9875	

From empty model to compare:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	
UN(1,1)	schoolID	17.8085	2.3063	
Residual		62.2296	1.5179	

Model 2a: Cluster-MC Level-1 Predictor

Model for the Means (relevant new parameters only):

- γ_{00} = **41.58** = fixed **intercept**: expected language for students in a school with homework=2 (~mean), mixgrd=0 (=not mixed), and school mean verbal = 10; for a student whose verbal = 10
- γ_{03} = **3.66*** = fixed **BC slope** of **school verbal**: difference in **school mean** language per unit higher mean verbal ability *than other schools*
- γ_{10} = **2.42*** = fixed **WC slope** of **student verbal**: difference in **student** language per unit higher verbal ability *than their school mean*

Model for the Variance:

- U_{0c} = level-2 random intercept = deviation of the original from predicted school mean language for school c (with variance $\tau^2_{U_0}$ = **8.39**), where "original" is from the empty means, random intercept model
  - Pseudo-$R^2_{U0} = \frac{17.809-8.394}{17.809}$ = **.529 → 52.9%** explained (of original 22.3% L2 BC)
- e_{pc} = level-1 residual = deviation of the observed outcome for student p from their outcome predicted by β_{0c} and β_{1c} (with variance σ^2_e = **40.55**)
  - Pseudo-$R^2_e = \frac{62.230-40.551}{62.230}$ = **.348 → 34.8%** explained (of original 77.7% L1 WC)

Model 3

- Add random slope of L1 within verbal across L2 schools *(~Model 2b in Lecture 4 slides 8–9)*

Example Random L1 Cluster-MC Within Slope: (2b) Syntax by Univariate MLM Program

SAS:

```
PROC MIXED DATA=work.Example COVTEST NOCLPRINT IC METHOD=REML;
     CLASS schoolID;                                    * GCORR = random effect correlations;
     MODEL langpost = hw2 mixgrd CMverb10 WCverb / GCORR SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT WCverb / TYPE=UN SUBJECT=schoolID;
     ESTIMATE "L2 Contextual Effect of Verbal"  CMverb10 1 WCverb -1;
RUN;
```

R lmer from lme4 package—using lmerTest package to get Satterthwaite denominator DF and contest1D:

```
name = lmer(data=Example, REML=TRUE,
            formula=langpost~1+hw2+mixgrd+CMverb10+WCverb+(1+WCverb|schoolID))
summary(name, ddf="Satterthwaite") # Shows random effect correlations already
contest1D(name, ddf="Satterthwaite", L=c(0,0,0,1,-1)) # L2 Contextual effect of verbal
```

STATA:

```
mixed langpost c.hw2 c.mixgrd c.CMverb10 c.WCverb, || schoolID: WCverb, ///
      covariance(un) reml dfmethod(satterthwaite) dftable(pvalue) nolog
estat recovariance, relevel(schoolID) correlation  // Random effect correlations
lincom c.CMverb10*1 + c.WCverb*-1, small  // L2 Contextual effect of verbal
```

SPSS:

```
MIXED langpost BY schoolID WITH hw2 mixgrd CMverb10 WCverb
      /METHOD    = REML
      /CRITERIA = DFMETHOD(SATTERTHWAITE)
      /PRINT     = SOLUTION TESTCOV
      /FIXED     = hw2 mixgrd CMverb10 WCverb
      /RANDOM    = INTERCEPT WCverb | COVTYPE(UN) SUBJECT(schoolID)
      /TEST      = "L2 Contextual effect of verbal" CMverb10 1 WCverb -1.
```

Electronic materials for this example from my 2023 APA training sessions are **here**

Model 2b: Cluster-MC Random Slope

Level-1: $Lang_{pc} = \beta_{0c} + \beta_{1c}(Verbal_{pc} - \overline{Verbal_c}) + e_{pc}$

Level-2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(HW_c - 2) + \gamma_{02}(MixGrd_c) + \gamma_{03}(\overline{Verbal_c} - 10) + U_{0c}$

$\beta_{1c} = \gamma_{10} + U_{1c}$

Adding L2 random slope variance of U_{1c} (as $\tau^2_{U_1}$) and L2 random intercept–slope covariance (as $\tau_{U_{01}}$)

Results from SAS MIXED:

L1 WCverb = $Verbal_{pc} - \overline{Verbal_c}$

L2 CMverb10 = $\overline{Verbal_c} - 10$

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	41.5281	0.3576	177	116.14	<.0001
hw2	-0.09509	0.4464	178	-0.21	0.8316
mixgrd	-0.9337	0.5052	201	-1.85	0.0660
CMverb10	3.6212	0.2647	209	13.68	<.0001
WCverb	2.4486	0.06831	151	35.85	<.0001

Btw, L2 Contextual = 1.173, SE = 0.273, $p < .0001$

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	schoolID	8.4655	1.1352	7	01
UN(2,1)	schoolID	-0.6943	0.2386	9	00
UN(2,2)	schoolID	0.2239	0.08630	2	0 7
Residual		39.7586	0.9910	40.12	<.0001

Estimated G Correlation Matrix

Row	Effect	schoolID	Col1	Col2
1	Intercept	1	1.0000	-0.5043
2	WCverb	1	-0.5043	1.0000

Likelihood ratio test of random slope variance (and intercept–slope covariance): **$-2\Delta LL(\sim 2) = 19.29$, $p < .0001$**

Model 4

- Add moderation of L1 within and L2 between verbal slopes by mixed grade *(~Model 4e in Lecture 4 slides 29–32)*

Model 4e: All Cluster-MC Version

L1: $Lang_{pc} = \beta_{0c} + \beta_{1c}(Verbal_{pc} - \overline{Verbal}_c) + e_{pc}$

L2: $\beta_{0c} = \gamma_{00} + \gamma_{02}(MixGrd_c) + \gamma_{03}(\overline{Verbal}_c - 10) + \gamma_{04}(MixGrd_c)(\overline{Verbal}_c - 10) + U_{0c}$

$\beta_{1c} = \gamma_{10} + \gamma_{12}(MixGrd_c) + U_{1c}$

Interpreting Fixed Effects:

- γ_{10} = ***simple* L1 within slope**: difference in student language per unit higher verbal than school mean, *specifically for schools without mixed grades*
- γ_{03} = ***simple* L2 between slope**: difference in school language per unit higher school mean verbal than other schools (NOT controlling for student verbal), *now specifically for schools without mixed grades*
- $\gamma_{03} - \gamma_{10}$ = ***simple* L2 contextual slope**: extra difference in school language per unit higher school mean verbal than other schools (controlling for student verbal), *now specifically for schools without mixed grades*
- γ_{12} = **guaranteed-to-be-unsmushed cross-level interaction**: how the **L1 within** verbal slope differs in schools with mixed grades
- γ_{04} = **level-2 interaction**: how the **L2 between** verbal slope differs in schools with mixed grades
- $\gamma_{04} - \gamma_{12}$ = **implied level-2 interaction**: how the **L2 contextual** verbal slope differs in schools with mixed grades (or how moderation differs: BC − WC)

Cluster-MC with Unsmushed Cross-Level Int: (4e) Syntax by Univariate MLM Program

SAS:

```
PROC MIXED DATA=work.Example COVTEST NOCLPRINT IC METHOD=REML;
     CLASS schoolID;          * In SAS, * creates interactions;
     MODEL langpost = ~~hw2~~ mixgrd CMverb10 WCverb mixgrd*WCverb mixgrd*CMverb10
                    / GCORR SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT WCverb / TYPE=UN SUBJECT=schoolID;
RUN;
```

Oops! Predictor hw2 should not be included.

R lmer from lme4 package—using lmerTest package to get Satterthwaite denominator DF and contest1D:

```
name = lmer(data=Example, REML=TRUE,
            formula=langpost~1+~~hw2~~+mixgrd+CMverb10+WCverb+ mixgrd:WCverb
                          +mixgrd:CMverb10+(1+WCverb|schoolID))
summary(name, ddf="Satterthwaite") # In R, : creates interactions
```

STATA:

```
mixed langpost ~~c.hw2~~ c.mixgrd c.CMverb10 c.WCverb c.mixgrd#c.WCverb ///
      c.mixgrd#c.CMverb10, || schoolID: WCverb, /// In STATA, # creates interactions
      covariance(un) reml dfmethod(satterthwaite) dftable(pvalue) nolog
estat recovariance, relevel(schoolID) correlation  // random effect correlations
```

SPSS: `* In SPSS, * creates interactions.`

```
MIXED langpost BY schoolID WITH ~~hw2~~ mixgrd CMverb10 WCverb
      /METHOD    = REML
      /CRITERIA = DFMETHOD(SATTERTHWAITE)
      /PRINT     = SOLUTION TESTCOV
      /FIXED     = ~~hw2~~ mixgrd CMverb10 WCverb mixgrd*WCverb mixgrd*CMverb10
      /RANDOM    = INTERCEPT WCverb | COVTYPE(UN) SUBJECT(schoolID).
```

Electronic materials for this example **added** to my 2023 APA training sessions are **here**

Hybrid vs. Cluster-MC: Different L2 Slopes!

Hybrid: $\boldsymbol{\beta_{1c}(Verbal_{pc} - 10)}$
→ Direct **L2 Context** Effects

CMC: $\boldsymbol{\beta_{1c}(Verbal_{pc} - \overline{Verbal_c})}$
→ Direct **L2 Between** Effects

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	41.6217	0.3592	176	115.86	<.0001
~~hw2~~	~~-0.06631~~	~~0.4457~~	~~178~~	~~-0.15~~	~~0.8819~~
mixgrd	-1.1165	0.5161	197	-2.16	0.0317
CMverb10	0.8255	0.4053	182	2.04	0.0431
verb10	2.3613	0.07856	124	30.06	<.0001
mixgrd*verb10	0.3362	0.1567	239	2.15	0.0329
mixgrd*CMverb10	0.4136	0.5536	235	0.75	0.4557

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Simple L2 between	3.1868	0.3992	172	7.98	<.0001
L2 between*mixgrd	0.7498	0.5308	199	1.41	0.1593

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	41.6217	0.3592	176	115.86	<.0001
~~hw2~~	~~0.06631~~	~~0.4457~~	~~178~~	~~0.15~~	~~0.8819~~
mixgrd	-1.1165	0.5161	197	-2.16	0.0317
CMverb10	3.1868	0.3992	172	7.98	<.0001
WCverb	2.3613	0.07856	124	30.06	<.0001
mixgrd*WCverb	0.3362	0.1567	239	2.15	0.0329
mixgrd*CMverb10	0.7498	0.5308	199	1.41	0.1593

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Simple L2 context	0.8255	0.4053	182	2.04	0.0431
L2 context*mixgrd	0.4136	0.5536	235	0.75	0.4557

L1 within verbal slope is signif more positive (stronger) by **0.3362** in mixed-grade schools
L2 between verbal slope is n.s. more positive (stronger) by **0.7498** in mixed-grade schools
L2 contextual verbal slope is n.s. more positive (stronger) by **0.4136** in mixed-grade schools

Same Model for the Variance Either Way

L1: $Lang_{pc} = \beta_{0c} + \beta_{1c}(Verbal_{pc} - 10) + \beta_{2c}(Verbal_{pc} - \overline{Verbal}_c) + e_{pc}$

L2: $\beta_{0c} = \gamma_{00} + \gamma_{02}(MixGrd_c) + \gamma_{03}(\overline{Verbal}_c - 10) + \gamma_{04}(MixGrd_c)(\overline{Verbal}_c - 10) + U_{0c}$

$\beta_{1c} = \gamma_{10} + \gamma_{12}(MixGrd_c)$; $\beta_{2c} = U_{2c}$

Hybrid → $\beta_{1c}(Verbal_{pc} - 10)$

Cluster-MC → $\beta_{1c}(Verbal_{pc} - \overline{Verbal}_c)$

Interpreting the Model for the Variance:

- U_{0c} = **level-2 random intercept** → deviation of original from predicted mean language for school *c* (where "original" is from an empty means, random intercept model), now **specifically where student verbal = their school mean** (with variance = $\tau^2_{U_0}$)
- U_{2c} = **level-2 random slope** → deviation of original from predicted **L1 within** verbal slope for school *c* (where "original" is from a model without cross-level interactions for β_{1c}), (with variance = $\tau^2_{U_2}$ and U_{0c} covariance = $\tau_{U_{02}}$)
  - *If applied to constant-centered student verbal instead, it would reflect both school differences in the L1 within verbal slope AND intercept heteroscedasticity (bad)*
- e_{pc} = **level-1 residual** = deviation of the observed outcome for student *p* from their outcome predicted by all fixed and random effects

Model 5

- Add moderation of L1 within and L2 between verbal slopes by L2 verbal intercept (L2 quadratic interaction) (~*Model 5b in Lecture 4 slides 34, 36–37*)

Model 5b: Intra-Variable Cross-Level Interactions

- To unsmush the cross-level interaction, we add the corresponding L2 interaction with the L2 moderator, just as we did before…

L1: $$Lang_{pc} = \beta_{0c} + \beta_{1c}(Verbal_{pc} - 10) + \beta_{2c}(Verbal_{pc} - \overline{Verbal}_c) + e_{pc}$$

L2: $$\beta_{0c} = \gamma_{00} + \gamma_{02}(MixGrd_c) + \gamma_{03}(\overline{Verbal}_c - 10) + \gamma_{04}(MixGrd_c)(\overline{Verbal}_c - 10) + \gamma_{05}(\overline{Verbal}_c - 10)(\overline{Verbal}_c - 10) + U_{0c}$$

$$\beta_{1c} = \gamma_{10} + \gamma_{12}(MixGrd_c) + \gamma_{13}(\overline{Verbal}_c - 10); \beta_{2c} = U_{2c}$$

- …the solution is a **quadratic slope** for L2 school mean verbal!
  - γ_{13} = how the **L1 within** verbal slope differs by school mean verbal
  - γ_{05} = how the **L2 contextual** verbal slope differs by school mean verbal
  - $\gamma_{13} + \gamma_{05}$ = how the **L2 between** verbal slope differs by school mean verbal

Cluster-MC with Intra-Variable Interaction: (5b) Syntax by Univariate MLM Program

SAS:

```
PROC MIXED DATA=work.Example COVTEST NOCLPRINT IC METHOD=REML;
     CLASS schoolID;          * In SAS, * creates interactions;
     MODEL langpost = hw2 mixgrd CMverb10 WCverb mixgrd*WCverb mixgrd*CMverb10
                      CMverb10*WCverb CMverb10*CMverb10 / GCORR SOLUTION DDFM=Satterth;
     RANDOM INTERCEPT WCverb / TYPE=UN SUBJECT=schoolID;
RUN;
```

Oops! Predictor hw2 should not be included.

R lmer from lme4 package—using lmerTest package to get Satterthwaite denominator DF:

```
name = lmer(data=Example, REML=TRUE,
            formula=langpost~1+hw2+mixgrd+CMverb10+WCverb+I(CMverb10^2)
            +mixgrd:WCverb+mixgrd:CMverb10+CMverb10:WCverb+(1+WCverb|schoolID))
summary(name, ddf="Satterthwaite") # In R, : creates interactions, I(^2) creates quad
```

STATA:

```
                                                /// In STATA, # creates interactions
mixed langpost c.hw2 c.mixgrd c.CMverb10 c.WCverb c.mixgrd#c.WCverb  ///
      c.mixgrd#c.CMverb10 c.CMverb10#c.WCverb c.CMverb10#c.CMverb10, ///
      || schoolID: WCverb, covariance(un) reml dfmethod(satterthwaite) dftable(pvalue)
estat recovariance, relevel(schoolID) correlation  // Random effect correlations
```

SPSS:

```
                                                 * In SPSS, * creates interactions.
MIXED langpost BY schoolID WITH hw2 mixgrd CMverb10 WCverb
      /METHOD    = REML
      /CRITERIA  = DFMETHOD(SATTERTHWAITE)
      /PRINT     = SOLUTION TESTCOV
      /FIXED     = hw2 mixgrd CMverb10 WCverb mixgrd*WCverb mixgrd*CMverb10
                   CMverb10*WCverb CMverb10*CMverb10
      /RANDOM    = INTERCEPT WCverb | COVTYPE(UN) SUBJECT(schoolID).
```

Electronic materials for this example **added** to my 2023 APA training sessions are **here**

Hybrid vs. Cluster-MC: Different L2 Slopes!

Hybrid: $\beta_{1c}(Verbal_{pc} - 10)$
→ Direct **L2 Context** Effects

CMC: $\beta_{1c}(Verbal_{pc} - \overline{Verbal_c})$
→ Direct **L2 Between** Effects

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	41.8383	0.3628	178	115.32	<.0001
~~hw2~~	~~-0.06701~~	~~0.4367~~	~~176~~	~~-0.15~~	~~0.8782~~
mixgrd	-0.8419	0.5213	196	-1.61	0.1079
CMverb10	0.8592	0.4061	192	2.12	0.0357
verb10	2.3589	0.07905	123	29.84	<.0001
mixgrd*verb10	0.3394	0.1578	231	2.15	0.0325
mixgrd*CMverb10	-0.1281	0.5772	231	-0.22	0.8246
CMverb10*verb10	-0.04328	0.07779	179	-0.56	0.5787
CMverb10*CMverb10	-0.3817	0.1671	344	-2.28	0.0229

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Simple L2 between	3.2181	0.4004	181	8.04	<.0001
L2 between*mixgrd	0.2113	0.5549	198	0.38	0.7038
L2 between*CMverbal	-0.4250	0.1486	233	-2.86	0.0046

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	41.8383	0.3628	178	115.32	<.0001
~~hw2~~	~~-0.06701~~	~~0.4367~~	~~176~~	~~-0.15~~	~~0.8782~~
mixgrd	-0.8419	0.5213	196	-1.61	0.1079
CMverb10	3.2181	0.4004	181	8.04	<.0001
WCverb	2.3589	0.07905	123	29.84	<.0001
mixgrd*WCverb	0.3394	0.1578	231	2.15	0.0325
mixgrd*CMverb10	0.2113	0.5549	198	0.38	0.7038
CMverb10*WCverb	-0.04328	0.07779	179	-0.56	0.5787
CMverb10*CMverb10	-0.4250	0.1486	233	-2.86	0.0046

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Simple L2 context	0.8592	0.4061	192	2.12	0.0357
L2 context*mixgrd	-0.1281	0.5772	231	-0.22	0.8246
L2 context*CMverbal	-0.3817	0.1671	344	-2.28	0.0229

L1 within verbal slope is n.s. less positive (weaker) by **0.0433** per unit school mean verbal
L2 between verbal slope is n.s. less positive by **0.4250** per unit school mean verbal
L2 contextual verbal slope is n.s. less positive by **0.3817** per unit school mean verbal

Model 6: Add Random Scale Factor

- Continuing with this simplified "location" model:

L1: $$Lang_{pc} = \beta_{0c} + \beta_{2c}(Verbal_{pc} - \overline{Verbal_c}) + e_{pc}$$

L2: $$\beta_{0c} = \gamma_{00} + \gamma_{01}(HW_c - 2) + \gamma_{02}(MixGrd_c) + \gamma_{03}\left(\overline{Verbal_c} - 10\right) + U_{0c}$$

$$\beta_{1c} = \gamma_{10} + \gamma_{12}(MixGrd_c) + U_{1c}$$

- Add a "scale-model" random intercept (in composite notation):
  - **Step A:** $\log\left(\sigma^2_{e_c}\right) = \tau_{00} + \omega_{0c}$
    - τ_{00} → fixed intercept for average amount of L1 residual variance
    - ω_{0c} → random scale factor → Does the amount of L1 residual variance vary randomly over clusters?
  - **Step B:** $\log\left(\sigma^2_{e_c}\right) = \tau_{00} + \tau_{01}(MixGrd_c) + \omega_{0c}$
    - τ_{01} → Does the amount of L1 residual variance differ in mixed grades?