Example 2: Empty Models and Level-2 Predictors in General Multilevel Models for Two-Level Nested Outcomes

(complete syntax, data, and output available for STATA, R, and SAS electronically)

The data for this example come from the High School and Beyond dataset (HSB4) used in McNeish (2023). Using 7,185 students from 160 schools, we will be examining the extent to which student math can be predicted from school-level variables of school size and public versus private. Note that this example computes total-R^2 and pseudo-R^2 in SAS using two custom macros (available in the SAS syntax file online) as well as in R using two custom functions and a general package (available in the R syntax file and function files online).

STATA Syntax for Importing and Preparing Data for Analysis:

```
// Define global variable for file location to be replaced in code below
// \\Client\ precedes path in Virtual Desktop outside H drive
   global filesave "C:\Dropbox\23_PSQF6272\PSQF6272_Example2"

// Open trimmed example excel data file from sheet "HSB4" and clear away any existing data
   clear // clear memory in case of open data
   import excel "$filesave\Example2_Data.xlsx", firstrow case(preserve) sheet("HSB4") clear

display "STATA Descriptive Statistics for Example 2 Variables"
summarize math size private
```

Variable	Label	N	Mean	Std Dev	Minimum	Maximum
math	math	7185	12.7478526	6.8782457	-2.8320000	24.9930000
size	size	7185	1056.86	604.1724993	100.0000000	2713.00
private	private	7185	0.4931106	0.4999873	0	1.0000000

```
// Center and re-scale school size
   gen size100 = (size-1000)/100  // private is already 0/1

// Filter to only cases complete on all variables to be used below
   egen nmiss=rowmiss(math size private)
   drop if nmiss>0
```

R Syntax for Importing and Preparing Data for Analysis (after loading packages *readxl, TeachingDemos, psych, lme4, lmerTest, performance, nlme, psychometric,* and *r2mlm*):

```
# Define variables for working directory and data name -- CHANGE THESE
filesave = "C:\\Dropbox/23_PSQF6272/PSQF6272_Example2/"
filename = "Example2_Data.xlsx"
setwd(dir=filesave)

# Load Jonathan's custom R functions from folder within working directory
functions = paste0("R functions/",dir("R functions/"))
temp = lapply(X=functions, FUN=source)

#  Import trimmed example excel data file in sheet "HSB4"
Example2 = read_excel(paste0(filesave,filename), sheet="HSB4")
# Convert to data frame to use in analysis
Example2 = as.data.frame(Example2)

print("R Descriptive Statistics for Example 2 Variables")
describe(x=Example2[ , c("math","size","private")])

# Center and re-scale school size
Example2$size100 = (Example2$size-1000)/100 # private is already 0/1

# Filter to only cases complete on all variables to be used below
Example2 = Example2[complete.cases(Example2[ , c("math","size","private")]),]
```

Model 0: Single-Level Empty Means, NO Random Intercept (2 parameters)

$Math_{pc} = \beta_0 + e_{pc}$

The `||  schoolID:  ,` indicates the level-2 nesting variable, where any random effects would go after the colon. The **noconstant** option removes the default random intercept variance (for now).

STATA Syntax and Output for Model 0:

```
display "STATA Model 0: Single-Level Empty Means, NO Random Intercept"
mixed math , || schoolID: , noconstant reml dfmethod(residual) dftable(pvalue) nolog

Mixed-effects REML regression                   Number of obs     =      7,185
DF method: Residual                             DF:           min =   7,184.00
                                                              avg =   7,184.00
                                                              max =   7,184.00
                                                F(0,   7184.00)   =          .
Log restricted-likelihood = -24051.459 = LL     Prob > F          =          .
------------------------------------------------------------------
        math |      Coef.   Std. Err.          DF       t    P>|t|
-------------+----------------------------------------------------
       _cons |   12.74785   .0811455       7184.0   157.10   0.000  Beta0 (overall mean)
------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
               var(Residual) |   47.31026    .789327      45.78823    48.88289  Var(e_pc)
------------------------------------------------------------------------------

display "-2LL = " e(ll)*-2      // Print -2LL for model
-2LL = 48102.917

estat ic, n(160)                // AIC and BIC for # level-2 units
Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |          Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |          160         .  -24051.46       2    48106.92   48113.07
-----------------------------------------------------------------------------
               Note: N=160 used in calculating BIC.
```

R Syntax and Output for Model 0 (using `gls` from `nlme` to omit random intercept variance for now):

```
print("R Model 0: Single-Level Empty Means, NO Random Intercept")
Model0 = gls(data=Example2, model=math~1)
print("Show results including -2LL and residual variance")
-2*logLik(Model0); summary(Model0)

'log Lik.' 48102.917 (df=2) → -2LL for model (with 2 parameters)

Generalized least squares fit by REML
  Model: math ~ 1
  Data: Example2
        AIC       BIC     logLik
  48106.917 48120.676 -24051.459

Coefficients:
                Value   Std.Error   t-value p-value
(Intercept) 12.747853 0.081145473 157.09875       0  Beta0 (overall mean)

Residual standard error: 6.8782457
Degrees of freedom: 7185 total; 7184 residual

summary(Model0)$sigma^2
[1] 47.310264 → Var(e_pc) as residual variance
```

Model 1: Two-Level Empty Means, WITH Random Intercept (3 parameters)

Level 1: $Math_{pc} = \beta_{0c} + e_{pc}$
Level 2: $\beta_{0c} = \gamma_{00} + U_{0c}$

Denominator degrees of freedom (DDF) now uses **Satterthwaite**, a good MLM option (although Kenward-Roger is better for really small samples).

STATA Syntax and Output for Model 1:

```
display "STATA Model 1: Two-Level Empty Means, WITH Random Intercept"
mixed math , || schoolID: , reml dfmethod(satterthwaite) dftable(pvalue) nolog

Mixed-effects REML regression                   Number of obs     =      7,185
Group variable: schoolID                        Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
DF method: Satterthwaite                        DF:           min =     158.82
                                                              avg =     158.82
                                                              max =     158.82
                                                F(0,      0.00)   =          .
Log restricted-likelihood = -23558.397 = LL     Prob > F          =          .
----------------------------------------------------------------------
        math |      Coef.   Std. Err.          DF       t    P>|t|
-------------+--------------------------------------------------------
       _cons |   12.63697   .2443943        158.8   51.71    0.000 gamma00
----------------------------------------------------------------------
```

The fixed intercept is now the grand mean of the school means, which will differ from the overall grand mean (as given by the model without a random intercept variance) whenever level-2 units have different level-1 sizes (i.e., are "unbalanced").

```
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolID: Identity           |
                  var(_cons) |   8.614081   1.078813      6.739162    11.01062 Var(U_0c)
-----------------------------+------------------------------------------------
               var(Residual) |   39.14832   .6606445      37.87466    40.46481 Var(e_pc)
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 986.12        Prob >= chibar2 = 0.0000
```

The **chibar2** test above is a likelihood ratio (LR) test comparing this model to a single-level regression (without a random intercept, as **linear model**) using a chi-square (χ^2) distribution with a mixture of DF=0 (for which $\chi^2 = 0$ always) and DF=1. Consequently, in this case you can obtain the mixture *p*-value by weighting each contribution to the χ^2 by 0.5, which means cutting the regular *p*-value in half. **Here, this LRT is a significance test of the intraclass correlation (ICC), which in turn provides an effect size for the amount of constant dependency attributed to school mean differences in math.**

```
display "-2LL = " e(ll)*-2       // Print -2LL for model
-2LL = 47116.793
```

Likelihood Ratio Test (LRT) Statistic:
$= 48{,}102.917 - 47{,}116.793 = 986.12$

```
estat ic, n(160)                 // AIC and BIC for # level-2 units
Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |        160         .   -23558.4       3    47122.79   47132.02
-----------------------------------------------------------------------------
Note: N=160 used in calculating BIC.

estat icc  // Intraclass correlation
Intraclass correlation
------------------------------------------------------------------------------
                       Level |        ICC   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
                    schoolID |   .1803528   .0187219      .1465168    .2199886
------------------------------------------------------------------------------
```

$$ICC = \frac{\tau^2_{U_0}}{\tau^2_{U_0} + \sigma^2_e} = \frac{8.614}{8.614 + 39.148} = \mathbf{.180}$$

```
display "ICC2 = " 8.614081/(8.614081+(39.14832/45))
ICC2 = .90827091 → Reliability of the school means using mean cluster size
```

R Syntax and Output for Model 1 (using `lmer` from `lme4` instead):

```
print("R Model 1: Two-Level Empty Means, WITH Random Intercept")
Model1 = lmer(data=Example2, REML=TRUE, formula=math~1+(1|schoolID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(Model1, chkREML=FALSE); summary(Model1, ddf="Satterthwaite")

'log Lik.' -23558.397 (df=3) → LL for model (with 3 parameters)
$AICtab
       AIC        BIC     logLik   deviance   df.resid
 47122.793  47143.433 -23558.397  47116.793   7182.000  → Deviance = −2LL for model

Random effects:
 Groups   Name        Variance Std.Dev.
 schoolID (Intercept)  8.614   2.9350   Var(U_pc)
 Residual             39.148   6.2569   Var(e_pc)
Number of obs: 7185, groups:  schoolID, 160

Fixed effects:
             Estimate Std. Error        df t value  Pr(>|t|)
(Intercept)  12.63697    0.24439 156.64732  51.708 < 2.2e-16 gamma00 = mean of school means

print("Show intraclass correlation and its LRT")
icc(Model1); ranova(Model1)

# Intraclass Correlation Coefficient
    Adjusted ICC: 0.180
  Unadjusted ICC: 0.180
```

$$ICC = \frac{\tau^2_{U_0}}{\tau^2_{U_0} + \sigma^2_e} = \frac{8.614}{8.614 + 39.148} = .\mathbf{180}$$

```
ANOVA-like table for random-effects: Single term deletions
               npar  logLik     AIC     LRT Df Pr(>Chisq)
<none>            3 -23558.4 47122.8
(1 | schoolID)    2 -24051.5 48106.9 986.124  1 < 2.22e-16
```

Likelihood Ratio Test (LRT) Statistic:
$= -2(-23{,}558.4 + 24{,}051.5) = 986.124$

The **`LRT`** above is a likelihood ratio test comparing this model to a single-level regression (without a random intercept) using a chi-square (χ^2) distribution with a regular DF=1 distribution, instead of a mixture of DF=0 (for which χ^2 = 0 always) and DF=1 as the default in STATA MIXED. Consequently, in this case you can obtain the mixture *p*-value by weighting each contribution to the χ^2 by 0.5, which means cutting the regular *p*-value in half. **Here, this LRT is a significance test of the intraclass correlation (ICC), which in turn provides an effect size for the amount of constant dependency attributed to school mean differences in math.**

```
print("Show ICC2 for Reliability of School Mean Math -- Weighted and Unweighted")
ICC2.lme(data=Example2, dv=math, grp=schoolID, weighted=TRUE)   [1] 0.90877647
ICC2.lme(data=Example2, dv=math, grp=schoolID, weighted=FALSE)  [1] 0.9013773
```

Design effect using mean #students per school: = 1 + ((*n* – 1) * ICC) → 1 + [(45−1)*.180] = **7.357**

Effective sample size: Effective N = (#Total Obs) / Design Effect → 7,185 / 7.357 = **977!!!**

This means that our power to detect effects of level-1 person predictors will be approximately that of an independent sample of 977 students—only if the ICC were 0 would we have power for level-1 effects based on the actual number of students. Power for level-2 effects is based on the number of schools (160).

Random intercept 95% confidence interval: $CI = \gamma_{00} \pm z_{crit}\sqrt{\tau^2_{U_0}} = 12.637 \pm 1.96\sqrt{8.614} = 6.884\ to\ 18.390$

This means that 95% of the schools are expected to have school mean math outcomes between 6.884 and 18.390 (around the average of 12.637).

Model 2: Add Main Effects of Level-2 School Size and Public vs Private (5 parameters)

Level 1: $Math_{pc} = \beta_{0c} + e_{pc}$
Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(size_c - 100) + \gamma_{02}(private_c) + U_{0c}$

STATA Syntax and Output for Model 2:

```
display "STATA Model 2: Add Main Effects of Level-2 School Size and Public vs Private"
mixed math c.size100 c.private, || schoolID: , ///
          reml dfmethod(satterthwaite) dftable(pvalue) nolog

Mixed-effects REML regression                   Number of obs     =      7,185
Group variable: schoolID                        Number of groups  =        160
                                                Obs per group:
                                                              min =         14
                                                              avg =       44.9
                                                              max =         67
DF method: Satterthwaite                        DF:           min =     154.86
                                                              avg =     157.66
                                                              max =     159.19
                                                F(2,   157.31)    =      21.80
Log restricted-likelihood = -23541.156          Prob > F          =     0.0000
-------------------------------------------------------------------
        math |      Coef.   Std. Err.          DF       t    P>|t|
-------------+-----------------------------------------------------
     size100 |   .0613509   .0389774        158.9     1.57   0.117 gamma01
     private |   3.151499   .4896777        154.9     6.44   0.000 gamma02
       _cons |   11.18194   .3210705        159.2    34.83   0.000 gamma00
-------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolID: Identity           |
                  var(_cons) |   6.618341   .8603554      5.129755    8.538894 Var(U_0c)
-----------------------------+------------------------------------------------
               var(Residual) |      39.15   .6607004      37.87623     40.4666 Var(e_pc)
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 712.02        Prob >= chibar2 = 0.0000

display "-2LL = " e(ll)*-2      // Print -2LL for model
-2LL = 47082.313
```

There is still significant school dependency (*conditional* ICC = .144 vs .180 before).

```
estat ic, n(160)                // AIC and BIC for # level-2 units
Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |          Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |          160         .  -23541.16       5    47092.31   47107.69
-----------------------------------------------------------------------------
               Note: N=160 used in calculating BIC.

predict predmain                // Save fixed-effect predicted outcomes
corr math predmain              // Get total r to make R2
             |     math predmain
-------------+------------------
        math |   1.0000
    predmain |   0.2076   1.0000

display "Total-R2 = " r(rho)^2  // Print total R2 relative to empty model
Total-R2 = .04310206
```

The model F-test printed in the first part of the output, $F(2, 157.31) = 21.80$, $p < .0001$, is analogous to the F-test of the model R^2 in single-level regression. Here, it provides a significance test for the total-$R^2 = .043$.

R Syntax and Output for Model 2 (using `lmer` from `lme4`):

```
print("R Model 2: Add Main Effects of Level-2 School Size and Public vs Private")
Model2 = lmer(data=Example2, REML=TRUE, formula=math~1+size100+private+(1|schoolID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(Model2, chkREML=FALSE); summary(Model2, ddf="Satterthwaite")

'log Lik.' -23541.156 (df=5) → LL for model (with 5 parameters)
$AICtab
       AIC        BIC      logLik   deviance    df.resid
 47092.313  47126.711 -23541.156  47082.313    7180.000 → Deviance = −2LL for model

Random effects:
 Groups   Name        Variance Std.Dev.
 schoolID (Intercept)  6.6183  2.5726  Var(U_0c)
 Residual             39.1500  6.2570  Var(e_pc)
Number of obs: 7185, groups:  schoolID, 160

Fixed effects:
              Estimate Std. Error         df t value       Pr(>|t|)
(Intercept)  11.181939   0.321070 156.319744 34.8271      < 2.2e-16 gamma00
size100       0.061351   0.038977 156.069668  1.5740         0.1175 gamma01
private       3.151499   0.489676 152.059267  6.4359 0.000000001516 gamma02
```

Intercept γ_{00} =

size100 γ_{01} =

private γ_{02} =

```
print("F-Test of Model Total-R2")
contestMD(Model2, ddf="Satterthwaite", L=rbind(c(0,1,0),c(0,0,1)))

     Sum Sq    Mean Sq NumDF      DenDF   F value               Pr(>F)
1 1707.2093 853.60464      2 154.47003 21.803438 0.0000000045647872

print("Psuedo-R2 relative to empty model using Jonathan's function")
pseudoRSquaredinator(smallerModel=Model1, largerModel=Model2)

R2 Random.(Intercept)           R2 L1.sigma2
       0.231683234632        -0.000042893403
```

Pseudo-R^2 Results: The two fixed slopes of our two level-2 predictors accounted for 23.2% of the level-2 random intercept variance and 0% of the level-1 residual variance (as expected for school-level predictors).

```
print("Rights & Sterba R2 suite")
r2mlm(model=Model2, bargraph=FALSE)

$R2s
          total within    between
f1   0.000000000      0          NA → Pseudo-R2 for level-1 residual
f2   0.044375381     NA 0.24306893 → Pseudo-R2 for level-2 random intercept
v    0.000000000      0          NA
m    0.138187567     NA 0.75693107
f    0.044375381     NA          NA
fv   0.044375381      0          NA
fvm  0.182562948     NA          NA
```

Total-R^2 Results: The 2 fixed slopes of our 2 level-2 predictors accounted for 4.33% of the total math variance (approximated by .232*. 180 = 4.2% using the ICC from the empty model).

```
print("Total-R2 relative to empty model using Jonathan's function")
totalRSquaredinator(model=Model2, dvName="math", data=Example2)
0.043102056

# Compute total-R2 the longer way instead by saving predicted outcomes
Example2$PredMain = predict(Model2, re.form=NA) # Do not include random intercept
rModel2 = cor.test(Example2$PredMain, Example2$math, method="pearson")
print("Total R2"); rModel2$estimate^2
0.043102056
```

Model 3: Add Interaction of Level-2 School Size and Public vs Private (6 parameters)

Level 1: $Math_{pc} = \beta_{0c} + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(size_c - 100) + \gamma_{02}(private_c) + \gamma_{03}(size_c - 100)(private_c) + U_{0c}$

Slope of school size for private schools = $[\gamma_{01} + \gamma_{03}(private_c)](size_c - 100)$

STATA Syntax and Output for Model 3:

```
display "STATA Model 3: Add Interaction of Level-2 School Size and Public vs Private"
mixed math c.size100##c.private , || schoolID: , ///
           reml dfmethod(satterthwaite) dftable(pvalue) nolog
```

The ## in the fixed effects means "give me the two-way interaction and all lower-order main effects" (for less typing).

```
                                                F(3,    154.92)    =      15.26
Log restricted-likelihood = -23541.699 = LL     Prob > F           =     0.0000
--------------------------------------------------------------------------
                math |      Coef.   Std. Err.          DF       t    P>|t|
--------------------+-----------------------------------------------------
             size100 |   .0306622   .0447862        160.8     0.68   0.495 gamma01
             private |   3.251838   .4935148        153.0     6.59   0.000 gamma02
c.size100#c.private |   .1239353   .0900336        152.3     1.38   0.171 gamma03
               _cons |   11.28755   .3290906        158.4    34.30   0.000 gamma00
--------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
schoolID: Identity           |
                  var(_cons) |   6.569508   .8582494      5.085466    8.486624 Var(U_0c)
-----------------------------+------------------------------------------------
               var(Residual) |    39.1507   .6607227      37.87689    40.46734 Var(e_pc)
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 698.09        Prob >= chibar2 = 0.0000

display "-2LL = " e(ll)*-2      // Print -2LL for model
-2LL = 47083.398

estat ic, n(160)                // AIC and BIC for # level-2 units
Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |        160         .   -23541.7       6     47095.4   47113.85
-----------------------------------------------------------------------------

lincom c.size100*1 + c.size100#c.private // Size slope for private schools
------------------------------------------------------------------------------
        math |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   .1545975   .0781041     1.98   0.048     .0015163    .3076787 g01 + g03(1)
------------------------------------------------------------------------------

predict predinteract            // Save fixed-effect predicted outcomes
corr math predinteract          // Get total r to make R2

             |     math predin~t
-------------+------------------
        math |   1.0000
predinteract |   0.2130   1.0000

display "Total-R2 = " r(rho)^2  // Print total R2 relative to empty model
Total-R2 = .04537771
```

R Syntax and Output for Model 3:

```
print("R Model 3: Add Interaction of Level-2 School Size and Public vs Private")
Model3 = lmer(data=Example2, REML=TRUE, formula=math~1+size100*private+(1|schoolID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(Model3, chkREML=FALSE); summary(Model3, ddf="Satterthwaite")
```

The * in the fixed effects means "give me the two-way interaction and all lower-order main effects" (for less typing).

```
'log Lik.' -23541.699 (df=6)   → LL for model (with 6 parameters)
$AICtab
       AIC        BIC     logLik   deviance   df.resid
 47095.398  47136.677 -23541.699  47083.398   7179.000  → deviance = −2LL for model

Random effects:
 Groups   Name        Variance Std.Dev.
 schoolID (Intercept)  6.5695  2.5631  Var(U_0c)
 Residual             39.1507  6.2571  Var(e_pc)
Number of obs: 7185, groups:  schoolID, 160

Fixed effects:
                 Estimate Std. Error         df t value        Pr(>|t|)
(Intercept)     11.287555   0.329090 155.263861 34.2993       < 2.2e-16 gamma00
size100          0.030662   0.044786 157.645142  0.6846          0.4946 gamma01
private          3.251838   0.493513 150.004059  6.5892 0.0000000007046 gamma02
size100:private  0.123935   0.090033 149.283221  1.3765          0.1707 gamma03
```

Intercept γ_{00} =

size100 γ_{01} =

private γ_{02} =

size100*private γ_{03} =

```
print("Size slope for private schools?"); contest1D(Model3, L=c(0,1,0,1))
    Estimate  Std. Error        df   t value     Pr(>|t|)
1 0.15459752 0.078103868 146.67772 1.9793837 0.049644738 gamma01 + gamma03(1)

print("F-Test of Model Total-R2")
contestMD(Model3, ddf="Satterthwaite", L=rbind(c(0,1,0,0),c(0,0,1,0),c(0,0,0,1)))
     Sum Sq   Mean Sq NumDF    DenDF   F value             Pr(>F)
1 1792.4576 597.48586     3 151.89378 15.261179 0.0000000099242283

print("Psuedo-R2 relative to empty model using Jonathan's function")
pseudoRSquaredinator(smallerModel=Model1, largerModel=Model3)
R2 Random.(Intercept)          R2 L1.sigma2
       0.237352159809       -0.000060781529

print("Rights & Sterba R2 suite -- f2 is for random intercept")
r2mlm(model=Model3, bargraph=FALSE)
$R2s
          total within    between
f1   0.000000000      0         NA
f2   0.046869466     NA 0.25496998
v    0.000000000      0         NA
m    0.136954000     NA 0.74503002
f    0.046869466     NA         NA
fv   0.046869466      0         NA
fvm  0.183823466     NA         NA
```

Pseudo-R^2 Results: The 3 fixed slopes of our 3 level-2 predictors accounted for 23.7% of the level-2 random intercept variance, an increase of 0.5%, using the usual way of computing it. The alternative (from just this model using **r2mlm**) provides a slightly higher estimate.

```
print("Total-R2 relative to empty model using Jonathan's function")
totalRSquaredinator(model=Model3, dvName="math", data=Example2)
0.04537771
```

Total-R^2 Results: The 3 fixed slopes of our 3 level-2 predictors accounted for 4.53% of the total math variance, as increase of 0.2% relative to the main effects model.

Sample Results Section [indicates notes about what to customize or also include; note that SE and *p*-values are not needed if you provide a table for the model solution]

The extent to which the extent to which student math outcomes (M = 12.75, SD = 6.88, range = −2.83 to 24.99) could be predicted from school-level variables of school size and public versus private status was examined in a series of multilevel models in which the 7,185 students were modeled as nested within their 160 schools. The number of students included per school ranged from 14 to 67 (M = 45). Restricted Maximum likelihood (REML) within SAS MIXED [or STATA MIXED or R `lmer`] was used in estimating and reporting all model parameters. The significance of fixed effects was evaluated with univariate and multivariate Wald tests using Satterthwaite denominator degrees of freedom, whereas random effects were evaluated via likelihood ratio tests (i.e., −2ΔLL with degrees of freedom equal to the number of new random effects variances and covariances). Alpha was chosen as .01. Model-implied fixed effects were requested via ESTIMATE [or LINCOM or `contest1D`] statements. Effect size for the fixed effects was evaluated via pseduo-R^2 values for the proportion reduction in each variance component relative to a nested model without the predictors in question, as well as with total-R^2, the squared correlation between the actual math outcomes and those predicted by the model fixed effects.

As derived from an empty means, random intercept model, student math had an intraclass correlation of ICC = .180, indicating that 18.0% of the variance in student math was between schools, a significant amount, −2ΔLL(1) = 986.12, p < .0001. Given an average of 45 students per school in this sample, the ICC = .180 translated into a design effect = 7.36, further indicating the need for a multilevel analysis. The school mean math outcomes had strong reliability, as evidenced by a weighted ICC2 = .908. The fixed intercept was 12.637 (SE = 0.244), which represented the expected average school mean math outcome. A random intercept confidence interval (computed as the fixed intercept ± 1.96*SQRT[random intercept variance]) indicated that 95% of the schools were expected to have school mean math outcomes between 6.884 and 18.390 (around the average of 12.637).

We then added the school-level predictors of school size (per 100 students, centered so that 0 = 1000 students) and status (public = 0 versus private = 1). These two fixed effects accounted for significant variance overall, F(2, 157.31) = 21.80, p < .0001, including 23.2% of the level-2 random intercept variance and 4.3% of the total variance. The fixed intercept was 11.182 (SE = 0.321), which represented the expected school mean math outcome for public schools with 1000 students. The slope for school size was nonsignificantly positive, indicating that school mean math was expected to be nonsignificantly higher by 0.061 (SE = 0.039, p = .118) per 100 more students. The slope for public vs. private was significantly positive, indicating school mean math was predicted to be higher by 3.151 (SE = 0. 490, p < .0001) for private schools relative to public schools.

We then added an interaction between school size and type. The three fixed effects still accounted for significant variance overall, F(3, 154.92) = 15.26, p < .0001, including 23.7% of the level-2 random intercept variance (an increase of 0.5%) and 4.5% of the total variance (an increase of 0.2%). The interaction slope was nonsignificantly positive, indicating that the slope of school size for private schools (Est = 0.155, SE = 0.078, p = .495) was 0.12 nonsignificantly more positive than the size slope for public schools (Est = 0.031, SE = 0.045, p = .050). The slope for public vs. private remained significantly positive, indicating school mean math was predicted to be higher by 3.252 (SE = 0.494, p < .0001) for private schools relative to public schools.